高知工科大学
Kochi University of Technology

数学 3

（2005年度版）

解答

< 1ページ. 複素数の定義 >

問の解答

(1) $a = \dfrac{1}{2}$, $b = \dfrac{3}{2}$

(2) $a = 0$, $b = \dfrac{1 - \sqrt{2}}{3}$

< 2 ページ. 複素数の四則演算 (1) >

問 1 の解答

(1) 5

(2) $-1+2i$

(3) $0.88-i$

(4) $\dfrac{1}{8}$

(5) $\sqrt{3}+1-3i$

(6) $-\dfrac{1}{12}-(\sqrt{2}+\sqrt{3})i$

問 2 の解答

(1) $12+3i$

(2) $\dfrac{3}{2}-3i$

(3) $10-2i$

(4) $\dfrac{4}{3}-5i$

< 3ページ. 複素数の四則演算 (2) >

問の解答

(1) $-i$

(2) 1

(3) i

(4) -1

(5) $-i$

(6) 1

(7) 2

(8) 7

(9) $\dfrac{3-i^2}{4}=1$

(10) $-2i$

(11) $2i$

(12) $8-8i-6i^2=14-8i$

(13) $-3-11i$

(14) $18-26i$

< 4 ページ. 複素数の四則演算 (3) >

問の解答

(1) $\dfrac{-1+i}{2}$

(2) $\dfrac{-1-i}{2}$

(3) $\dfrac{1-i}{2}$

(4) $\dfrac{\sqrt{5}+i}{2}$

(5) $\dfrac{3-\sqrt{5}i}{2}$

(6) $\dfrac{-1-i}{2}$

(7) $\dfrac{1+\sqrt{3}i}{4}$

(8) $\dfrac{2+\sqrt{2}i}{3}$

(9) $\dfrac{1+2\sqrt{2}i}{9}$

(10) $-\dfrac{i}{4}$

< 5 ページ. 負の数の平方根 >

問の解答

(1) $2\sqrt{15}i$

(2) $-2\sqrt{15}i$

(3) $-\sqrt{3}i$

(4) $\sqrt{3}i$

＜ 6 ページ.2 次方程式 ＞

問の解答

(1) $x = \dfrac{-1 \pm \sqrt{7}i}{2}$

(2) $x = \dfrac{-3 \pm \sqrt{23}i}{2}$

(3) $x = \dfrac{5 \pm \sqrt{23}i}{6}$

