高知工科大学

Kochi University of Technology

数学 2

（2007年度版）

解答

＜ 1 ページ. 不定積分 1 ＞

問の解答

(1) $\left(\dfrac{1}{\alpha+1}x^{\alpha+1}\right)' = x^{\alpha} \quad \Rightarrow \quad \displaystyle\int x^{\alpha}dx = \frac{1}{\alpha+1}x^{\alpha+1} + C$

(2) $\big(\log|x|\big)' = \dfrac{1}{x} \quad \Rightarrow \quad \displaystyle\int \frac{1}{x}dx = \log|x| + C$

(3) $(\sin x)' = \cos x \quad \Rightarrow \quad \displaystyle\int \cos x dx = \sin x + C$

(4) $(-\cos x)' = \sin x \quad \Rightarrow \quad \displaystyle\int \sin x dx = -\cos x + C$

(5) $(e^x)' = e^x \quad \Rightarrow \quad \displaystyle\int e^x dx = e^x + C$

< 2 ページ. 不定積分 2 >

問の解答

(1) $\displaystyle\int x^{10}dx = \frac{1}{11}x^{11} + C$

(2) $\displaystyle\int \frac{dx}{x^3} = -\frac{1}{2x^2} + C$

(3) $\displaystyle\int \frac{dx}{\sqrt[4]{x^3}} = \frac{4}{3}x^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{x^3} + C$

(4) $\displaystyle\int \sqrt{x}dx = \frac{2}{3}x^{\frac{3}{2}} + C = \frac{2}{3}x\sqrt{x} + C$

(5) $\displaystyle\int \sqrt[3]{x^2}dx = \frac{3}{5}x^{\frac{2}{3}+1} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$

(6) $\displaystyle\int \frac{dx}{x\sqrt{x}} = \int x^{-\frac{3}{2}}dx = -\frac{2}{\sqrt{x}} + C$

＜ 3 ページ. 不定積分 3 ＞

問の解答

(1) $\displaystyle\int\left(\frac{1}{x}-\frac{4}{x^2}+\frac{1}{x^3}\right)dx=\log|x|+\frac{4}{x}-\frac{1}{2x^2}+C$

(2) $\displaystyle\int\left(1-\frac{4}{x^2}+\frac{3}{x^4}\right)dx=x+\frac{4}{x}-\frac{1}{x^3}+C$

(3) $\displaystyle\int\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)dx=\frac{2}{3}x\sqrt{x}+4\sqrt{x}+C$

(4) $\displaystyle\int\left(1-\frac{2}{\sqrt{x}}+\frac{1}{x}\right)dx=x-4\sqrt{x}+\log|x|+C$

＜ 4 ページ. 不定積分 4 ＞

問 1 の解答

(1) $(\tan x)' = \dfrac{1}{\cos^2 x}$ $\Rightarrow$ $\displaystyle\int \frac{dx}{\cos^2 x} = \tan x + C$

(2) $(\cot x)' = -\dfrac{1}{\sin^2 x}$ $\Rightarrow$ $\displaystyle\int \frac{dx}{\sin^2 x} = -\cot x + C$

(3) $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$ $\Rightarrow$ $\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

(4) $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$ $\Rightarrow$ $\displaystyle\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$

