高知工科大学
Kochi University of Technology

# 数学 1

（2008年度版）

# 解答

# ＜ 1 ページ. 弧度法の復習 ＞

## 問 1 の解答

| 度数法 | $180°/\pi$ | $45°$ | $60°$ | $90°$ | $120°$ | $180°$ | $360°$ | |
|---|---|---|---|---|---|---|---|---|
| 弧度法 $\theta$ | $1$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\pi$ | $2\pi$ | $\theta$ |
| 弧の長さ $\ell$ | $r$ | $\frac{1}{4}\pi r$ | $\frac{\pi}{3}r$ | $\frac{\pi}{2}r$ | $\frac{2\pi}{3}r$ | $\pi r$ | $2\pi r$ | $\theta r$ |
| 面積 $S$ | $\frac{1}{2}r^2$ | $\frac{\pi}{8}r^2$ | $\frac{\pi}{6}r^2$ | $\frac{1}{4}\pi r^2$ | $\frac{\pi}{3}r^2$ | $\frac{1}{2}\pi r^2$ | $\pi r^2$ | $\frac{\theta}{2}r^2$ |

## 問 2 の解答

$\ell = \theta$

$S = \dfrac{\theta}{2}$

# ＜ 3 ページ. 三角関数の極限 2 ＞

## 問の解答

(1) $\displaystyle \lim_{x\to 0}\frac{\sin(3x)}{2x} = \lim_{x\to 0}\frac{3}{2}\times\frac{\sin(3x)}{3x} = \frac{3}{2}$

(2) $\displaystyle \lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\frac{\sin x}{x}\times\frac{1}{\cos x} = 1\times\frac{1}{\cos 0} = 1$

(3) $\displaystyle \lim_{x\to 0}\frac{\cos x - 1}{x} = \lim_{x\to 0}\frac{\cos^2 x - 1}{x(\cos x + 1)} = \lim_{x\to 0}\frac{-\sin^2 x}{x(\cos x + 1)}$

$$= \lim_{x\to 0}\frac{\sin x}{x}\times\frac{-\sin x}{\cos x + 1} = 1\times\frac{-0}{1+1} = 0$$

# < 4ページ. 接線の傾き 1 >

## 問の解答

(1) 曲線 $y = \sin x$ の $x = \dfrac{\pi}{2}$ における接線の傾き

(2) 曲線 $y = \cos x$ の $x = \dfrac{\pi}{3}$ における接線の傾き

(3) 曲線 $y = \sqrt{x}$ の $x = 3$ における接線の傾き

(4) 曲線 $y = x^5$ の $x = 2$ における接線の傾き

# ＜ 5 ページ. 接線の傾き 2 ＞

**問 1 の解答**

$$\lim_{h\to 0}\frac{\sin(a+h)-\sin a}{h}=\lim_{h\to 0}\frac{\sin a\cos h+\cos a\sin h-\sin a}{h}$$

$$=\lim_{h\to 0}\frac{(\sin a)(\cos h-1)+(\cos a)(\sin h)}{h}$$

$$=\lim_{h\to 0}\left\{(\sin a)\times\left(\frac{\cos h-1}{h}\right)+(\cos a)\times\left(\frac{\sin h}{h}\right)\right\}$$

$$=(\sin a)\times 0+(\cos a)\times 1=\cos a$$

**問 2 の解答**

$$\lim_{h\to 0}\frac{\cos(a+h)-\cos a}{h}=\lim_{h\to 0}\frac{\cos a\cos h-\sin a\sin h-\cos a}{h}$$

$$=\lim_{h\to 0}\frac{\cos a(\cos h-1)-(\sin a)(\sin h)}{h}$$

$$=\lim_{h\to 0}\left\{(\cos a)\times\left(\frac{\cos h-1}{h}\right)-(\sin a)\times\left(\frac{\sin h}{h}\right)\right\}$$

$$=(\cos a)\times 0-(\sin a)\times 1=-\sin a$$

# ＜ 6 ページ. 導関数 1 ＞

## 問の解答

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{2-2}{h} = 0$

(2) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{(x+h) - x}{h} = \lim_{h \to 0} \dfrac{h}{h} = 1$

(3) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0}(2x + h) = 2x$$

# ＜ 7ページ. 導関数 2 ＞

## 問 1 の解答

(1) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h \to 0} \left(4x^3 + 6x^2h + 4xh^2 + h^3\right) = 4x^3$$

(2) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^5 - x^5}{h}$$

$$= \lim_{h \to 0} \frac{x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - x^5}{h}$$

$$= \lim_{h \to 0} \left(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4\right) = 5x^4$$

## 問 2 の解答

$f'(x) = nx^{n-1}$

# ＜ 8ページ. 導関数 3 ＞

## 問の解答

(1) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h+1} - \sqrt{x+1})(\sqrt{x+h+1} + \sqrt{x+1})}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \frac{1}{2\sqrt{x+1}}$$

(2) $$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

(3) $$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left\{ (\sin x) \times \left( \frac{\cos h - 1}{h} \right) + (\cos x) \times \left( \frac{\sin h}{h} \right) \right\} = \cos x$$

(4) $$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left\{ (\cos x) \times \left( \frac{\cos h - 1}{h} \right) - (\sin x) \times \left( \frac{\sin h}{h} \right) \right\} = -\sin x$$

# < 9 ページ. 導関数 4 >

## 問 1 の解答

$$\{f(x)-g(x)\}' = \lim_{h\to 0}\frac{\{f(x+h)-g(x+h)\}-\{f(x)-g(x)\}}{h}$$

$$= \lim_{h\to 0}\left\{\frac{f(x+h)-f(x)}{h}-\frac{g(x+h)-g(x)}{h}\right\}$$

$$= f'(x)-g'(x)$$

## 問 2 の解答

(1) $(x^5+4)'=5x^4$

(2) $(2x^6-3x^3)'=12x^5-9x^2$

(3) $((x-1)^2)'=(x^2-2x+1)'=2x-2$

(4) $((x+1)(x^2-x))'=(x^3-x)'=3x^2-1$

# ＜ 10 ページ. 積の微分 1 ＞

## 問の解答

(1) $\big((x-1)\sin x\big)' = \sin x + (x-1)\cos x$

(2) $\big((x^2+1)\cos x\big)' = 2x\cos x - (x^2+1)\sin x$

(3) $\big(\sin x\cos x\big)' = \cos^2 x - \sin^2 x$

(4) $\big((x+1)^4\big)' = \big((x+1)^2 \times (x+1)^2\big)'$

$$= \{(x+1)^2\}' \times (x+1)^2 + (x+1)^2 \times \{(x+1)^2\}'$$

$$= 2(x+1)(x+1)^2 + (x+1)^2 \times 2(x+1) = 4(x+1)^3$$

# < 11ページ.積の微分 2 >

## 問 1 の解答

(1) $\left(x\sqrt{x}\right)' = (x)' \times \sqrt{x} + x \times (\sqrt{x})' = 1 \times \sqrt{x} + x \times \dfrac{1}{2\sqrt{x}}$