< 7ページ. 高次方程式 >

問の解答

(1) $x = 1,\ x = \dfrac{-1 \pm \sqrt{3}i}{2}$

(2) $x = -2,\ x = 1 \pm \sqrt{3}i$

(3) $x = \pm 2,\ x = \pm 2i$

＜ 8 ページ. 共役複素数 ＞

問 1 の解答

(1) 1

(2) $-i$

(3) $1+i$

(4) $\dfrac{1-i}{2}$

問 2 の解答

(1) 4

(2) 3

(3) 25

問 3 の解答

(1) a

(2) b

(3) a^2+b^2

＜ 9 ページ. 絶対値 ＞

問 1 の解答

(1) 1

(2) 7

(3) 5

(4) $\dfrac{\sqrt{2}}{2}$

問 2 の解答

(1) $|z|^2 = 4^2 + (-3)^2 = 25$

$z^2 = 7 - 24i$

$|z^2| = \sqrt{7^2 + (-24)^2} = \sqrt{625} = 25$

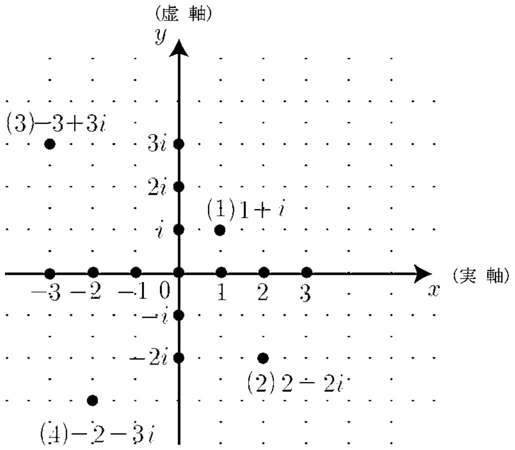
(2) $|z|^2 = 1^2 + 1^2 = 2$

$z^2 = (1 + i)^2 = 2i$

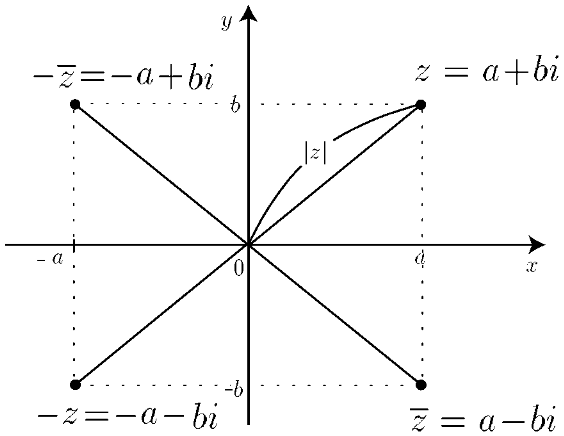
$|z^2| = \sqrt{2^2} = 2$

＜ 10ページ. 複素平面 (1) ＞

問１の解答



問２の解答



＜ 11 ページ. 複素平面 (2) ＞

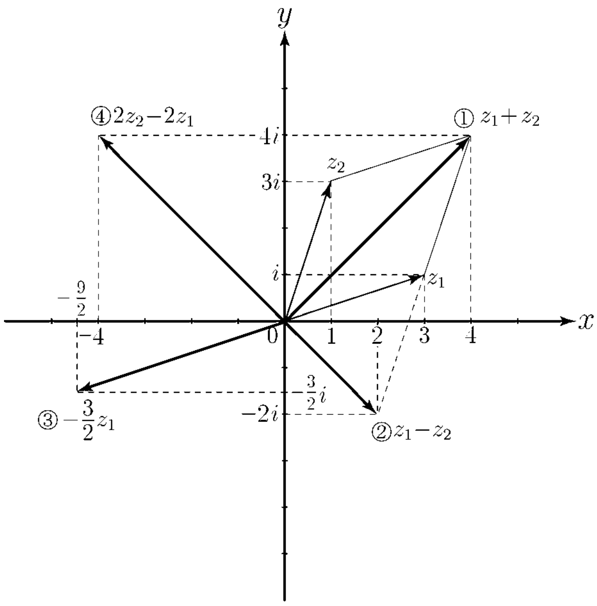
問の解答

① $4+4i$

② $2-2i$

③ $-\dfrac{9}{2}-\dfrac{3}{2}i$

④ $-4+4i$



＜ 12 ページ. 複素数の i 倍 ＞

問の解答

(1) $z = 1 + i$

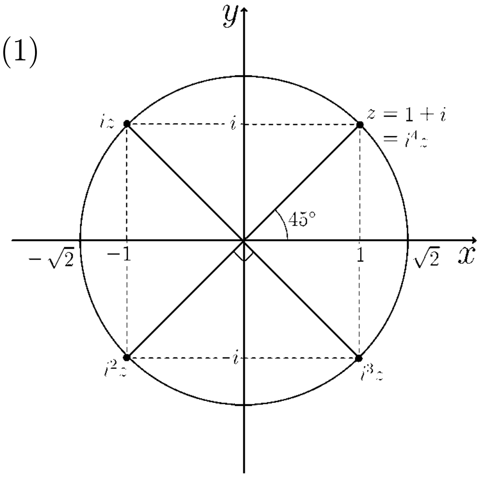
$iz = -1 + i$

$i^2 z = -1 - i$

$i^3 z = 1 - i$

$i^4 z = 1 + i$

(1)



(2) $z = 1 + \sqrt{3}i$

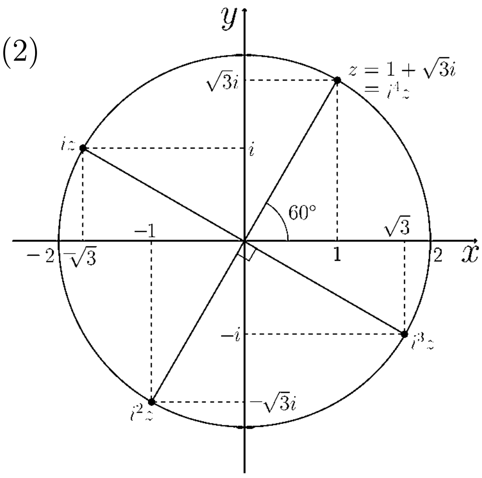
$iz = -\sqrt{3} + i$

$i^2 z = -1 - \sqrt{3}i$

$i^3 z = \sqrt{3} - i$

$i^4 z = 1 + \sqrt{3}i$

(2)