問 2 の解答

(1) $\sin^2 x + \cos^2 x = 1$

(2) $\dfrac{1}{\cos^2 x} - \tan^2 x = \dfrac{1-\sin^2 x}{\cos^2 x} = 1$

問 3 の解答

(1) $-4\cos x - 3\sin x + C$

(2) $3x - \tan x + c$

(3) $2\sin x + \cos x + C$

(4) $\tan x + C$

(5) $-\cot x - x + C$

(6) $\tan x + C$

(7) $3\sin^{-1} x + C$

(8) $5\tan^{-1} x + C$

< 5 ページ. 積分記号 >

問の解答

(1) $10t - 4.9t^2 + C$

(2) $\dfrac{4}{3}\pi r^3 + C$

(3) $e^u + C$

(4) $\log|y| + C$

(5) $\sin u + C$

＜ 6 ページ. 置換積分法 1 ＞

問の解答

(1) $\displaystyle\int \cos(4x-3)dx = \frac{1}{4}\sin(4x-3)+C$

(2) $\displaystyle\int \sin(3x+4)dx = -\frac{1}{3}\cos(3x+4)+C$

＜ 7ページ. 置換積分法 2 ＞

問の解答

(1) $\displaystyle\int (5x+3)^6 dx = \frac{1}{35}(5x+3)^7 + C$

(2) $\displaystyle\int \frac{dx}{6x+5} = \frac{1}{6}\log|6x+5| + C$

(3) $\displaystyle\int \frac{dx}{\cos^2(3x-4)} = \frac{1}{3}\tan(3x-4) + C$

(4) $\displaystyle\int e^{4x+1} dx = \frac{1}{4}e^{4x+1} + C$

＜ 8 ページ. 置換積分法 3 ＞

問の解答

(1) $\displaystyle\int \sqrt{4x-3}\,dx = \frac{1}{6}(4x-3)\sqrt{4x-3} + C$

(2) $\displaystyle\int \frac{1}{(5x-3)^4}dx = -\frac{1}{15(5x-3)^3} + C$

(3) $\displaystyle\int \frac{1}{\sqrt{2x+1}}dx = \sqrt{2x+1} + C$

＜ 9 ページ. 置換積分法 4 ＞

問の解答

(1) $\displaystyle\int x^3 e^{x^4+1} dx = \frac{1}{4} e^{x^4+1} + C$

(2) $\displaystyle\int x^2 \cos(x^3+2) dx = \frac{1}{3} \sin(x^3+2) + C$

(3) $\displaystyle\int x \sin(x^2+3) dx = -\frac{1}{2} \cos(x^2+3) + C$

(4) $\displaystyle\int x(x^2+1)^5 dx = \frac{1}{12}(x^2+1)^6 + C$

< 10 ページ. 置換積分法 5 >

問の解答

(1) $\displaystyle\int \frac{x^2}{x^3+1}dx = \frac{1}{3}\ \log|\ x^3+1\ | + C$

(2) $\displaystyle\int \frac{3x}{x^2+4}dx = \frac{3}{2}\ \log(x^2+4) + C$

(3) $\displaystyle\int \cot x\ dx = \log|\sin x| + C$

(4) $\displaystyle\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

< 11 ページ. 置換積分法 6 >

問の解答

(1) $$\int \frac{dx}{(5x-4)^2+1} = \frac{1}{5}\tan^{-1}(5x-4)+C$$

(2) $$\int \frac{dx}{(8x+10)^2+4} = \frac{1}{16}\tan^{-1}(4x+5)+C$$

< 12ページ. 不定積分の練習 1 >

問 1 の解答

(1) $\sin u + C$

(1)′ $\dfrac{1}{a}\sin(ax+b) + C$

(2) $-\cos u + C$

(2)′ $-\dfrac{1}{a}\cos(ax+b) + C$

(3) $\tan u + C$

(3)′ $\dfrac{1}{a}\tan(ax+b) + C$

(4) $e^u + C$

(4)′ $\dfrac{1}{a}e^{ax+b} + C$

(5) $\log|u| + C$

(5)′ $\dfrac{1}{a}\log|ax+b| + C$

(6) $\dfrac{1}{n+1}u^{n+1} + C$

(6)′ $\dfrac{1}{a(n+1)}(ax+b)^{n+1} + C$

(7) $\tan^{-1} u + C$

(7)′ $\dfrac{1}{a}\tan^{-1}(ax+b) + C$

問 2 の解答

(1) $\dfrac{1}{3}\sin(3x+4) + C$

(2) $-\dfrac{1}{4}\cos(4x+3) + C$

(3) $\dfrac{1}{5}\tan(5x+1) + C$

(4) $\dfrac{1}{3}e^{3x-5} + C$

(5) $\dfrac{1}{7}\log|7x+10| + C$

(6) $\dfrac{1}{28}(7x-5)^4 + C$

(7) $-\dfrac{1}{64}(-8x+10)^8 + C$

(8) $\dfrac{1}{5}\tan^{-1}(5x-3) + C$

＜ 13 ページ. 不定積分の練習 2 ＞

問の解答

(1) $\displaystyle\int \frac{1}{(3x+4)^6}dx = -\frac{1}{15(3x+4)^5} + C$

(2) $\displaystyle\int \sqrt{5x+4}dx = \frac{2}{15}(5x+4)\sqrt{5x+4} + C$

(3) $\displaystyle\int \sqrt{7x+3}dx = \frac{2}{7}\sqrt{7x-3} + C$

(4) $\displaystyle\int x\cos(x^2+3)dx = \frac{1}{2}\sin(x^2+3) + C$

(5) $\displaystyle\int xe^{-x^2}dx = -\frac{1}{2}e^{-x^2} + C$

(6) $\displaystyle\int xe^{-x^2}dx = \frac{1}{2}\log(1+x^2) + C$

(7) $\displaystyle\int \tan x dx = -\log|\cos x| + C$

< 14 ページ. 分数関数の積分 >

問の解答

(1) $\log\left|\dfrac{x}{x+1}\right| + C$

(2) $\dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right| + C$

(3) $\log\left|\dfrac{x-3}{x-2}\right| + C$

(4) $\dfrac{1}{7}\log\left|\dfrac{x-3}{x+4}\right| + C$

(5) $\dfrac{1}{5}\log\left|\dfrac{2x+1}{3x+4}\right| + C$

< 15 ページ. 部分積分法 1 >

問 1 の解答

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

問 2 の解答

(1) $2x\sin x + 2\cos x + C$

(2) $-x\cos x + \sin x + C$

(3) $-(3x+2)\cos x + 3\sin x + C$

(4) $xe^x - e^x + C$

(5) $(4x-7)e^x + C$

＜ 16 ページ. 部分積分法 2 ＞

問の解答

(1) $\dfrac{x}{4}\sin(4x-3)+\dfrac{1}{16}\cos(4x-3)+C$

(2) $-\dfrac{x}{2}\cos(2x+3)+\dfrac{1}{4}\sin(2x+3)+C$

(3) $\dfrac{x}{2}e^{2x}-\dfrac{1}{4}e^{2x}+C$

(4) $-\dfrac{x}{3}e^{-3x+5}-\dfrac{1}{9}e^{-3x+5}+C$

< 17ページ.部分積分法 3 >

問 1 の解答

(1) $\dfrac{x^2}{2}\log x - \dfrac{x^2}{4} + C$

(2) $\dfrac{x^3}{3}\log x - \dfrac{x^3}{9} + C$

問 2 の解答

(1) $-x^2\cos x + 2x\sin x + 2\cos x + C$

(2) $x^2e^x - 2xe^x + 2e^x + C$

< 18 ページ. 三角関数の不定積分 >

問の解答

(1) $\dfrac{1}{2}x - \dfrac{1}{4}\sin(2x) + C$

(2) $\dfrac{1}{10}\sin(5x) + \dfrac{1}{2}\sin x + C$

(3) $\dfrac{1}{6}\sin(3x) - \dfrac{1}{10}\sin(5x) + C$

(4) $-\dfrac{1}{14}\cos(7x) - \dfrac{1}{2}\cos x + C$

(5) $\dfrac{1}{2}x + \dfrac{1}{12}\sin(6x) + C$

(6) $\dfrac{1}{2}x - \dfrac{1}{16}\sin(8x) + C$

＜ 19 ページ. 不定積分の検証 ＞

問の解答

(1) $\left\{ \dfrac{1}{4}(x^4-1)^4 \right\}' = 4x^3(x^4-1)^3$ より正しくない (×)

(2) $\left(\dfrac{1}{2}\log|x^2-1|\right)' = \dfrac{x}{x^2-1}$ より正しい (○)

(3) $(x^2e^x - 2xe^x + 2e^x)' = x^2e^x$ より正しい (○)

＜ 20 ページ. 不定積分の練習 3 ＞

問 1 の解答

(1) $\dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right|+C$

(2) $\dfrac{1}{3}\log\left|\dfrac{x-2}{x+1}\right|+C$

(3) $(3x+4)\sin x+3\cos x+C$

(4) $-\dfrac{x}{2}\cos(2x)+\dfrac{1}{4}\sin(2x)+C$

(5) $\dfrac{x}{3}e^{3x-1}-\dfrac{1}{9}e^{3x-1}+C$

(6) $2x\log x-2x+C$

(7) $\dfrac{x^4}{4}\log x-\dfrac{x^4}{16}+C$

(8) $\dfrac{x}{2}+\dfrac{1}{4}\sin(2x)+C$

(9) $\dfrac{x}{2}-\dfrac{1}{4}\sin(2x)+C$

問 2 の解答

(1) $(-x^2\cos x+2x\sin x+2\cos x)'=x^2\sin x$ より正しい (○)

(2) $\left(\dfrac{1}{2}(\log|x-2|-\log|x+2|)\right)'=\dfrac{2}{x^2-4}$ より正しくない (×)

$<$ 23 ページ. 和の記号 $\sum$ $>$

問 1 の解答

(1) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum_{k=1}^{n} k^3$

(2) $2^2 + 3^2 + 4^2 + \cdots + (n-1)^2 = \sum_{k=2}^{n-1} k^2$

(3) $1 + 2 + 3 + \cdots + n = \sum_{k=1}^{n} k$

(4) $\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \cdots + \frac{n}{n} = \sum_{k=1}^{n} \frac{k}{n}$

問 2 の解答

(1) $1 + 2 + 3 + \cdots + 1000 = \sum_{k=1}^{1000} k = \frac{1000 \times 1001}{2} = 500500$

(2) $1^2 + 2^2 + 3^2 + \cdots + 20^2 = \sum_{k=1}^{20} k^2 = \frac{1}{6} \times 20 \times 21 \times 41 = 2870$

(3) $1^3 + 2^3 + 3^3 + \cdots + 10^3 = \sum_{k=1}^{10} k^3 = \left(\frac{10 \times 11}{2}\right)^2 = 3025$