$= \sqrt{x} + \dfrac{1}{2}\sqrt{x} = \dfrac{3}{2}\sqrt{x}$

(2) $\left(k\sqrt{x}\right)' = (k)' \times \sqrt{x} + k \times (\sqrt{x})' = 0 \times \sqrt{x} + k \times \dfrac{1}{2\sqrt{x}}$

$= \dfrac{k}{2\sqrt{x}}$

## 問 2 の解答

$\left(k \times f(x)\right)' = (k)' \times f(x) + k \times \left(f(x)\right)'$

$= 0 \times f(x) + k \times f'(x) = k \times f'(x)$

## 問 3 の解答

$\left(f(x)g(x)h(x)\right)' = \left(f(x)g(x) \times h(x)\right)' = \{f(x)g(x)\}' \times h(x) + \{f(x)g(x)\} \times \left(h(x)\right)'$

$= \{f'(x)g(x) + f(x)g'(x)\} \times h(x) + f(x)g(x) \times h'(x)$

$= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

# < 12 ページ. 商の微分 >

## 問 1 の解答

$$
\begin{aligned}
\left\{\frac{f(x)}{g(x)}\right\}' &= \bigl(f(x)\bigr)' \times \frac{1}{g(x)} + f(x) \times \left(\frac{1}{g(x)}\right)' \\
&= f'(x) \times \frac{1}{g(x)} + f(x) \times \left\{-\frac{g'(x)}{(g(x))^2}\right\} \\
&= \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}
\end{aligned}
$$

## 問 2 の解答

(1) $\left(\dfrac{1}{x^2}\right)' = -\dfrac{2x}{x^4} = -\dfrac{2}{x^3}$

(2) $\left(\dfrac{1}{x^4}\right)' = -\dfrac{4x^3}{x^8} = -\dfrac{4}{x^5}$

(3) $\left(\dfrac{x^3}{x+1}\right)' = \dfrac{3x^2 \times (x+1) - x^3 \times 1}{(x+1)^2} = \dfrac{2x^3 + 3x^2}{(x+1)^2}$

(4) $\left(\dfrac{x}{\sin x}\right)' = \dfrac{(x)' \times \sin x - x \times (\sin x)'}{(\sin x)^2} = \dfrac{\sin x - x\cos x}{\sin^2 x}$

# ＜ 13 ページ. 三角関数の微分 ＞

## 問 1 の解答

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)' \times \cos x - \sin x \times (\cos x)'}{\cos^2 x}$$
$$= \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

## 問 2 の解答

(1) $(3\sin x + 4\cos x)' = 3\cos x - 4\sin x$

(2) $(-3\cos x + 5\tan x)' = 3\sin x + \dfrac{5}{\cos^2 x}$

(3) $\big((x + \sin x)\cos x\big)' = (1 + \cos x)\cos x - (x + \sin x)\sin x$

(4) $(\sin^2 x)' = (\sin x)' \times \sin x + \sin x \times (\sin x)' = 2\sin x \cos x$

(5) $(\cos^2 x)' = (\cos x)' \times \cos x + \cos x \times (\cos x)' = -2\sin x \cos x$

(6) $(x\tan x)' = \tan x + \dfrac{x}{\cos^2 x}$

(7) $\left(\dfrac{\sin x}{x}\right)' = \dfrac{x\cos x - \sin x}{x^2}$

(8) $\left(\dfrac{\cos x}{x}\right)' = \dfrac{-x\sin x - \cos x}{x^2}$

## 問 3 の解答

(1) $(\mathrm{cosec}\ x)' = \left(\dfrac{1}{\sin x}\right)' = -\dfrac{\cos x}{(\sin x)^2}$

(2) $(\sec x)' = \left(\dfrac{1}{\cos x}\right)' = -\dfrac{-\sin x}{(\cos x)^2} = \dfrac{\sin x}{\cos^2 x}$

(3) $(\cot x)' = \left(\dfrac{1}{\tan x}\right)' = -\dfrac{\frac{1}{\cos^2 x}}{\tan^2 x} = -\dfrac{1}{\cos^2 x \tan^2 x} = -\dfrac{1}{\sin^2 x}$

# ＜ 14 ページ. 微分の練習 1 ＞

## 問 1 の解答

(1) $\displaystyle\lim_{h\to 0}\frac{\sin h}{h}=1$ (2) $\displaystyle\lim_{h\to 0}\frac{\cos h-1}{h}=0$

## 問 2 の解答

(1) $\displaystyle f'(x)=\lim_{h\to 0}\frac{4-4}{h}=0$

(2) $$f'(x)=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$
$$=\lim_{h\to 0}(3x^2+3xh+h^2)=3x^2$$

(3) $$f'(x)=\lim_{h\to 0}\frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}=\lim_{h\to 0}\frac{(\sqrt{3(x+h)})^2-(\sqrt{3x})^2}{h(\sqrt{3(x+h)}+\sqrt{3x})}$$
$$=\lim_{h\to 0}\frac{3(x+h)-3x}{h(\sqrt{3(x+h)}+\sqrt{3x})}=\lim_{h\to 0}\frac{3}{\sqrt{3(x+h)}+\sqrt{3x})}=\frac{3}{2\sqrt{3x}}=\frac{\sqrt{3}}{2\sqrt{x}}$$

(4) $$f'(x)=\lim_{h\to 0}\frac{\frac{5}{x+h}-\frac{5}{x}}{h}=\lim_{h\to 0}\frac{\frac{5x-5(x+h)}{(x+h)x}}{h}=\lim_{h\to 0}\frac{-5}{(x+h)x}=-\frac{5}{x^2}$$

(5) $$f'(x)=\lim_{h\to 0}\frac{2\sin(x+h)-2\sin x}{h}=\lim_{h\to 0}\frac{2\sin x\cos h+2\cos x\sin h-2\sin x}{h}$$
$$=\lim_{h\to 0}\left\{(2\sin x)\left(\frac{\cos h-1}{h}\right)+(2\cos x)\left(\frac{\sin h}{h}\right)\right\}=2\cos x$$

(6) $$f'(x)=\lim_{h\to 0}\frac{3\cos(x+h)-3\cos x}{h}=\lim_{h\to 0}\frac{3\cos x\cos h-3\sin x\sin h-3\cos x}{h}$$
$$=\lim_{h\to 0}\left\{(3\cos x)\left(\frac{\cos h-1}{h}\right)-(3\sin x)\left(\frac{\sin h}{h}\right)\right\}=-3\sin x$$

## 問 2 の解答

(1) $(4x^3-6x^5-18)'=12x^2-30x^4$ (2) $\left((x^2-1)(x^2+1)\right)'=(x^4-1)'=4x^3$

(3) $(5\sin x+6\cos x)'=5\cos x-6\sin x$ (4) $(3\sin x-4\tan x)'=3\cos x-\frac{4}{\cos^2 x}$

