< 常微分方程式への応用 5 >

例題

$$\begin{cases} \frac{dx}{dt} = 2x + y \\ \frac{dy}{dt} = -x + 4y \\ x(0) = 0\,,\ y(0) = 1 \end{cases}$$

(解) $\mathscr{L}[x(t)] = X(s)$, $\mathscr{L}[y(t)] = Y(s)$ とおくと

$$\mathscr{L}\left[\frac{dx}{dt}\right] = sX(s) - x(0) = sX(s)\,,\quad \mathscr{L}\left[\frac{dy}{dt}\right] = sY(s) - y(0) = sY(s) - 1$$

であるから，微分方程式の両辺をラプラス変換すると

$$\begin{cases} sX(s) = 2X(s) + Y(s) \\ sY(s) - 1 = -X(s) + 4Y(s) \end{cases} \Rightarrow \begin{cases} (s-2)X(s) - Y(s) = 0 & \cdots ① \\ X(s) + (s-4)Y(s) = 1 & \cdots ② \end{cases}$$

① ×$(s-4)$+ ②より

$$\begin{array}{lr} & (s-2)(s-4)X(s) - (s-4)Y(s) = 0 \\ +) & X(s) + (s-4)Y(s) = 1 \\ \hline & (s^2-6s+9)X(s) \qquad = 1 \end{array}$$

よって

$$X(s) = \frac{1}{(s-3)^2}\ ,\ Y(s) = (s-2)X(s) = \frac{(s-2)}{(s-3)^2} = \frac{s-3+1}{(s-3)^2} = \frac{1}{s-3} + \frac{1}{(s-3)^2}$$

であるから

(答) $x(t) = \mathscr{L}^{-1}[X(s)] = \mathscr{L}^{-1}\left[\frac{1}{(s-3)^2}\right] = te^{3t}$

$$\underline{y(t) = \mathscr{L}^{-1}[Y(s)] = \mathscr{L}^{-1}\left[\frac{1}{s-3} + \frac{1}{(s-3)^2}\right] = e^{3t} + te^{3t}}$$

問　次の連立微分方程式の初期値問題をラプラス変換を用いて解け。

$$\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = -x + 3y \\ x(0) = 0\,,\ y(0) = 1 \end{cases}$$