(4) $\sum_{k=1}^{n}(k-1) = \sum_{k=1}^{n-1} k = \frac{(n-1)n}{2}$

(5) $\sum_{k=1}^{n}(k-1)^2 = \sum_{k=1}^{n-1} k^2 = \frac{(n-1)\,n\,(2n-1)}{6}$

＜ 27 ページ. 定積分の計算 1 ＞

問の解答

(1) $\displaystyle\int_4^{10} dx = \Big[x\Big]_4^{10} = 6$

(2) $\displaystyle\int_{-1}^{2} x^3 dx = \left[\frac{x^4}{4}\right]_{-1}^{2} = \frac{15}{4}$

(3) $\displaystyle\int_1^5 \frac{1}{x^2} dx = \left[-\frac{1}{x}\right]_1^5 = \frac{4}{5}$

(4) $\displaystyle\int_1^4 \sqrt{x} dx = \left[\frac{2}{3}x\sqrt{x}\right]_1^4 = \frac{14}{3}$

(5) $\displaystyle\int_1^3 \frac{1}{x} dx = \Big[\log|x|\Big]_1^3 = \log 3$

(6) $\displaystyle\int_0^2 e^x dx = \Big[e^x\Big]_0^2 = e^2 - 1$

(7) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x dx = \Big[\sin x\Big]_0^{\frac{\pi}{2}} = 1$

(8) $\displaystyle\int_0^{\pi} \sin x dx = \Big[-\cos x\Big]_0^{\pi} = 2$

(9) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = \Big[\tan x\Big]_0^{\frac{\pi}{4}} = 1$

(10) $\displaystyle\int_0^1 \frac{dx}{1+x^2} dx = \Big[\tan^{-1} x\Big]_0^1 = \frac{\pi}{4}$

＜28 ページ. 定積分の計算 2＞

問の解答

(1) $\displaystyle\int_1^2 \frac{2x^2-3x+1}{x^2}dx = \Big[2x - 3\log|x| - \frac{1}{x}\Big]_1^2 = \frac{5}{2} - 3\log 2$

(2) $\displaystyle\int_0^2 \frac{dx}{x^2+4x+3}dx = \left[\frac{1}{2}\log\left|\frac{x+1}{x+3}\right|\right]_0^2 = \frac{1}{2}\log\frac{9}{5}$

(3) $\displaystyle\int_0^\pi \cos^2 x dx = \left[\frac{x}{2} + \frac{1}{4}\sin(2x)\right]_0^\pi = \frac{\pi}{2}$

(4) $\displaystyle\int_2^\pi \sin x\cos x dx = \left[-\frac{1}{4}\cos(2x)\right]_0^{\frac{\pi}{2}} = \frac{1}{2}$

$<$ 29 ページ. 定積分の性質 $>$

問の解答

(1) $\displaystyle\int_3^3 e^{-x^2}dx = 0$

(2) $\displaystyle\int_{-1}^3 (x^2+3x+4)dx - \int_{-1}^3 (x^2-3x-4)dx = \int_{-1}^3 (6x+8)dx = 56$

(3) $\displaystyle\int_{-2}^1 (x^2+x^3)dx + \int_1^2 (x^2+x^3)dx = \int_{-2}^2 (x^2+x^3)dx = \frac{16}{3}$

< 30 ページ. 定積分の積分変数 >

問の解答

(1) $\Big[4t - 5t^2\Big]_{t=1}^{t=3} = -32$

(2) $\Big[\pi r^2\Big]_{r=0}^{r=R} = \pi R^2$

(3) $\Big[-\cos\theta\Big]_{\theta=0}^{\theta=\pi} = 2$

(4) $\left[\dfrac{u^{n+1}}{n+1}\right]_{u=a}^{u=b} = \dfrac{b^{n+1}}{n+1} - \dfrac{a^{n+1}}{n+1}$

(5) $\left[\dfrac{2}{3}u^{\frac{3}{2}}\right]_{u=1}^{u=9} = \dfrac{52}{3}$

< 31 ページ. 定積分の置換積分法 1 >

問の解答

$u = 2x + 1$ とおくと $\dfrac{du}{dx} = 2$

x	-1	$\rightarrow$	1
u	-1	$\rightarrow$	3

より

$$\int_{-1}^{1} (2x+1)^4 dx = \frac{1}{2} \int_{x=-1}^{x=1} (2x+1)^4 2dx = \frac{1}{2} \int_{x=-1}^{x=1} u^4 \frac{du}{dx} dx$$

$$= \frac{1}{2} \int_{u=-1}^{u=3} u^4 du = \frac{1}{2} \left[\frac{u^5}{5} \right]_{u=-1}^{u=3} = \frac{1}{2} \left(\frac{243}{5} - \frac{(-1)}{5} \right)$$

$$= \frac{1}{2} \times \frac{244}{5} = \frac{122}{5}$$

< 32 ページ. 定積分の置換積分法 2 >

問の解答

(1) $u = 3x - 2$

x	$0 \to 2$
u	$-2 \to 4$

$$\int_0^2 (3x-2)^4 dx = \frac{1}{3}\int_{-2}^4 u^4 du = \frac{1}{3}\Big[\frac{u^5}{5}\Big]_{-2}^4$$
$$= \frac{1}{3}\Big\{\frac{1024}{5} - \frac{-32}{5}\Big\} = \frac{352}{5}$$

(2) $u = 2x + 1$

x	$0 \to 4$
u	$1 \to 9$

$$\int_0^4 \sqrt{2x+1}dx = \frac{1}{2}\int_1^9 u^{\frac{1}{2}}du = \frac{1}{2}\Big[\frac{2}{3}u^{\frac{3}{2}}\Big]_1^9$$
$$= \frac{1}{2} \times \frac{2}{3} \times \Big(9^{\frac{3}{2}} - 1\Big) = \frac{26}{3}$$

(3) $u = 4x + 1$

x	$0 \to 1$
u	$1 \to 5$

$$\int_0^1 \frac{1}{(4x+1)^3}dx = \frac{1}{4}\int_1^5 u^{-3}du = \frac{1}{4}\Big[\frac{1}{-2}u^{-2}\Big]_1^5$$
$$= -\frac{1}{8} \times \Big(\frac{1}{25} - 1\Big) = \frac{3}{25}$$

(4)
$$\int_0^2 \frac{3}{5x+2}dx = \frac{3}{5}\int_0^2 \frac{5}{5x+2}dx$$
$$= \frac{3}{5}\Big[\log|5x+2|\Big]_0^2$$
$$= \frac{3}{5}\Big(\log(12) - \log 2\Big) = \frac{3}{5}\log 6$$

＜ 33 ページ. 定積分の置換積分法 3 ＞

問の解答

(1) $u = x^2 + 2$ とおくと

$\dfrac{du}{dx} = 2x$ より $xdx = \dfrac{1}{2}du$

x	$0 \to 1$
u	$2 \to 3$

より

$$\int_{x=0}^{x=1} x(x^2+2)^3 dx = \int_{u=2}^{u=3} u^3 \frac{1}{2} du = \Big[\ \frac{1}{8}u^4\ \Big]_{u=2}^{u=3}$$

$$= \frac{1}{8}(3^4 - 2^4) = \frac{65}{8}$$

(2) $u = x^2 + 1$ とおくと

$\dfrac{du}{dx} = 2x$ より $xdx = \dfrac{1}{2}du$

x	$0 \to 2$
u	$1 \to 5$

より

$$\int_{x=0}^{x=2} \frac{x}{(x^2+1)^3} dx = \int_{u=1}^{u=5} \frac{1}{u^3} \times \frac{1}{2} du = \Big[\ -\frac{1}{4}u^{-2}\ \Big]_{u=1}^{u=5} = \frac{6}{25}$$