(5) $(x^2\sin x)'=2x\sin x+x^2\cos x$ (6) $(x^3\cos x)'=3x^2\cos x-x^3\sin x$

(7) $(\sin x\tan x)'=\cos x\tan x+\sin x\times\frac{1}{\cos^2 x}=\sin x+\frac{\sin x}{\cos^2 x}$

(8) $\left(\frac{x}{x+1}\right)'=\frac{(x+1)-x\times 1}{(x+1)^2}=\frac{1}{(x+1)^2}$ (9) $(x^4\tan x)'=4x^3\tan x+\frac{x^4}{\cos^2 x}$

(10) $$\left(\frac{\sin x}{x^2}\right)'=\frac{x^2\cos x-2x\sin x}{x^4}=\frac{x\cos x-2\sin x}{x^3}$$

# ＜ 15 ページ. 微分記号 ＞

## 問 1 の解答

(1) $\dfrac{dy}{dx} = 3x^2 - 8x$

(2) $\dfrac{dy}{du} = -\sin u$

(3) $\dfrac{d\ell}{dt} = 6t - 2$

(4) $\dfrac{dS}{dr} = 2\pi r$

(5) $\dfrac{dV}{dr} = 4\pi r^2$

## 問 2 の解答

(1) $5x^4$

(2) $7t^6 - 20t^3$

(3) $\dfrac{1}{2\sqrt{u}}$

(4) $-\sin t$

(5) $\dfrac{1}{\cos^2 u}$

(6) $\cos^2 u - \sin^2 u$

# < 16 ページ. 微分と極限 1 >

## 問の解答

(1) $\dfrac{d}{dx}(x^5) = 5x^4$

(2) $\dfrac{d}{dt}(\sin t) = \cos t$

(3) $\dfrac{d}{du}(\cos u) = -\sin u$

(4) $\dfrac{d}{dr}(\tan r) = \dfrac{1}{\cos^2 r}$

# < 17 ページ. 微分と極限 2 >

## 問の解答

(1) $\dfrac{d}{dx}(\cos x) = -\sin x$

(2) $\dfrac{d}{du}(\sin u) = \cos u$

(3) $\dfrac{d}{du}(u^4) = 4u^3$

(4) $\dfrac{d}{dt}(t^6) = 6t^5$

(5) $\dfrac{d}{dt}(\tan t) = \dfrac{1}{\cos^2 t}$

# < 18ページ. 合成関数の微分 1 >

## 問の解答

(1) $\left\{\dfrac{d}{du}\cos u\right\} \times \left\{\dfrac{d}{dx}x^4\right\} = -\sin(u) \times 4x^3$

$(u = x^4$ とおく $)$ $\quad = -4x^3 \sin(x^4)$

(2) $\left\{\dfrac{d}{du}\tan u\right\} \times \left\{\dfrac{d}{dx}x^5\right\} = \dfrac{1}{\cos^2 u} \times 5x^4$

$(u = x^5$ とおく $)$ $\quad = \dfrac{5x^4}{\cos^2(x^5)}$

# < 19 ページ. 合成関数の微分 2 >

## 問の解答

(1) $\dfrac{d}{dx}\{\sin(5x)\} = \left\{\dfrac{d}{du}\sin u\right\} \times \left\{\dfrac{d}{dx}(5x)\right\} = \cos(u) \times 5 = 5\cos(5x)$

$(u = 5x)$

(2) $\dfrac{d}{dx}\{\cos(7x)\} = \left\{\dfrac{d}{du}\cos u\right\} \times \left\{\dfrac{d}{dx}(7x)\right\} = -\sin(u) \times 7 = -7\sin(7x)$

$(u = 7x)$

(3) $\dfrac{d}{dx}\{\sin(4x-5)\} = \left\{\dfrac{d}{du}\sin u\right\} \times \left\{\dfrac{d}{dx}(4x-5)\right\} = \cos(u) \times 4 = 4\cos(4x-5)$

$(u = 4x-5)$

(4) $\dfrac{d}{dx}\{\cos(2x+3)\} = \left\{\dfrac{d}{du}\cos u\right\} \times \left\{\dfrac{d}{dx}(2x+3)\right\} = -\sin(u) \times 2 = -2\sin(2x+3)$

$(u = 2x+3)$

(5) $\dfrac{d}{dx}\{\tan(8x-7)\} = \left\{\dfrac{d}{du}\tan u\right\} \times \left\{\dfrac{d}{dx}(8x-7)\right\} = \dfrac{1}{\cos^2 u} \times 8 = \dfrac{8}{\cos^2(8x-7)}$

$(u = 8x-7)$

(6) $\dfrac{d}{dx}\left\{\sin(x^3+2x^4)\right\} = \left\{\dfrac{d}{du}\sin u\right\} \times \left\{\dfrac{d}{dx}(x^3+2x^4)\right\} = \cos(u) \times (3x^2+8x^3)$

$(u = x^3+2x^4)$　　　　$= (3x^2+8x^3)\cos(x^3+2x^4)$

# ＜ 20 ページ. 合成関数の微分 3 ＞

## 問の解答

(1) $u = 3x + 4$ とおくと $y = u^5$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \left\{\frac{d}{du}(u^5)\right\} \times \left\{\frac{d}{dx}(3x+4)\right\} = 5u^4 \times 3$$

$$= 15(3x+4)^4$$

(2) $u = 4x - 5$ とおくと $y = u^{10}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \left\{\frac{d}{du}(u^{10})\right\} \times \left\{\frac{d}{dx}(4x-5)\right\} = 10u^9 \times 4$$

$$= 40u^9 = 40(4x-5)^9$$

(3) $u = x^2 + 3x$ とおくと $y = u^6$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \left\{\frac{d}{du}(u^6)\right\} \times \left\{\frac{d}{dx}(x^2+3x)\right\} = 6u^5 \times (2x+3)$$

$$= 6(2x+3)(x^2+3x)^5$$

(4) $u = x^2 - 3x$ とおくと $y = \cos(u)$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \left\{\frac{d}{du}\cos u\right\} \times \left\{\frac{d}{dx}(x^2-3x)\right\}$$

