

< 積分 3 >

部分積分の公式

$$\int_a^b f(t)g'(t)dt = \left[f(t)g(t) \right]_a^b - \int_a^b f'(t)g(t)dt$$

例 1

$$\begin{aligned} \int_0^\pi t \cos(3t)dt &= \int_0^\pi t \times \left(\frac{1}{3} \sin(3t) \right)' dt \\ &= \left[t \times \frac{1}{3} \sin(3t) \right]_0^\pi - \int_0^\pi (t)' \times \frac{1}{3} \sin(3t)dt \\ &= \left\{ \frac{\pi}{3} \sin(3\pi) - 0 \right\} - \int_0^\pi \frac{1}{3} \sin(3t)dt \\ &= 0 - \left[-\frac{1}{9} \cos(3t) \right]_0^\pi = \left[\frac{1}{9} \cos(3t) \right]_0^\pi \\ &= \frac{1}{9} \cos(3\pi) - \frac{1}{9} \cos 0 = -\frac{2}{9} \end{aligned}$$

例 2

$$\begin{aligned} \int_0^\pi t \sin(3t)dt &= \int_0^\pi t \left(-\frac{1}{3} \cos(3t) \right)' dt \\ &= \left[t \left(-\frac{1}{3} \cos(3t) \right) \right]_0^\pi - \int_0^\pi (t)' \times \left(-\frac{1}{3} \cos(3t) \right) dt \\ &= -\frac{\pi}{3} \cos(3\pi) - 0 + \int_0^\pi \frac{1}{3} \cos(3t)dt \\ &= \frac{\pi}{3} + \left[\frac{1}{9} \sin(3t) \right]_0^\pi = \frac{\pi}{3} + \frac{1}{9} \sin(3\pi) - \frac{1}{9} \sin(0) = \frac{\pi}{3} \end{aligned}$$

問 次の定積分の値を求めよ。

$$(1) \int_0^\pi t \cos(4t)dt =$$

$$(2) \int_0^\pi t \sin(4t)dt =$$

$$(3) \int_0^\pi t \cos(5t)dt =$$

$$(4) \int_0^\pi t \sin(5t)dt =$$