(3) $u = x^3 + 2$ とおくと

$\dfrac{du}{dx} = 3x^2$ より $x^2dx = \dfrac{1}{3}du$

x	$-1 \to 2$
u	$1 \to 10$

より

$$\int_{x=-1}^{x=2} \frac{x^2}{x^3+2} dx = \int_{u=1}^{u=10} \frac{1}{u} \times \frac{1}{3} du = \Big[\ \frac{1}{3}\log|\,u\,|\ \Big]_{u=1}^{u=10} = \frac{1}{3}\log 10$$

(4) $u = x^3 + 1$ とおくと

$\dfrac{du}{dx} = 3x^2$ より $x^2dx = \dfrac{1}{3}du$

x	$-1 \to 1$
u	$0 \to 2$

より

$$\int_{x=-1}^{x=1} x^2 e^{x^3+1} dx = \int_{u=0}^{u=2} e^u \frac{1}{3} du = \Big[\ \frac{1}{3}e^u\ \Big]_{u=0}^{u=2} = \frac{1}{3}e^2 - \frac{1}{3}$$

＜ 34 ページ. 定積分の置換積分法 4 ＞

問の解答

(1) $\displaystyle\int_0^a \sqrt{a^2-x^2}dx = \int_0^{\frac{\pi}{2}} \sqrt{a^2-a^2\sin^2\theta}\ a\cos\theta\ d\theta$

($x = a\sin\theta$ とおく)

$$= \int_0^{\frac{\pi}{2}} a^2\cos^2\theta\ d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left\{ \frac{a^2}{2} + \frac{a^2}{2}\cos(2\theta) \right\} d\theta$$

$$= \left[\frac{a^2}{2}\theta + \frac{a^2}{4}\sin(2\theta)\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}a^2$$

(2) $\displaystyle\int_0^{\sqrt{3}} \sqrt{4-x^2}dx = \int_0^{\frac{\pi}{3}} \sqrt{4-4\sin^2\theta}\ 2\cos\theta\ d\theta$

($x = 2\sin\theta$ とおく)

$$= \int_0^{\frac{\pi}{3}} 4\cos^2\theta\ d\theta$$

$$= \int_0^{\frac{\pi}{3}} \Big\{2 + 2\cos(2\theta)\Big\} d\theta$$

$$= \Big[2\theta + \sin(2\theta)\Big]_0^{\frac{\pi}{3}} = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$$

＜ 35 ページ. 定積分の部分積分法 1 ＞

問の解答

(1) $\displaystyle\int_0^1 x(x-1)^3\ dx = \left[x\frac{(x-1)^4}{4}\right]_0^1 - \int_0^1 \frac{(x-1)^4}{4}\ dx$

$\displaystyle = 0 - 0 - \left[\frac{(x-1)^5}{20}\right]_0^1 = -\frac{1}{20}$

(2) $\displaystyle\int_0^{\pi} x\cos x\ dx = \Big[x\sin x\Big]_0^{\pi} - \int_0^{\pi} \sin x\ dx$

$\displaystyle = 0 - 0 + \Big[\cos x\Big]_0^{\pi} = -2$

(3) $\displaystyle\int_0^{\frac{\pi}{2}} x\sin x\ dx = \Big[x(-\cos x)\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (-\cos x)\ dx$

$\displaystyle = 0 - 0 + \Big[\sin x\Big]_0^{\frac{\pi}{2}} = 1$

(4) $\displaystyle\int_0^1 xe^x\ dx = \Big[xe^x\Big]_0^1 - \int_0^1 e^x\ dx = e - 0 - \Big[e^x\Big]_0^1 = 1$

＜ 36 ページ. 定積分の部分積分法 2 ＞

問の解答

(1) $\displaystyle\int_0^{\pi} \cos(2x)\ dx = \left[\frac{x}{2}\sin(2x)\right]_0^{\pi} - \int_0^{\pi} \frac{1}{2}\sin(2x)\ dx$

$\displaystyle= 0 + \left[\frac{1}{4}\cos(2x)\right]_0^{\pi} = \frac{1}{4} - \frac{1}{4} = 0$

(2) $\displaystyle\int_0^{\frac{\pi}{2}} x\sin(2x)\ dx = \left[-\frac{x}{2}\cos(2x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\frac{1}{2}\cos(2x)\ dx$

$\displaystyle= -\frac{\pi}{4}\cos(\pi) - 0 + \left[\frac{1}{4}\sin(2x)\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

(3) $\displaystyle\int_0^{\frac{\pi}{4}} x\cos(4x)\ dx = \left[\frac{x}{4}\sin(4x)\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{4}\sin(4x)\ dx$

$\displaystyle= 0 + \left[\frac{1}{16}\cos(4x)\right]_0^{\frac{\pi}{4}} = \frac{\pi}{16}\cos(\pi) - \frac{1}{16}\cos 0 = -\frac{1}{8}$

(4) $\displaystyle\int_0^{3} xe^{2x}\ dx = \left[\frac{x}{2}e^{2x}\right]_0^{3} - \int_0^{3} \frac{1}{2}e^{2x}\ dx$

$\displaystyle= \frac{3}{2}e^6 - 0 - \left[\frac{1}{4}e^{2x}\right]_0^{3} = \frac{3}{2}e^6 - \frac{1}{4}(e^6 - 1) = \frac{5}{4}e^6 + \frac{1}{4}$

＜ 37 ページ. 定積分の部分積分法 3 ＞

問の解答

(1) $$\int_1^e x\log x\ dx = \left[\ \frac{x^2}{2}\log x\ \right]_1^e - \int_1^e \frac{x}{2}\ dx$$

$$= \frac{e^2}{2} - 0 - \left[\ \frac{x^2}{4}\ \right]_1^e = \frac{e^2}{4} + \frac{1}{4}$$

(2) $$\int_1^e x^2\log x\ dx = \left[\frac{x^3}{3}\log x\right]_1^e - \int_1^e \frac{x^2}{3}\ dx$$

$$= \frac{e^3}{3} - 0 - \left[\ \frac{x^3}{9}\ \right]_1^e = \frac{2}{9}e^3 + \frac{1}{9}$$

(3) $$\int_1^{\sqrt{e}} x^3\log x\ dx = \left[\ \frac{x^4}{4}\log x\ \right]_1^{\sqrt{e}} - \int_1^{\sqrt{e}} \frac{x^3}{4}\ dx$$

$$= \frac{e^2}{4}\log\sqrt{e} - 0 - \left[\ \frac{x^4}{16}\ \right]_1^{\sqrt{e}}$$

$$= \frac{e^2}{16} + \frac{1}{16}$$

(4) $$\int_1^e \log x\ dx = \int_1^e 1\times\log x\ dx$$

$$= \left[\ x\log x\ \right]_1^e - \int_1^e 1\ dx$$

$$= e - 0 - \left[\ x\ \right]_1^e = 1$$

< 38 ページ. 定積分の部分積分法 4 >

問の解答

(1) $$\int_0^{\pi} x^2 \sin \pi \ dx = \Big[x^2(-\cos x)\Big]_0^{\pi} - \int_0^{\pi} 2x(-\cos x)\ dx$$

$$= -\pi^2 \cos \pi - 0 + \Big[2x \sin x\Big]_0^{\pi} - \int_0^{\pi} 2 \sin x \ dx$$

$$= \pi^2 + 2\pi \sin \pi - 0 + \Big[2 \cos x\Big]_0^{\pi} = \pi^2 - 4$$

(2) $$\int_0^{\frac{\pi}{2}} x^2 \cos x \ dx = \Big[x^2 \sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x \sin x \ dx$$

$$= \frac{\pi^2}{4} - 0 + \Big[2x \cos x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2 \cos x \ dx$$

$$= \frac{\pi^2}{4} + \pi \cos\left(\frac{\pi}{2}\right) - 0 - \Big[2 \sin x\Big]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4} - 2$$