$$= -\sin(u) \times (2x-3)$$

$$= -(2x-3)\sin(x^2-3x)$$

# < 21 ページ. 微分の練習 2 >

## 問 1 の解答

(1) $20x^4 - 63x^8$

(2) $8t - 8$

(3) $\cos u$

(4) $-\sin u$

(5) $\dfrac{1}{\cos^2 u}$

(6) $nu^{n-1}$

## 問 2 の解答

(1) $2x\cos(x^2)$

(2) $-3x^2\sin(x^3)$

(3) $\dfrac{4x^3}{\cos^2(x^4)}$

(4) $4\cos(4x)$

(5) $-5\sin(5x)$

(6) $\dfrac{6}{\cos^2(6x)}$

(7) $2\cos(2x-3)$

(8) $-3\sin(3x+5)$

(9) $\dfrac{7}{\cos^2(7x+6)}$

(10) $2(x+1)\cos(x^2+2x)$

(11) $18(3x+4)^5$

(12) $28(4x-3)^6$

(13) $50(5x+8)^9$

(14) $5(2x-3)(x^2-3x)^4$

(15) $8(\cos x)(1+\sin x)^7$

(16) $-9(\sin x)(2+\cos x)^8$

## 問 3 の解答

(1) $2x\sin(4x) + 4x^2\cos(4x)$

(2) $3x^2\cos(5x) - 5x^3\sin(5x)$

(3) $2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$

## 問 4 の解答

(1) $f'(x)\{g(x) - h(x)\} + f(x)\{g'(x) - h'(x)\}$

(2) $f'(g(h(x))) \times g'(h(x)) \times h'(x)$

(3) $\dfrac{\{f'(x)h(x) + f(x)h'(x)\}g(x) - f(x)h(x)g'(x)}{\{g(x)\}^2}$

# < 22 ページ. ネピアの数 >

## 問の解答

(1) $\displaystyle\lim_{h\to 0}(1+h)^{\frac{1}{h}} = e$

(2) $\displaystyle\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

(3) $\displaystyle\lim_{k\to 0}\frac{1}{k}\log_a(1+k) = \log_a e$

# < 23 ページ. 対数関数の導関数 >

## 問 1 の解答

(1) $f'(3) = \lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \dfrac{\log_{10}(3+h) - \log_{10} 3}{h} = \lim_{h \to 0} \dfrac{1}{h} \log_{10}\left(\dfrac{3+h}{3}\right)$

$= \lim_{h \to 0} \dfrac{1}{h} \log_{10}\left(1 + \dfrac{h}{3}\right)$

ここで $k = \dfrac{h}{3}$ とおくと、$h \to 0$ のとき $k \to 0$ より

$$f'(x) = \lim_{k \to 0} \frac{1}{3k} \log_{10}(1+k) = \lim_{k \to 0} \frac{1}{3} \log_{10}(1+k)^{\frac{1}{k}} = \frac{1}{3} \log_{10} e$$

(2) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{\log_{10}(x+h) - \log_{10} x}{h}$

$= \lim_{h \to 0} \dfrac{1}{h} \log_{10}\left(1 + \dfrac{h}{x}\right)$

ここで $k = \dfrac{h}{x}$ とおくと、$h \to 0$ のとき $k \to 0$ より

$$f'(x) = \lim_{k \to 0} \frac{1}{xk} \log_{10}(1+k) = \lim_{k \to 0} \frac{1}{x} \log_{10}(1+k)^{\frac{1}{k}} = \frac{1}{x} \log_{10} e$$

## 問 2 の解答

$$f'(x) = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h} = \lim_{h \to 0} \frac{1}{h} \log_a\left(1 + \frac{h}{x}\right)$$

ここで $k = \dfrac{h}{x}$ とおくと、$h \to 0$ のとき $k \to 0$ より

$$f'(x) = \lim_{k \to 0} \frac{1}{x} \log_a(1+k)^{\frac{1}{k}} = \frac{1}{x} \log_a e$$

# < 24 ページ. 自然対数 >

## 問 1 の解答

(1) $(\log_{10} x)' = \dfrac{1}{x}\log_{10} e$

(2) $(\log_a x)' = \dfrac{1}{x}\log_a e$

## 問 2 の解答

(答) $(\log_e x)' = \dfrac{1}{x}\log_e e = \dfrac{1}{x}$

## 問 3 の解答

(1) $\log e = 1$ (2) $\log(\sqrt[3]{e}) = \dfrac{1}{3}$ (3) $\log\left(\dfrac{1}{e}\right) = -1$ (4) $\log 1 = 0$

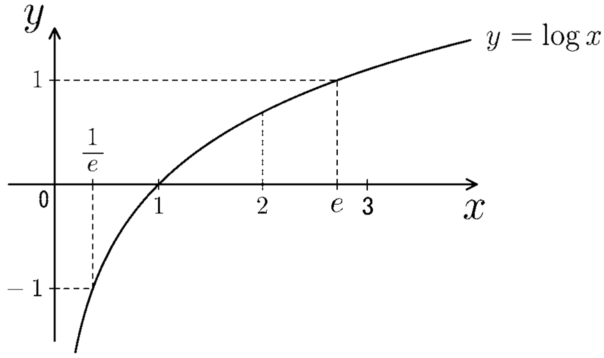
(5) $\ln\left(\dfrac{1}{e}\right) = -1$ (6) $\ln(\sqrt[4]{e}) = \dfrac{1}{4}$ (7) $\ln(e) = 1$ (8) $\ln(e\sqrt{e}) = \dfrac{3}{2}$

## 問 4 の解答

(1) $(\log x)' = \dfrac{1}{x}$

(2) $(\ln x)' = \dfrac{1}{x}$

## 問 5 の解答



# < 25 ページ.$\log f(x)$ の導関数 >

## 問 1 の解答

(1) $\dfrac{dy}{dx} = \dfrac{3x^2+2}{x^3+2x-5}$

(2) $\dfrac{dy}{dx} = \dfrac{\cos x}{1+\sin x}$

(3) $\dfrac{dy}{dx} = \dfrac{\sin x}{5-\cos x}$

## 問 2 の解答

$$\Big(\log(f(x))\Big)' = \frac{f'(x)}{f(x)}$$

## 問 3 の解答

(1) $\Big(\log(x^2+2x)\Big)' = \dfrac{2x+2}{x^2+2x}$

(2) $\Big(\log(x^6+3x^4)\Big)' = \dfrac{6x^5+12x^3}{x^6+3x^4} = \dfrac{6x^2+12}{x^3+3x}$

(3) $\Big(\log(\sin x)\Big)' = \dfrac{\cos x}{\sin x}$

# < 26 ページ. 指数関数の導関数 1 >

## 問の解答

(1) $f'(3) = \lim_{h \to 0} \dfrac{e^{3+h} - e^3}{h} = \lim_{h \to 0} e^3 \times \dfrac{e^h - 1}{h} = e^3 \times 1 = e^3$

(2) $f'(x) = \lim_{h \to 0} \dfrac{e^{x+h} - e^x}{h} = \lim_{h \to 0} e^x \times \dfrac{e^h - 1}{h} = e^x \times 1 = e^x$

# < 27 ページ. 指数関数の導関数 2 >

## 問 1 の解答

(1) $\dfrac{dy}{dx} = 2e^{2x}$

(2) $\dfrac{dy}{dx} = -3e^{-3x}$

(3) $\dfrac{dy}{dx} = 2e^{2x-1}$

(4) $\dfrac{dy}{dx} = -xe^{-\frac{x^2}{2}}$

(5) $\dfrac{dy}{dx} = Ke^{Kx}$

(6) $\dfrac{dy}{dx} = e^{x \log a} \log a$

## 問 2 の解答

$$\frac{d}{dx}a^x = \frac{d}{dx}\left(e^{\log a}\right)^x = \frac{d}{dx}\left\{e^{x \log a}\right\} = e^{x \log a} \times \log a$$

$$= \left(e^{\log a}\right)^x \times \log a = \underline{a^x \log a}$$

# < 28 ページ. 逆関数の微分 1 >

## 問の解答

(1) $y = \sqrt{x} \iff x = y^2$ より

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{d}{dy}(y^2)} = \frac{1}{2y} = \frac{1}{2\sqrt{x}}$$