< 13 ページ. 絶対値 1 の複素数 >

問の解答

(1) $\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$,
(2) $\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)=\frac{1}{2}+\frac{\sqrt{3}}{2}i$,
(3) $\cos\left(\frac{2}{3}\pi\right)+i\sin\left(\frac{2}{3}\pi\right)=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

(4) $\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$,
(5) $\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$,
(6) $\cos(\pi)+i\sin(\pi)=-1$

(7) $\cos\left(\frac{7}{6}\pi\right)+i\sin\left(\frac{7}{6}\pi\right)=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$,
(8) $\cos\left(\frac{5}{4}\pi\right)+i\sin\left(\frac{5}{4}\pi\right)=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$,
(9) $\cos\left(\frac{4}{3}\pi\right)+i\sin\left(\frac{4}{3}\pi\right)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

(10) $\cos\left(\frac{3}{2}\pi\right)+i\sin\left(\frac{3}{2}\pi\right)=-i$,
(11) $\cos\left(\frac{5}{3}\pi\right)+i\sin\left(\frac{5}{3}\pi\right)=\frac{1}{2}-\frac{\sqrt{3}}{2}i$,
(12) $\cos\left(\frac{7}{4}\pi\right)+i\sin\left(\frac{7}{4}\pi\right)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

< 14 ページ. 極形式 (1) >

問の解答

(1) $4i = 4\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

(2) $-2 = 2(\cos\pi + i\sin\pi)$

(3) $$-\sqrt{2}i = \sqrt{2}\left(\cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right)\right)$$

$$= \sqrt{2}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$$

< 15 ページ. 極形式 (2) >

問の解答

(1) $z = 1 + i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(2) $z = -1 + \sqrt{3}i = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

(3) $z = -\sqrt{6} - \sqrt{6}i = \sqrt{12}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = 2\sqrt{3}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$$= 2\sqrt{3}\left(\cos(-\frac{3\pi}{4}) + i\sin(-\frac{3\pi}{4})\right)$$

(4) $z = -3 - \sqrt{3}i = \sqrt{12}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2\sqrt{3}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

$$= 2\sqrt{3}\left(\cos(-\frac{5\pi}{6}) + i\sin(-\frac{5\pi}{6})\right)$$

(5) $z = \sqrt{6} - \sqrt{2}i = \sqrt{8}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2\sqrt{2}\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

$$= 2\sqrt{2}\left(\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})\right)$$

< 16 ページ. 複素数の積 >

問の解答

(1) $\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)z=\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)r(\cos\theta+i\sin\theta)$

$=r\left(\cos\left(\theta+\dfrac{\pi}{3}\right)+i\sin\left(\theta+\dfrac{\pi}{3}\right)\right)$

原点を中心として反時計まわりに $\dfrac{\pi}{3}(=60^\circ)$ 回転する

(2) $\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}i\right)z=\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)r(\cos\theta+i\sin\theta)$

$=r\left(\cos\left(\theta+\dfrac{\pi}{4}\right)+i\sin\left(\theta+\dfrac{\pi}{4}\right)\right)$

原点を中心として反時計まわりに $\dfrac{\pi}{4}(=45^\circ)$ 回転する

(3) $\left(-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)z=r\left(\cos\left(\theta+\dfrac{5\pi}{6}\right)+i\sin\left(\theta+\dfrac{5\pi}{6}\right)\right)$

原点を中心として $\dfrac{5\pi}{6}(=150^\circ)$ 回転する。

＜ 17 ページ. 複素数の商 ＞

問の解答

(1) $$\frac{1+\sqrt{3}i}{\sqrt{3}+i} = \frac{2(\frac{1}{2}+\frac{\sqrt{3}}{2}i)}{2(\frac{\sqrt{3}}{2}+\frac{1}{2}i)} = \frac{\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)} = \cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)$$

(2) $$\frac{-2+2i}{1+i} = \frac{2\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})}{\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})} = 2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$$

(3) $$\frac{-1-i}{-\sqrt{3}+i} = \frac{\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)}{2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)} = \frac{\sqrt{2}}{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$$

< 18ページ. ド・モアブルの定理 >

問の解答

(1) $$(-\sqrt{3}+i)^3 = \left(2\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\right)^3 = 2^3\left(\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)\right)^3$$

$$= 8\left(\cos\left(\frac{5}{2}\pi\right)+i\sin\left(\frac{5}{2}\pi\right)\right) = 8i$$