(3) $$\int_0^{1} x^2 e^x \ dx = \Big[x^2 e^x\Big]_0^{1} - \int_0^{1} 2x e^x \ dx$$

$$= 1e^1 - 0 - \left\{\Big[2xe^x\Big]_0^1 - \int_0^1 2e^x \ dx\right\}$$

$$= e - \left\{2e^1 - 0 - \Big[2e^x\Big]_0^1\right\}$$

$$= e - 2e + \left(2e^1 - 2e^0\right) = e - 2$$

＜ 39 ページ. 定積分の練習 1 ＞

問の解答

(1) $\Big[\ x\ \Big]_{-1}^{3} = 4$

(2) $\Big[\ \log|x|\ \Big]_{1}^{\sqrt{e}} = \dfrac{1}{2}$

(3) $\Big[\ \dfrac{3}{4}x^{\frac{4}{3}}\ \Big]_{0}^{1} = \dfrac{3}{4}$

(4) $\Big[\ -3\cos x - 4\sin x\ \Big]_{0}^{\pi} = 6$

(5) $\Big[\ 3x - 4\log|x| - \dfrac{1}{x}\ \Big]_{1}^{2} = \dfrac{7}{2} - 4\log 2$

(6) $\Big[\ 2\sqrt{x}\ \Big]_{1}^{9} = 4$

(7) $\Big[\ \tan x\ \Big]_{-\frac{\pi}{3}}^{\frac{\pi}{4}} = 1 + \sqrt{3}$

(8) $\Big[\ \dfrac{1}{3}\log|3x+1|\ \Big]_{0}^{2} = \dfrac{1}{3}\log 7$

(9) $\left[\ \dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right|\ \right]_{2}^{3} = \dfrac{1}{2}\log\left(\dfrac{3}{2}\right)$

(10) $\displaystyle\int_{0}^{\frac{\pi}{2}}\left\{\frac{1}{2} - \frac{1}{2}\cos(2x)\right\}dx = \Big[\ \frac{1}{2}x - \frac{1}{4}\sin(2x)\ \Big]_{0}^{\frac{\pi}{2}} = \frac{\pi}{4}$

(11) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left\{\frac{1}{2} + \frac{1}{2}\cos(4x)\right\}dx = \Big[\ \frac{1}{2}x + \frac{1}{8}\sin(4x)\ \Big]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{4}$

(12) $\Big[\ \dfrac{1}{3}e^{3x-1}\ \Big]_{-2}^{2} = \dfrac{1}{3}e^{5} - \dfrac{1}{3}e^{-7}$

(13) $\Big[\ -\dfrac{1}{2}e^{-x^2}\ \Big]_{0}^{1} = \dfrac{1}{2} - \dfrac{1}{2e}$

＜ 40 ページ. 定積分の練習 2 ＞

問の解答

(1) $\displaystyle\int_1^4 \frac{1}{u^5}\times\frac{1}{3}\ du = \Big[-\frac{1}{12}u^{-4} \Big]_1^4 = \frac{85}{1024}$

$(u = 3x+1$ とおく$)$

(2) $\displaystyle\int_4^{49} \sqrt{u}\times\frac{1}{5}\ du = \Big[\frac{2}{15}u^{\frac{3}{2}} \Big]_4^{49} = \frac{134}{3}$

$(u = 5x-1$ とおく$)$

(3) $\displaystyle\int_1^2 \frac{1}{u^4}\times\frac{1}{3}\ du = \Big[-\frac{1}{9}u^{-3} \Big]_1^2 = \frac{7}{72}$

$(u = x^3+1$ とおく$)$

(4) $\displaystyle\int_1^2 \frac{3}{u}\times\frac{1}{2}\ du = \Big[\frac{3}{2}\log|u| \Big]_1^2 = \frac{3}{2}\log 2$

$(u = x^2+1$ とおく$)$

(5) $\displaystyle\Big[(3x+2)\sin x \Big]_0^{\pi} - \int_0^{\pi} 3\sin x\ dx = (3\pi+2)\sin\pi - 2\sin 0 + \Big[3\cos x \Big]_0^{\pi} = -6$

(6) $\displaystyle\Big[x\times(-\frac{1}{3}\cos 3x) \Big]_0^{\pi} - \int_0^{\pi} 1\times(-\frac{1}{3}\cos 3x)\ dx = -\frac{\pi}{3}\cos 3\pi - 0 + \Big[\frac{1}{9}\sin 3x \Big]_0^{\pi} = \frac{\pi}{3}$

(7) $\displaystyle\Big[x\log x \Big]_1^4 - \int_1^4 1\ dx = 4\log 4 - 1\log 1 - \Big[x \Big]_1^4 = 4\log 4 - 3$

(8) $\displaystyle\Big[(2x+1)\,e^x \Big]_{-1}^{1} - \int_{-1}^{1} 2e^x\ dx = 3e^1 - (-1)e^{-1} - \Big[2e^x \Big]_{-1}^{1} = 3e + \frac{1}{e} - 2e + \frac{2}{e} = e + \frac{3}{e}$

(9) $\displaystyle\Big[x^2\sin x \Big]_0^{\pi} - \int_0^{\pi} 2x\sin x\ dx$

$\displaystyle= \pi^2\sin\pi - 0 + \Big[2x\cos x \Big]_0^{\pi} - \int_0^{\pi} 2\cos x\ dx$

$\displaystyle= 2\pi\cos\pi - 0 - \Big[2\sin x \Big]_0^{\pi} = -2\pi$

＜ 42 ページ. 面積 2 ＞

問の解答

(1) $\displaystyle\int_0^1 e^x dx = \Big[\, e^x \,\Big]_0^1 = e - 1$

(2) $\displaystyle\int_1^9 \sqrt{x}\, dx = \Big[\, \frac{2}{3} x^{\frac{3}{2}} \,\Big]_1^9 = \frac{2}{3}(27 - 1) = \frac{52}{3}$

(3) $\displaystyle\int_1^2 \frac{1}{x^2} dx = \Big[\, -\frac{1}{x} \,\Big]_1^2 = \Big(-\frac{1}{2} - (-1) \Big) = \frac{1}{2}$

(4) $\displaystyle\int_1^2 \frac{1}{x} dx = \Big[\, \log|\, x \,| \,\Big]_1^2 = \log 2 - \log 1 = \log 2$