(2) $y = \sqrt[4]{x} \iff x = y^4$ より

$$\frac{dy}{dx} = \frac{1}{\frac{d}{dy}(y^4)} = \frac{1}{4y^3} = \frac{1}{4(\sqrt[4]{x})^3} = \frac{1}{4\sqrt[4]{x^3}}$$

# < 29 ページ. 逆関数の微分 2 >

## 問 1 の解答

$y = \cos^{-1} x \iff x = \cos y$ より

$$\frac{d}{dx}\left\{\cos^{-1}(x)\right\} = \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{d}{dy}(\cos y)} = \frac{1}{-\sin y}$$

ここで $0 \leqq y \leqq \pi$ だから $0 \leqq \sin y$ より

$\cos^2 y + \sin^2 y = 1 \Rightarrow \sin y = \sqrt{1-\cos^2 y}$

よって、

$$\frac{d}{dx}\{\cos^{-1}(x)\} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$$

## 問 2 の解答

$y = \tan^{-1} x \iff x = \tan y$ より

$$\frac{d}{dx}\left\{\tan^{-1} x\right\} = \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{d}{dy}(\tan y)} = \frac{1}{\frac{1}{\cos^2 y}}$$

$$= \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

# < 30 ページ. 対数微分法 1 >

## 問 1 の解答

(解) $y = 3^x$ の両辺の自然対数をとると、

$$\log y = x \log 3$$

両辺を $x$ で微分すると

$$\frac{y'}{y} = \log 3 \Rightarrow y' = y \times \log 3 = 3^x \log 3$$

## 問 2 の解答

(解) $y = a^x$ の両辺の自然対数をとると、

$$\log y = x \log a$$

両辺を $y$ で微分すると

$$\frac{y'}{y} = \log a \Rightarrow y' = y \log a = a^x \log a$$

## 問 3 の解答

(解) $y = x^x$ の両辺の自然対数をとると、

$$\log y = x \log x$$

両辺を $x$ で微分すると

$$\frac{y'}{y} = (x)' \times \log x + x \times (\log x)' = 1 \times \log x + x \times \frac{1}{x} = \log x + 1$$

$$y' = y \times (1 + \log x) = x^x(1 + \log x)$$

# < 31 ページ. 対数微分法 2 >

## 問 1 の解答

(解)　$y = x^{\frac{4}{3}}$ の両辺の自然対数をとると、

$\log y = \dfrac{4}{3}\log x$ である. この両辺を $x$ で微分すると

$$\frac{y'}{y} = \frac{4}{3} \times \frac{1}{x} \Rightarrow y' = \frac{4}{3} \times \frac{1}{x} \times y = \frac{4}{3} \times \frac{1}{x} \times x^{\frac{4}{3}} = \frac{4}{3}x^{\frac{1}{3}}$$

(答)　$\left(x^{\frac{4}{3}}\right)' = \dfrac{4}{3} \times x^{\frac{1}{3}} \left(= \dfrac{4}{3}\sqrt[3]{x}\right)$

## 問 2 の解答

(解)　$y = x^r$ の両辺の自然対数をとると

$\log y = r\log x$ であり，この両辺を $x$ で微分すると

$$\frac{y'}{y} = \frac{r}{x} \Rightarrow y' = \frac{r}{x} \times y = \frac{r}{x} \times x^r = rx^{r-1}$$

(答)　$(x^r)' = rx^{r-1}$

# < 32ページ.$x^r$ の導関数 >

## 問 1 の解答

(1) $\left(\sqrt[4]{x^5}\right)' = \dfrac{5}{4}x^{\frac{1}{4}} = \dfrac{5}{4}\sqrt[4]{x}$

(2) $\left(\sqrt[5]{x^7}\right)' = \dfrac{7}{5}x^{\frac{2}{5}} = \dfrac{7}{5}\sqrt[5]{x^2}$

(3) $\left(\sqrt{x^3}\right)' = \dfrac{3}{2}x^{\frac{1}{2}} = \dfrac{3}{2}\sqrt{x}$

## 問 2 の解答

(1) $\left(\dfrac{1}{x^3}\right)' = -3x^{-4} = -\dfrac{3}{x^4}$

(2) $\left(\dfrac{1}{x^4}\right)' = -4x^{-5} = -\dfrac{4}{x^5}$

(3) $\left(\dfrac{1}{x}\right)' = -x^{-2} = -\dfrac{1}{x^2}$

## 問 3 の解答

(1) $\left(\sqrt[4]{x}\right)' = \dfrac{1}{4}x^{-\frac{3}{4}} = \dfrac{1}{4\sqrt[4]{x^3}}$

(2) $\left(\sqrt[5]{x^4}\right)' = \dfrac{4}{5}x^{-\frac{1}{5}} = \dfrac{4}{5\sqrt[5]{x^4}}$

(3) $(\sqrt{x})' = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

## 問 4 の解答

(1) $\left(\dfrac{1}{\sqrt[3]{x^2}}\right)' = -\dfrac{2}{3}x^{-\frac{2}{3}-1} = -\dfrac{2}{3x\sqrt[3]{x^2}}$

(2) $\left(\dfrac{1}{\sqrt[4]{x}}\right)' = -\dfrac{1}{4}x^{-\frac{1}{4}-1} = -\dfrac{1}{4x\sqrt[4]{x}}$

(3) $\left(\dfrac{1}{\sqrt{x}}\right)' = -\dfrac{1}{2}x^{-\frac{1}{2}-1} = -\dfrac{1}{2x\sqrt{x}}$

# < 33 ページ.$\log|x|$ の導関数 >

## 問の解答

(1) $\dfrac{dy}{dx} = \dfrac{\frac{1}{\cos^2 x}}{\tan x} = \dfrac{1}{\sin x \cos x}$

(2) $\dfrac{dy}{dx} = \dfrac{2x+3}{x^2+3x}$

(3) $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

# ＜ 34 ページ. 微分の練習 3 ＞

## 問 1 の解答

(1) $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$　(2) $\lim_{k\to\infty}\frac{1}{k}\log(1+k)=\log e=1$　(3) $\lim_{h\to 0}\frac{e^h-1}{h}=1$

## 問 2 の解答

$$f'(x)=\lim_{h\to\infty}\frac{\log_2(x+h)-\log_2 x}{h}=\lim_{h\to\infty}\frac{1}{h}\log_2(1+\frac{h}{x})$$

ここで $\frac{h}{x}=k$ とおくと $h\to 0$ のとき $k\to 0$ より

$$f'(x)=\lim_{k\to 0}\frac{1}{xk}\log_2(1+k)=\lim_{k\to 0}\frac{1}{x}\log_2(1+k)^{\frac{1}{k}}=\frac{1}{x}\log_2 e$$