(2) $$\left(\frac{-1+\sqrt{3}i}{2}\right)^6 = \left(\cos\left(\frac{2}{3}\pi\right)+i\sin\left(\frac{2}{3}\pi\right)\right)^6$$

$$= \cos(4\pi)+i\sin(4\pi) = 1$$

(3) $$(1-i)^4 = \left\{\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)\right\}^4$$

$$= \left(\sqrt{2}\right)^4\left\{\cos(-\pi)+i\sin(-\pi)\right\} = -4$$

(4) $$\left(\frac{-1+i}{\sqrt{3}+i}\right)^{12} = \left(\frac{\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)}{2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}\right)^{12} = \left(\frac{\sqrt{2}}{2}\right)^{12}\times\left(\frac{\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)}{\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)}\right)^{12}$$

$$= \left(\frac{1}{\sqrt{2}}\right)^{12}\times\left(\cos\left(\frac{7}{12}\pi\right)+i\sin\left(\frac{7}{12}\pi\right)\right)^{12}$$

$$= \frac{1}{2^6}\times(\cos(7\pi)+i\sin(7\pi)) = -\frac{1}{64}$$

＜ 19 ページ.1 の累乗根 ＞

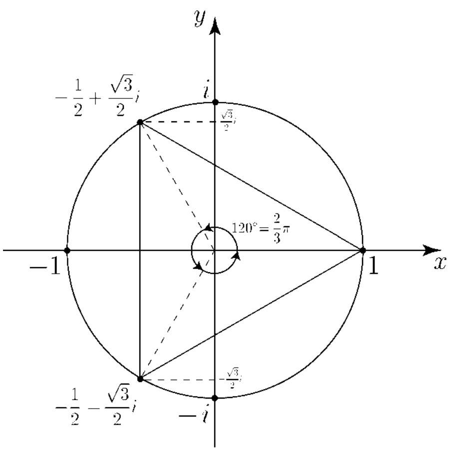
問の解答

(1) $z^3 = 1$

$\cos(3\theta) + i\sin(3\theta) = 1$

$\theta = 0\ ,\ \dfrac{2}{3}\pi\ ,\ \dfrac{4}{3}\pi$

(答) $z\ =\ 1\ ,\ -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\ ,\ -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

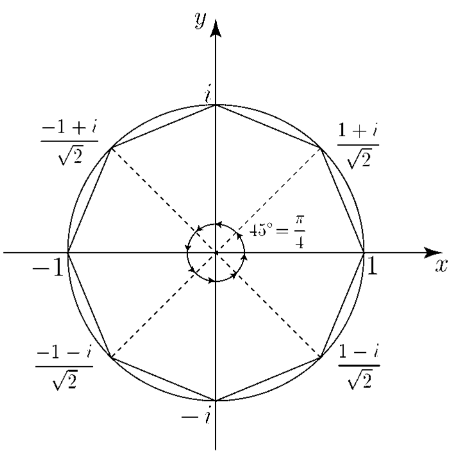


(2) $z^8 = 1$

$\cos(8\theta) + i\sin(8\theta) = 1$

$\theta = 0\ ,\ \dfrac{\pi}{4}\ ,\ \dfrac{\pi}{2}\ ,\ \dfrac{3}{4}\pi\ ,\ \pi\ ,\ \dfrac{5}{4}\pi\ ,\ \dfrac{3}{2}\pi\ ,\ \dfrac{7}{4}\pi$

(答) $z\ =\ 1\ ,\ \dfrac{1+i}{\sqrt{2}}\ ,\ i\ ,\ \dfrac{-1+i}{\sqrt{2}}\ ,\ -1\ ,\ \dfrac{-1-i}{\sqrt{2}}\ ,\ -i\ ,\ \dfrac{1-i}{\sqrt{2}}$



< 20 ページ. オイラーの公式 (1) >

問の解答

(1) $e^{2\pi i} = 1$

(2) $e^{-\frac{\pi}{2}i} = -i$

(3) $e^{\frac{3}{4}\pi i} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

(4) $e^{\frac{5}{3}\pi i} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

(5) $e^{-\frac{3}{4}\pi i} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

(6) $e^{-\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

< 21 ページ. オイラーの公式 (2) >

問の解答

(1) $e^{2-2\pi i} = e^2$

(2) $e^{0+\frac{\pi}{3}i} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

(3) $e^{2+\frac{3}{4}\pi i} = e^2\left(-\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)$