< 43 ページ. 面積 3 >

問 1 の解答

$$S=\int_a^b\{0-g(x)\}dx=-\int_a^b g(x)dx$$

問 2 の解答

$$S=\int_{\frac{\pi}{4}}^{\frac{5}{4}\pi}(\sin x-\cos x)dx=\Big[-\cos x-\sin x\Big]_{\frac{\pi}{4}}^{\frac{5}{4}\pi}$$

$$=2\sqrt{2}$$

問 3 の解答

(1) $\displaystyle\int_0^1\left(\sqrt{x}-x^2\right)dx=\left[\frac{2}{3}x^{\frac{3}{2}}-\frac{x^3}{3}\right]_0^1=\frac{1}{3}$

(2) $\displaystyle\int_1^4\left(-\frac{1}{4}x+\frac{5}{4}-\frac{1}{x}\right)dx=\left[-\frac{x^2}{8}+\frac{5}{4}x-\log|x|\right]_1^4$

$$=\frac{15}{8}-\log 4$$

< 44 ページ. 面積 4 >

問 1 の解答

求める面積を S とおくと

$$\frac{S}{4} = \int_0^a \sqrt{a^2 - x^2}\ dx = \int_0^{\frac{\pi}{2}} a^2 \cos^2\theta\ d\theta$$

$(x = a\sin\theta$ とおく$)$

$$= \int_0^{\frac{\pi}{2}} \left\{ \frac{a^2}{2} + \frac{a^2}{2}\cos(2\theta) \right\} d\theta = \left[\frac{a^2}{2}\theta + \frac{a^2}{4}\sin(2\theta) \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}a^2$$

よって $S = \left(\frac{\pi}{4}a^2 \right) \times 4 = \pi a^2$ (答) $\underline{S = \pi a^2}$

問 2 の解答

$$S = \int_0^1 \sqrt{4 - x^2}\ dx \qquad (x = 2\sin\theta \text{ とおく})$$

$$= \int_0^{\frac{\pi}{6}} \sqrt{4 - 4\sin^2\theta}\ 2\cos\theta\ d\theta = \int_0^{\frac{\pi}{6}} 4\cos^2\theta\ d\theta$$

$$= \int_0^{\frac{\pi}{6}} \left\{ 2 + 2\cos(2\theta) \right\} d\theta = \left[2\theta + \sin(2\theta) \right]_0^{\frac{\pi}{6}} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

＜ 45 ページ. 偶関数・奇関数の定積分 ＞

問の解答

(1) $\displaystyle\int_{-2}^{2}\left(x^3+x^4+x^5\right)dx = 2\int_0^2 x^4\,dx = 2\left[\frac{x^5}{5}\right]_0^2 = \frac{64}{5}$

(2) $\displaystyle\int_{-1}^{1}\left(x+x^3+x^6\right)dx = 2\int_0^1 x^6\,dx = 2\left[\frac{x^7}{7}\right]_0^1 = \frac{2}{7}$

(3) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin x+\cos x)\,dx = 2\int_0^{\frac{\pi}{2}}\cos x\,dx = 2\big[\sin x\big]_0^{\frac{\pi}{2}} = 2$

(4) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{1}{\cos^2 x}+\tan x\right)dx = 2\int_0^{\frac{\pi}{4}}\frac{1}{\cos^2 x}dx = 2\big[\tan x\big]_0^{\frac{\pi}{4}} = 2$

＜ 46 ページ. 定積分の応用問題 ＞

問 1 の解答

(1) $\displaystyle\int_{-1}^{1}(x+x^2+x^3+x^4+x^5)dx = 2\int_{0}^{1}(x^2+x^4)dx$

$\displaystyle = 2\left[\,\frac{x^3}{3}+\frac{x^5}{5}\,\right]_0^1 = 2\left(\frac{1}{3}+\frac{1}{5}\right) = \frac{16}{15}$

(2) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\sin x+\cos x+\tan x+\frac{1}{\cos^2 x}\right)dx$

$\displaystyle = 2\int_{0}^{\frac{\pi}{4}}\left(\cos x+\frac{1}{\cos^2 x}\right)dx = 2\left[\,\sin x+\tan x\,\right]_0^{\frac{\pi}{4}}$

$\displaystyle = 2\left(\frac{\sqrt{2}}{2}+1-0\right) = \sqrt{2}+2$

問 2 の解答

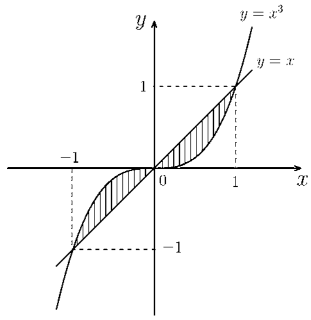
(1) $\displaystyle\int_{1}^{4}\frac{1}{\sqrt{x}}dx = \left[\,2\sqrt{x}\,\right]_1^4 = 2\sqrt{4}-2\sqrt{1} = 2$

(2) $\displaystyle\int_{-1}^{2}\left\{(-x^2+3)-(x^2-2x-1)\right\}dx = \left[-\frac{2}{3}x^3+x^2+4x\right]_{-1}^{2} = 9$

(3) $\displaystyle\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}(x-x^3)dx$

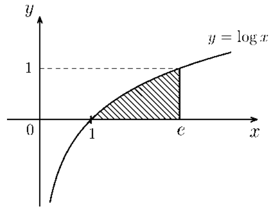
$\displaystyle = \left[\,\frac{x^4}{4}-\frac{x^2}{2}\,\right]_{-1}^{0}+\left[\,\frac{x^2}{2}-\frac{x^4}{4}\,\right]_0^1$

$\displaystyle = 0-\left(\frac{1}{4}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)-0 = \frac{1}{2}$



(4) $\displaystyle\int_{1}^{e}\log x dx = \left[\,x\log x\,\right]_1^e-\int_{1}^{e}x\times\frac{1}{x}dx$

$\displaystyle = e\log e-1\log 1-\left[\,x\,\right]_1^e = e-(e-1) = 1$



< 47 ページ. 関数の極限 >

問の解答

(1) $\displaystyle\lim_{x\to 1}\frac{x^2-16}{x-4}$

$\displaystyle=\frac{1-16}{1-4}=5$

(2) $\displaystyle\lim_{x\to 4}\frac{x^2-16}{x-4}$

$\displaystyle=\lim_{x\to 4}\frac{(x-4)(x+4)}{x-4}=\lim_{x\to 4}(x+4)=8$

(3) $\displaystyle\lim_{x\to 2}\frac{x^3-27}{x-3}$

$\displaystyle=\frac{8-27}{2-3}=19$

(4) $\displaystyle\lim_{x\to 3}\frac{x^3-27}{x-3}$

$\displaystyle=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{x-3}=\lim_{x\to 3}(x^2+3x+9)=27$

(5) $\displaystyle\lim_{x\to 2}\frac{x^4-16}{x-2}$

$\displaystyle=\lim_{x\to 2}\frac{(x-2)(x^3+2x^2+4x+8)}{(x-2)}=\lim_{x\to 2}(x^3+2x^2+4x+8)=32$

(6) $\displaystyle\lim_{x\to 1}\frac{x^5-1}{x-1}$

$\displaystyle=\lim_{x\to 1}\frac{(x-1)(x^4+x^3+x^2+x+1)}{(x-1)}=\lim_{x\to 1}(x^4+x^3+x^2+x+1)=5$