## 問 3 の解答

(1) $(2e^x)'=2e^x$

(2) $(3\log x)'=\frac{3}{x}$

(3) $(\sqrt[3]{x})'=\frac{1}{3\sqrt[3]{x^2}}$

(4) $\left(\frac{1}{x^3}\right)'=-\frac{3}{x^4}$

(5) $\left(\frac{1}{\sqrt{x}}\right)'=-\frac{1}{2x\sqrt{x}}$

(6) $\left(e^{4x+1}\right)'=4e^{4x+1}$

(7) $\left(\log(5x)\right)'=\frac{1}{x}$

(8) $\left(e^{-\frac{x^2}{2}}\right)'=-xe^{-\frac{x^2}{2}}$

(9) $\left(\log\left(x^3\right)\right)'=\frac{3}{x}$

(10) $(\log|4x|)'=\frac{1}{x}$

(11) $(\log|\sin x|)'=\frac{\cos x}{\sin x}$

(12) $(x\sqrt{x})'=\frac{3}{2}\sqrt{x}$

(13) $\left(e^x\sin x\right)'=e^x\sin x+e^x\cos x$

(14) $\left(e^{3x}\cos(4x)\right)'=3e^{3x}\cos(4x)-4e^{3x}\sin(4x)$

(15) $\left(xe^{-x}\right)'=e^{-x}-xe^{-x}$

(16) $\left(x^2\log|x|\right)'=2x\log|x|+x$

(17) $\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$

(18) $\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$

## 問 4 の解答

(1) $y=4^x$ の両辺の自然対数をとる

$\log y=\log 4^x=x\log 4$

両辺を $x$ で微分すると

$\frac{y'}{y}=1\times\log 4\Leftrightarrow y'=y\times\log 4$

(答) $y'=4^x\log 4$

(2) $y=(x+1)^x$ の両辺の自然対数をとる

$\log y=x\log(x+1)$

両辺を $x$ で微分すると

$\frac{y'}{y}=\log(x+1)+\frac{x}{x+1}\Leftrightarrow y'=y\times\left(\frac{x}{x+1}+\log(x+1)\right)$

(答) $y'=(x+1)^x\left\{\frac{x}{x+1}+\log(x+1)\right\}$

# < 35 ページ. 微分係数と傾き >

## 問 1

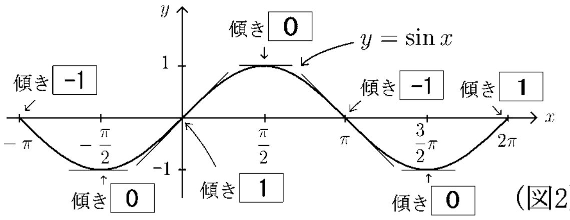
$f'(x) = \cos x$

$f'(-\pi) = -1$ $f'\left(-\dfrac{\pi}{2}\right) = 0$

$f'(0) = 1$ $f'\left(\dfrac{\pi}{2}\right) = 0$

$f'(\pi) = -1$ $f'\left(\dfrac{3}{2}\pi\right) = 0$

$f'(2\pi) = 1$




(図2)

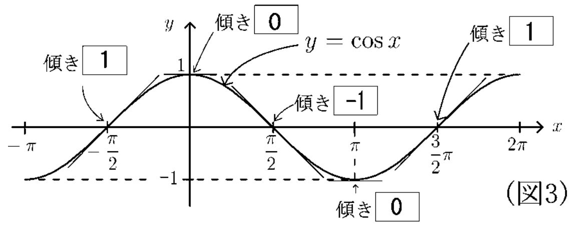
## 問 2

$f'(x) = -\sin x$

$f'\left(-\dfrac{\pi}{2}\right) = 1$ $f'(0) = 0$

$f'\left(\dfrac{\pi}{2}\right) = -1$ $f'(\pi) = 0$

$f'\left(\dfrac{3}{2}\pi\right) = 1$




(図3)

## 問 3 $f(x) = e^x$ とする。

(1) $f^{-1}(x) = \log x$

(2)

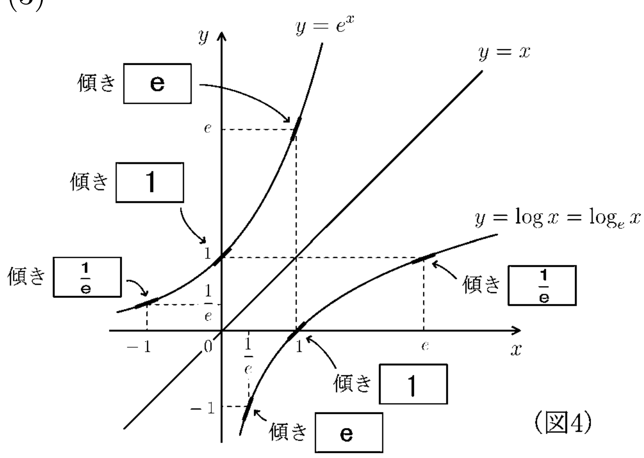
$f'(x) = e^x$ $g'(x) = \dfrac{1}{x}$

$f'(-1) = \dfrac{1}{e}$ $g'\left(\dfrac{1}{e}\right) = e$

$f'(0) = 1$ $g'(1) = 1$

$f'(1) = e$ $g'(e) = \dfrac{1}{e}$

(3)




(図4)

# < 36 ページ. 接線の方程式 >

## 問の解答

(1) $y = x + 1$

(2) $y = x - 1$

(3) $y = x$

(4) $y = \frac{1}{4}x + 1$

(5) $y = -x + 2$

# < 37 ページ. 関数の増減 1 >

## 問の解答

(1) $y' = 2x - 2$

頂点 $(1, 2)$

| $x$ | $x < 1$ | $1$ | $1 < x$ |
|---|---|---|---|
| $y'$ | $-$ | $0$ | $+$ |
| $y$ | ↘ | $2$ | ↗ |

(2) $y' = -4x + 8$

頂点 $(2, 7)$

| $x$ | $x < 2$ | $2$ | $2 < x$ |
|---|---|---|---|
| $y'$ | $+$ | $0$ | $-$ |
| $y$ | ↗ | $7$ | ↘ |

# < 38 ページ. 関数の増減 2 >

## 問の解答

(1) $y' = -3x^2 + 6x = -3x(x-2)$

| $x$ | $\cdots$ | 0 | $\cdots$ | 2 | $\cdots$ |
|---|---|---|---|---|---|
| $y'$ | $-$ | 0 | $+$ | 0 | $-$ |
| $y$ | $\searrow$ | 0 | $\nearrow$ | 4 | $\searrow$ |

(2) $y' = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$

| $x$ | $\cdots$ | 1 | $\cdots$ | 3 | $\cdots$ |
|---|---|---|---|---|---|
| $y'$ | $+$ | 0 | $-$ | 0 | $+$ |
| $y$ | $\nearrow$ | 4 | $\searrow$ | 0 | $\nearrow$ |