(4) $e^{\frac{1}{2}-\frac{3}{2}\pi i} = \sqrt{e}\,i$

(5) $e^{-1-\frac{\pi i}{6}} = \dfrac{\sqrt{3}}{2e} - \dfrac{i}{2e}$

(6) $e^{-\frac{1}{2}-\frac{\pi}{4}i} = \dfrac{1}{\sqrt{2e}} - \dfrac{i}{\sqrt{2e}}$

< 22 ページ. 複素数の指数表示 >

問 1 の解答

$e^{i\theta_1} \times e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$

問 2 の解答

(1) $e^{\frac{3}{2}\pi i} \times e^{\frac{\pi}{2}i} = e^{2\pi i} = 1$

(2) $e^{\frac{4}{3}\pi i} \div e^{\frac{\pi}{6}i} = e^{\frac{7}{6}\pi i} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

(3) $\left(e^{\frac{\pi}{8}i}\right)^4 = e^{\frac{\pi}{2}i} = i$

(4) $\left(e^{\frac{\pi}{48}i}\right)^{12} = e^{\frac{\pi}{4}i} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

< 23 ページ. 指数法則 >

問 1 の解答

(2) $\dfrac{e^{z_1}}{e^{z_2}} = e^{\boxed{z_1 - z_2}}$ (3) $\left(e^{z}\right)^{n} = e^{\boxed{nz}}$

問 2 の解答

(1) $e^{5+\pi i} \times e^{-1+\pi i} = e^{4+2\pi i} = e^{4}$

(2) $e^{2+\frac{\pi}{2}i} \div e^{4+\frac{\pi}{6}i} = e^{-2+\frac{\pi}{3}i}$

$$= \frac{1}{2e^2} + \frac{\sqrt{3}}{2e^2}i$$

(3) $\left(e^{\frac{1}{8}-\frac{\pi}{16}i}\right)^{4} = e^{\frac{1}{2}-\frac{\pi}{4}i} = \sqrt{\dfrac{e}{2}} - \sqrt{\dfrac{e}{2}}i$

問 3 の解答

$$\frac{\left(e^{\frac{\pi}{6}}i\right)^{8} \times \left(e^{\frac{\pi}{12}}i\right)^{4}}{\left(e^{\frac{\pi}{4}}i\right)^{6}} = \frac{e^{\frac{4\pi}{3}i} \times e^{\frac{\pi}{3}i}}{e^{\frac{3}{2}\pi i}} = e^{\left(\frac{4}{3}+\frac{1}{3}-\frac{3}{2}\right)\pi i} = e^{\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} + \frac{i}{2}i$$

＜ 24 ページ. 複素数の簡易表示 ＞

問 1 の解答

(1) $z_1 = \sqrt{3} + i$

$$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= 2e^{\frac{\pi}{6}i}$$

(2) $z_2 = -1 + i$

$$= \sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$

$$= \sqrt{2}e^{\frac{3}{4}\pi i}$$

(3) $z_3 = -\sqrt{3} - 3i$

$$= 2\sqrt{3}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

または $\left(2\sqrt{3}e^{\frac{4}{3}\pi i}\right)$

問 2 の解答

(1) $z_1 z_2$

$$= 2e^{\frac{\pi}{6}i} \times \sqrt{2}e^{\frac{3}{4}\pi i}$$

$$= 2\sqrt{2}e^{\frac{11}{12}\pi i}$$

(2) $z_2 z_3$

$$= \sqrt{2}e^{\frac{3}{4}\pi i} \times 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

$$= 2\sqrt{6}e^{\frac{\pi}{12}i}$$

$$\left(= 2\sqrt{6}e^{\frac{25}{12}\pi i}\right)$$

(3) $\dfrac{z_3}{z_1} = \dfrac{2\sqrt{3}e^{\frac{4}{3}\pi i}}{2e^{\frac{\pi}{6}i}}$

$$= \sqrt{3}e^{\frac{7}{6}\pi i}$$

$$\left(= \sqrt{3}e^{-\frac{5}{6}\pi i}\right)$$

＜ 25 ページ. 複素数の練習 ＞

問 1 の解答

(1) $\sqrt{6}+\sqrt{2}i = 2\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(2) $3-\sqrt{3}i = 2\sqrt{3}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = 2\sqrt{3}\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