＜ 48 ページ. ロピタルの定理 1 ＞

問の解答

(1) $\displaystyle\lim_{x\to 2}\frac{x^4-16}{x-2}=\lim_{x\to 2}\frac{4x^3-0}{1-0}=4\times 2^3=32$

(2) $\displaystyle\lim_{x\to 1}\frac{x^7-1}{x-1}=\lim_{x\to 1}\frac{7x^6-0}{1-0}=7$

(3) $\displaystyle\lim_{x\to 1}\frac{e^x-e}{x-1}=\lim_{x\to 1}\frac{e^x-0}{1-0}=e^1=e$

(4) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\cos x}{1}=\cos 0=1$

< 49 ページ. ロピタルの定理 2 >

問の解答

(1) $\displaystyle \lim_{x\to 2}\frac{x^5-32-80(x-2)}{(x-1)^2}=\lim_{x\to 2}\frac{5x^4-80}{2(x-2)}=\lim_{x\to 2}\frac{20x^3}{2}=80$

(2) $\displaystyle \lim_{x\to 1}\frac{\sqrt{x}-1-\frac{1}{2}(x-1)}{(x-1)^2}=\lim_{x\to 1}\frac{\frac{1}{2\sqrt{x}}-\frac{1}{2}}{2(x-1)}=\lim_{x\to 1}\frac{-\frac{1}{4x\sqrt{x}}}{2}=-\frac{1}{8}$

(3) $\displaystyle \lim_{x\to e}\frac{\log x-1-\frac{1}{2}(x-e)}{(x-e)^2}=\lim_{x\to e}\frac{\frac{1}{x}-\frac{1}{e}}{2(x-e)}=\lim_{x\to e}\frac{-\frac{1}{x^2}}{2}=-\frac{1}{2e^2}$

(4) $\displaystyle \lim_{x\to 0}\frac{\sin x-x}{x^2}=\lim_{x\to 0}\frac{\cos x-1}{2x}=\lim_{x\to 0}\frac{-\sin x}{2}=0$

(5) $\displaystyle \lim_{x\to 0}\frac{\cos x-1+\frac{1}{2}x^2}{x^3}=\lim_{x\to 0}\frac{-\sin x+x}{3x^2}=\lim_{x\to 0}\frac{-\cos x+1}{6x}=\lim_{x\to 0}\frac{\sin x}{6}=0$

(6) $\displaystyle \lim_{x\to 0}\frac{e^x-1-x-\frac{1}{2}x^2}{x^3}=\lim_{x\to 0}\frac{e^x-1-x}{3x^2}=\lim_{x\to 0}\frac{e^x-1}{6x}=\lim_{x\to 0}\frac{e^x}{6}=\frac{1}{6}$

＜ 50 ページ. ロピタルの定理 3 ＞

問の解答

(1) $$\lim_{x\to a}\frac{x^n-a^n-na^{n-1}(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{nx^{n-1}-na^{n-1}}{2(x-a)}=\lim_{x\to a}\frac{n(n-1)x^{n-2}}{2}=\frac{n(n-1)}{2}a^{n-2}$$

(2) $$\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}-\frac{1}{2\sqrt{a}}(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{\frac{1}{2\sqrt{x}}-\frac{1}{2\sqrt{a}}}{2(x-a)}=\lim_{x\to a}\frac{-\frac{1}{4x\sqrt{x}}}{2}=-\frac{1}{8a\sqrt{a}}$$

(3) $$\lim_{x\to a}\frac{\log x-\log a-\frac{1}{a}(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{\frac{1}{x}-\frac{1}{a}}{2(x-a)}=\lim_{x\to a}\frac{-\frac{1}{x^2}}{2}=-\frac{1}{2a^2}$$

(4) $$\lim_{x\to a}\frac{\sin x-\sin a-(\cos a)(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{\cos x-\cos a}{2(x-a)}=\lim_{x\to a}\frac{-\sin x}{2}=-\frac{\sin a}{2}$$

(5) $$\lim_{x\to a}\frac{\cos x-\cos a+(\sin a)(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{-\sin x+\sin a}{2(x-a)}=\lim_{x\to a}\frac{-\cos x}{2}=-\frac{\cos a}{2}$$

(6) $$\lim_{x\to a}\frac{e^x-e^a-e^a(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{e^x-e^a}{2(x-a)}=\lim_{x\to a}\frac{e^x}{2}=\frac{e^a}{2}$$

＜ 51 ページ. 高階導関数 ＞

問の解答

(1) $f(x) = x^5$

$f^{(1)}(x) = 5x^4$

$f^{(2)}(x) = 20x^3$

$f^{(3)}(x) = 60x^2$

$f^{(4)}(x) = 120x$

(2) $f(x) = e^x$

$f^{(1)}(x) = e^x$

$f^{(2)}(x) = e^x$

$f^{(3)}(x) = e^x$

$f^{(4)}(x) = e^x$

< 52ページ. 高階微分係数 >

問1の解答

$f^{(1)}(x) = \cos x \qquad f^{(2)}(x) = -\sin x \qquad f^{(3)}(x) = -\cos x \qquad f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x \qquad f^{(6)}(x) = -\sin x \qquad f^{(7)}(x) = -\cos x \qquad f^{(8)}(x) = \sin x$

$f^{(1)}(0) = 1 \qquad f^{(2)}(0) = 0 \qquad f^{(3)}(0) = -1 \qquad f^{(4)}(0) = 0$

$f^{(5)}(0) = 1 \qquad f^{(6)}(0) = 0 \qquad f^{(7)}(0) = -1 \qquad f^{(8)}(0) = 0$

問2の解答

$f^{(1)}(x) = -\sin x \qquad f^{(2)}(x) = -\cos x \qquad f^{(3)}(x) = \sin x \qquad f^{(4)}(x) = \cos x$

$f^{(5)}(x) = -\sin x \qquad f^{(6)}(x) = -\cos x \qquad f^{(7)}(x) = \sin x \qquad f^{(8)}(x) = \cos x$

$f^{(1)}(0) = 0 \qquad f^{(2)}(0) = -1 \qquad f^{(3)}(0) = 0 \qquad f^{(4)}(0) = 1$

$f^{(5)}(0) = 0 \qquad f^{(6)}(0) = -1 \qquad f^{(7)}(0) = 0 \qquad f^{(8)}(0) = 1$

問3の解答

$f^{(n)}(x) = e^x \qquad f^{(n)}(0) = e^0 = 1$

＜ 54 ページ. 関数の 1 次近似 ＞

問の解答

(1) $f'(x) = \dfrac{1}{2\sqrt{x}}$ より (答) $x \fallingdotseq a$ のとき $\sqrt{x} \fallingdotseq \sqrt{a} + \dfrac{1}{2\sqrt{a}}(x-a)$

(2) $f'(x) = \dfrac{1}{4\sqrt[4]{x^3}}$ より (答) $x \fallingdotseq a$ のとき $\sqrt[4]{x} \fallingdotseq \sqrt[4]{a} + \dfrac{1}{4\sqrt[4]{a^3}}(x-a)$