# < 39 ページ. 極大・極小 1 >

## 問の解答

$$y = 2x^3 + 3x^2 - 12x$$

$$\begin{aligned} y' &= 6x^2 + 6x - 12 \\ &= 6(x^2 + x - 2) \\ &= 6(x-1)(x+2) \end{aligned}$$

| $x$ | $\cdots$ | $-2$ | $\cdots$ | $1$ | $\cdots$ |
|---|---|---|---|---|---|
| $y'$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $y$ | $\nearrow$ | $20$ | $\searrow$ | $-7$ | $\nearrow$ |

$x = -2$ のとき極大値 $y = 20$

$x = 1$ のとき極小値 $y = -7$

## < 40 ページ. 極大・極小 2 >

### 問の解答

(1) $y = -x^4 + 2x^2 + 5$

$$\begin{aligned} y' &= -4x^3 + 4x \\ &= -4x(x^2 - 1) \\ &= -4x(x-1)(x+1) \end{aligned}$$

| $x$ | $\cdots$ | $-1$ | $\cdots$ | $0$ | $\cdots$ | $1$ | $\cdots$ |
|---|---|---|---|---|---|---|---|
| $y'$ | $+$ | $0$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| $y$ | $\nearrow$ | $6$ | $\searrow$ | $5$ | $\nearrow$ | $6$ | $\searrow$ |

$x = \pm 1$ のとき極大値 $y = 6$

$x = 0$ のとき極小値 $y = 5$

(2) $y = 3x^4 - 8x^3 - 18x^2$

$$\begin{aligned} y' &= 12x^3 - 24x^2 - 36x \\ &= 12x(x^2 - 2x - 3) \\ &= 12x(x-3)(x+1) \end{aligned}$$

| $x$ | $\cdots$ | $-1$ | $\cdots$ | $0$ | $\cdots$ | $3$ | $\cdots$ |
|---|---|---|---|---|---|---|---|
| $y'$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $y$ | $\searrow$ | $-7$ | $\nearrow$ | $0$ | $\searrow$ | $-135$ | $\nearrow$ |

$x = 0$ のとき極大値 $y = 0$

$x = -1$ のとき極小値 $y = -7$

$x = 3$ のとき極小値 $y = -135$

# < 41 ページ. 極大・極小 3 >

## 問の解答

(1) $y' = e^x - 1$

| $x$ | $\cdots$ | $0$ | $\cdots$ |
|---|---|---|---|
| $y'$ | $-$ | $0$ | $+$ |
| $y$ | $\searrow$ | $1$ | $\nearrow$ |

$x = 0$ のとき極小値 $1$

(2) $y' = 2xe^{-x} - x^2e^{-x}$

$= x(2 - x)e^{-x}$

| $x$ | $\cdots$ | $0$ | $\cdots$ | $2$ | $\cdots$ |
|---|---|---|---|---|---|
| $y'$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| $y$ | $\searrow$ | $0$ | $\nearrow$ | $\frac{4}{e^2}$ | $\searrow$ |

$x = 0$ のとき極小値 $0$

$x = 2$ のとき極大値 $\frac{4}{e^2}$

(3) $y' = \log x + 1$

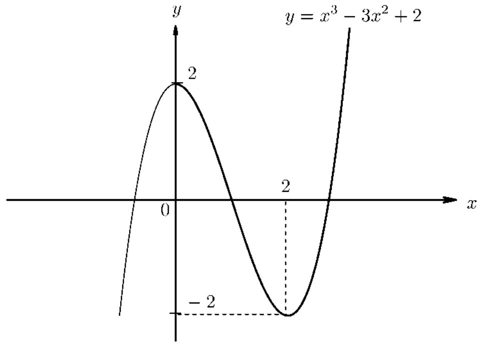
| $x$ | $0$ | $\cdots$ | $\frac{1}{e}$ | $\cdots$ |
|---|---|---|---|---|
| $y'$ | $\times$ | $-$ | $0$ | $+$ |
| $y$ | $\times$ | $\searrow$ | $-\frac{1}{e}$ | $\nearrow$ |

$x = \frac{1}{e}$ のとき極小値 $-\frac{1}{e}$

# ＜42 ページ. 関数のグラフ＞

## 問の解答

(1)　$y' = 3x^2 - 6x = 3x(x-2)$




| | | | | | |
|---|---|---|---|---|---|
| $x$ | $\cdots$ | 0 | $\cdots$ | 2 | $\cdots$ |
| $y'$ | $+$ | 0 | $-$ | 0 | $+$ |
| $y$ | $\nearrow$ | 2 | $\searrow$ | $-2$ | $\nearrow$ |

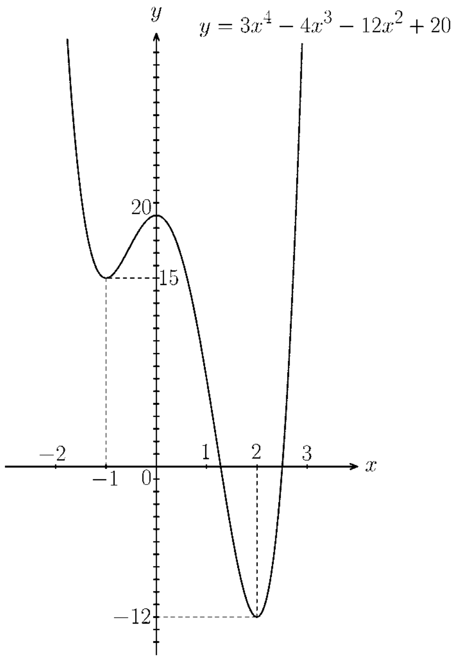
$x = 0$ のとき極大値 $y = 2$

$x = 2$ のとき極小値 $y = -2$

(2)　$y' = 12^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

$= 12x(x-2)(x+1)$



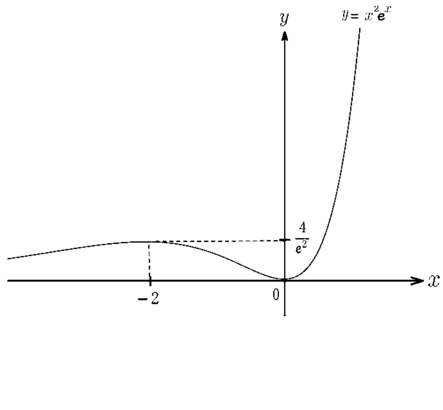

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | $\cdots$ | $-1$ | $\cdots$ | 0 | $\cdots$ | 2 | $\cdots$ |
| $y'$ | $-$ | 0 | $+$ | 0 | $-$ | 0 | $+$ |
| $y$ | $\searrow$ | 15 | $\nearrow$ | 20 | $\searrow$ | $-12$ | $\nearrow$ |