(3) $f'(x) = \dfrac{1}{x}$ より (答) $x \fallingdotseq a$ のとき $\log x \fallingdotseq \log a + \dfrac{1}{a}(x-a)$

(4) $f'(x) = \cos x$ より (答) $x \fallingdotseq a$ のとき $\sin x \fallingdotseq \sin a + (\cos a)(x-a)$

(5) $f'(x) = -\sin x$ より (答) $x \fallingdotseq a$ のとき $\cos x \fallingdotseq \cos a - (\sin a)(x-a)$

(6) $f'(x) = e^x$ より (答) $x \fallingdotseq a$ のとき $e^x \fallingdotseq e^a + e^a(x-a)$

< 55 ページ.1 次近似値 >

問の解答

(1) $\sqrt{4.1} \fallingdotseq \sqrt{4} + \dfrac{1}{2\sqrt{4}}(4.1-4) = 2 + \dfrac{1}{4} \times 0.1 = \underline{2.025}$

(2) $\sqrt[4]{16.1} \fallingdotseq \sqrt{16} + \dfrac{1}{4\left(\sqrt[4]{16}\right)^3}(16.1-16) = 2 + \dfrac{1}{4 \times 2^3} \times 0.1 = \underline{2.003125}$

(3) $\log 1.1 \fallingdotseq \log 1 + \dfrac{1}{1}(1.1-1) = 0 + 0.1 = \underline{0.1}$

＜ 56 ページ. 関数の 2 次近似 ＞

問の解答

(1) $x \fallingdotseq a$ のとき $\quad x^n \fallingdotseq a^n + na^{n-1}(x-a) + \dfrac{n(n-1)}{2}a^{n-2}(x-a)^2$

(2) $x \fallingdotseq a$ のとき $\quad \sqrt{x} \fallingdotseq \sqrt{a} + \dfrac{1}{2\sqrt{a}}(x-a) - \dfrac{1}{8a\sqrt{a}}(x-a)^2$

(3) $x \fallingdotseq a$ のとき $\quad \log x \fallingdotseq \log a + \dfrac{1}{a}(x-a) - \dfrac{1}{2a^2}(x-a)^2$

(4) $x \fallingdotseq a$ のとき $\quad \sin x \fallingdotseq \sin a + (\cos a)(x-a) - \dfrac{\sin a}{2}(x-a)^2$

(5) $x \fallingdotseq a$ のとき $\quad \cos x \fallingdotseq \cos a - (\sin a)(x-a) - \dfrac{\cos a}{2}(x-a)^2$

(6) $x \fallingdotseq a$ のとき $\quad e^x \fallingdotseq e^a + e^a(x-a) + \dfrac{e^a}{2}(x-a)^2$

< 57 ページ. テーラー展開 >

問 1 の解答

$$e^x = e^a + e^a(x-a) + \frac{e^a}{2!}(x-a)^2 + \frac{e^a}{3!}(x-a)^3 + \cdots + \frac{e^a}{n!}(x-a)^n + \cdots$$

問 2 の解答

$$f(a) = A_0 \quad f^{(1)}(a) = A_1 \quad f^{(2)}(a) = 2!A_2 \quad f^{(3)}(a) = 3!A_3 \quad f^{(n)}(a) = n!A_n$$

$$A_0 = f(a) \quad A_1 = f^{(1)}(a) \quad A_2 = \frac{f^{(2)}(a)}{2!} \quad A_3 = \frac{f^{(3)}(a)}{3!} \quad A_n = \frac{f^{(n)}(a)}{n!}$$

＜ 58 ページ. マクローリン展開 1 ＞

問 1 の解答

$f(0) = 1 \quad f^{(n)}(0) = 1$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

問 2 の解答

$f(0) = 0 \quad f^{(1)}(0) = 1 \quad f^{(2)}(0) = 0 \quad f^{(3)}(0) = -1 \quad f^{(4)}(0) = 0$

$f^{(5)}(0) = 1 \quad f^{(6)}(0) = 0 \quad f^{(7)}(0) = -1 \quad f^{(8)}(0) = 0$

$f^{(9)}(0) = 1 \quad f^{(10)}(0) = 0 \quad f^{(11)}(0) = -1 \quad f^{(12)}(0) = 0$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots$$

問 3 の解答

$f(0) = 1 \quad f^{(1)}(0) = 0 \quad f^{(2)}(0) = -1 \quad f^{(3)}(0) = 0 \quad f^{(4)}(0) = 1$

$f^{(5)}(0) = 0 \quad f^{(6)}(0) = -1 \quad f^{(7)}(0) = 0 \quad f^{(8)}(0) = 1$

$f^{(9)}(0) = 0 \quad f^{(10)}(0) = -1 \quad f^{(11)}(0) = 0 \quad f^{(12)}(0) = 1$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$$

< 61 ページ. 練習問題 >

問 1 の解答

(1) $\displaystyle\lim_{x\to 1}\frac{x^6-64}{x-2}=\frac{1-64}{1-2}=63$

(2) $\displaystyle\lim_{x\to 2}\frac{x^6-64}{x-2}=\lim_{x\to 2}\frac{6x^5}{1}=192$

(3) $\displaystyle\lim_{x\to 0}\frac{\cos x-1}{x^2}=\lim_{x\to 0}\frac{-\sin x}{2x}=\lim_{x\to 0}\frac{-\cos x}{2}=-\frac{1}{2}$

(4) $\displaystyle\lim_{x\to a}\frac{e^x-e^a-e^a(x-a)-\frac{1}{2}e^a(x-a)^2}{(x-a)^3}=\lim_{x\to a}\frac{e^x-e^a-e^a(x-a)}{3(x-a)^2}$

$\displaystyle=\lim_{x\to a}\frac{e^x-e^a}{6(x-a)}=\lim_{x\to a}\frac{e^x}{6}=\frac{e^a}{6}$

問 2 の解答

$$e^x=e^a+e^a(x-a)+\frac{e^a}{2!}(x-a)^2+\frac{e^a}{3!}(x-a)^n+\cdots+\frac{e^a}{n!}(x-a)^n+\cdots$$

問 3 の解答

(1) $x\fallingdotseq a$ のとき $\displaystyle f(x)\fallingdotseq f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2$

(2) $x\fallingdotseq a$ のとき $\displaystyle\sqrt{x}\fallingdotseq\sqrt{a}+\frac{1}{2\sqrt{a}}(x-a)$

(3) $x\fallingdotseq a$ のとき $\displaystyle\log x\fallingdotseq\log a+\frac{1}{a}(x-a)$

(4) $x\fallingdotseq a$ のとき $\displaystyle\sin x\fallingdotseq\sin a+(\cos a)(x-a)+\frac{\sin a}{2}(x-a)^2$

問 4 の解答

(1) $\displaystyle\sqrt{16.1}\fallingdotseq\sqrt{16}+\frac{1}{2\sqrt{16}}(16.1-16)=4+\frac{1}{8}\times 0.1=4.0125$

(2) $\displaystyle\log 1.05\fallingdotseq\log 1+\frac{1}{1}(1.05-1)=0+0.05=0.05$

問 5 の解答

(1) $\displaystyle\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\frac{x^{11}}{11!}+\cdots$

(2) $\displaystyle\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\frac{x^{10}}{10!}+\cdots$