$x = -1$ のとき極小値 $y = 15$

$x = 0$　のとき極大値 $y = 20$

$x = 2$　のとき極小値 $y = -12$

(3)　$y' = x(2+x)e^x$




| | | | | | |
|---|---|---|---|---|---|
| $x$ | $\cdots$ | $-2$ | $\cdots$ | 0 | $\cdots$ |
| $y'$ | $+$ | 0 | $-$ | 0 | $+$ |
| $y$ | $\nearrow$ | $\frac{4}{e^2}$ | $\searrow$ | 0 | $\nearrow$ |

$x = -2$ のとき極大値 $y = \dfrac{4}{e^2}$

$x = 0$ のとき極小値 $y = 0$

# < 43 ページ. 最大・最小 1 >

## 問の解答

$$
\begin{aligned}
y' &= 3x^2 - 12x + 9 \\
&= 3(x^2 - 4x + 3) \\
&= 3(x - 3)(x - 1)
\end{aligned}
$$

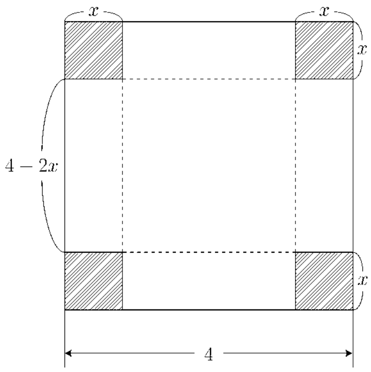
| $x$ | $-1$ | $\cdots$ | $1$ | $\cdots$ | $3$ |
|---|---|---|---|---|---|
| $y'$ | | $+$ | $0$ | $-$ | |
| $y$ | $-19$ | $\nearrow$ | $1$ | $\searrow$ | $-3$ |

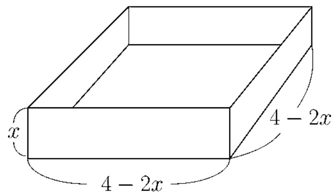
$x = 1$ のとき最大値 $y = 1$

$x = -1$ のとき最小値 $y = -19$

# ＜ 44 ページ. 最大・最小 2 ＞

## 問の解答




$x$ の範囲は $\underline{0 < x < 2}$ である

$$y = x(4-2x)^2$$
$$= x(16 - 16x + 4x^2)$$
$$= 4x^3 - 16x^2 + 16x$$
$$y' = 12x^2 - 32x + 16$$
$$= 4(3x^2 - 8x + 4)$$
$$= 4(3x-2)(x-2)$$

| $x$ | 0 | $\cdots$ | $\frac{2}{3}$ | $\cdots$ | 2 |
|---|---|---|---|---|---|
| $y'$ | ╳ | $+$ | 0 | $-$ | ╳ |
| $y$ | 0 | ↗ | $\frac{128}{27}$ | ↘ | 0 |

$$x = \frac{2}{3} \Rightarrow y = \frac{128}{27}$$

(答) $x = \dfrac{2}{3}$(cm) のとき最大容積 $y = \dfrac{128}{27}(\mathrm{cm}^3)$

# ＜ 45 ページ. 微分の応用問題 ＞

## 問 1 の解答

(1) $y = ex$　(2)$y = \dfrac{1}{e}x$　(3)$y = -x + \pi$　(4)$y = x + \dfrac{\pi}{2}$　(5)$y = \dfrac{1}{6}x + \dfrac{3}{2}$　(6)$y = -2x + 3$

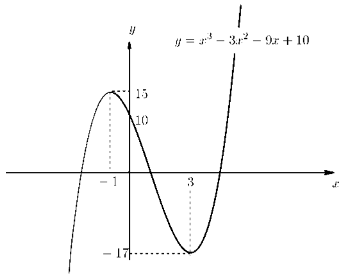
## 問 2 の解答

(1) $y' = 3(x-3)(x+1)$

$x = -1$ のとき極大値 $y = 15$

$x = 3$　のとき極小値 $y = -17$

| $x$ | $\cdots$ | $-1$ | $\cdots$ | $3$ | $\cdots$ |
|---|---|---|---|---|---|
| $y'$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $y$ | ↗ | $15$ | ↘ | $-17$ | ↗ |



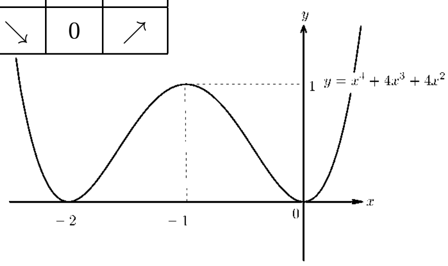

(2) $y' = 4x(x+1)(x+2)$

$x = -1$　のとき極大値 $y = 1$

$x = 0$ または $-2$ のとき

極小値 $y = 0$

| $x$ | $\cdots$ | $-2$ | $\cdots$ | $-1$ | $\cdots$ | $0$ | $\cdots$ |
|---|---|---|---|---|---|---|---|
| $y'$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $y$ | ↘ | $0$ | ↗ | $1$ | ↘ | $0$ | ↗ |

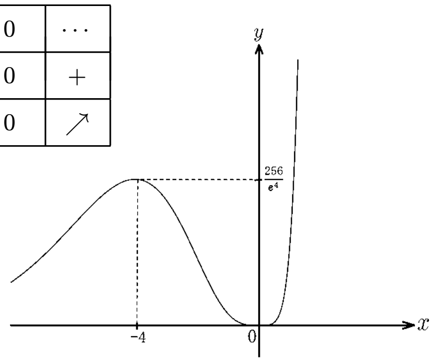



(3) $y' = (4+x)x^3e^x$

$x = -4$ のとき極大値 $y = \dfrac{256}{e^4}$

$x = 0$　のとき極小値 $y = 0$

| $x$ | $\cdots$ | $-4$ | $\cdots$ | $0$ | $\cdots$ |
|---|---|---|---|---|---|
| $y'$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $y$ | ↗ | $\frac{256}{e^4}$ | ↘ | $0$ | ↗ |




## 問 3 の解答

$$
\begin{aligned}
y &= x(9-2x)(24-2x) \\
&= 4x^3 - 66x^2 + 216x \\
y' &= 12x^2 - 132x + 216x \\
&= 12(x^2 - 11x + 18) \\
&= 12(x-2)(x-9)
\end{aligned}
$$

| $x$ | $0$ | $\cdots$ | $2$ | $\cdots$ | $\frac{9}{2}$ |
|---|---|---|---|---|---|
| $y'$ | ╳ | $+$ | $0$ | $-$ | ╳ |
| $y$ | $0$ | ↗ | $200$ | ↘ | $0$ |

$x > 0$、$24 - 2x > 0$、$9 - 2x > 0$ がすべてなりたつ範囲は、$\underline{0 < x < \dfrac{9}{2}}$ である。

$x = 2$ のとき、$y = 2 \times 5 \times 20 = 200$

(答)$x = 2\text{cm}$ のとき最大容積 $y = 200\text{cm}^3$