(3) $-2+2i = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

問 2 の解答

(1) $\left(\frac{\sqrt{3}+i}{2}\right)^{12} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{12} = 1$

(2) $(1-i)^8 = \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right)^8 = 16$

(3) $\left(\frac{1+\sqrt{3}i}{1+i}\right)^9 = \left\{\frac{2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})}{\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})}\right\}^9 = \left(\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right)^9$

$$= 2^{\frac{9}{2}}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = 2^4\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = -16+16i$$

問 3 の解答

(1) $e^{\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ (2) $e^{\frac{2\pi}{3}i} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ (3) $e^{1-\frac{\pi}{6}i} = \frac{\sqrt{3}}{2}e - \frac{e}{2}i$

(4) $e^{\frac{2+3\pi i}{4}} = -\frac{\sqrt{2e}}{2} + \frac{\sqrt{2e}}{2}i$ (5) $e^{\frac{\pi}{3}i} \div e^{\frac{\pi}{2}i} = \frac{\sqrt{3}}{2} - \frac{i}{2}$

(6) $\left(e^{\frac{\pi}{6}i}\right)^7 \div \left(e^{\frac{3\pi}{8}i}\right)^4 = e^{(\frac{7}{6}-\frac{3}{2})\pi i} = e^{-\frac{\pi}{3}i} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

問 4 の解答

(1) $\frac{1-\sqrt{3}i}{2} = e^{-\frac{\pi}{3}i}\ \left(= e^{\frac{5\pi}{3}i}\right)$ (2) $-\frac{\sqrt{2}e}{2} + \frac{\sqrt{2}e}{2}i = e^{1+\frac{3\pi}{4}i}$

問 5 の解答

(1) $1+\sqrt{3}i = 2e^{\frac{\pi}{3}i}$ (2) $-3+\sqrt{3}i = 2\sqrt{3}\ e^{\frac{5\pi}{6}i}$ (3) $\sqrt{2}-\sqrt{6}i = 2\sqrt{2}e^{-\frac{\pi}{3}i}$

$\left(= 2\sqrt{2}e^{\frac{5\pi}{3}i}\right)$

問 6 の解答

(1) $\frac{e^{i\theta}+e^{-i\theta}}{2} = \cos\theta$ (2) $\frac{e^{i\theta}-e^{-i\theta}}{2i} = \sin\theta$

＜ 26 ページ. 複素数値関数の微分 (1) ＞

問の解答

(1) $z(t) = 3t^4 + ie^{2t}$

$$\frac{dz}{dt} = 12t^3 + 2e^{2t}i$$

(2) $z(t) = \cos(bt) + i\sin(bt)$

$$\frac{dz}{dt} = -b\sin(bt) + b\cos(bt)i$$

(3) $z(t) = e^{(3+4i)t} = e^{3t}\left(\cos(4t) + i\sin(4t)\right)$

$$\frac{dz}{dt} = \left(3e^{3t}\cos(4t) - 4e^{3t}\sin(4t)\right) + i(3e^{3t}\sin(4t) + 4e^{3t}\cos(4t)\big)$$

(4) $z(t) = e^{(a+bi)t} = e^{at}\cos(bt) + ie^{at}\sin(bt)$

$$\frac{dz}{dt} = \left(ae^{at}\cos(bt) - be^{at}\sin(bt)\right) + i\left(ae^{at}\sin(bt) + be^{at}\cos(bt)\right)$$

< 27ページ.複素数値関数の微分 (2) >

問の解答

(1) $\dfrac{d}{dt}e^{3it} = 3ie^{3ti}$

(2) $\dfrac{d}{dt}e^{-2it} = -2ie^{-2ti}$

(3) $\dfrac{d}{dt}e^{bit} = bie^{bit}$

(4) $\dfrac{d}{dt}e^{(1+i)t} = (1+i)e^{(1+i)t}$

(5) $\dfrac{d}{dt}e^{(2-i)t} = (2-i)e^{(2-i)t}$

(6) $\dfrac{d}{dt}e^{(-3+2i)t} = (-3+2i)e^{(-3+2i)t}$

(7) $\dfrac{d}{dt}e^{(a-i)t} = (a-i)e^{(a-i)t}$

(8) $\dfrac{d}{dt}e^{(a-bi)t} = (a-bi)e^{(a-bi)t}$

(9) $\dfrac{d}{dt}\left(\dfrac{1}{a+bi}e^{(a+bi)t}\right) = e^{(a+bi)t}$

(10) $\dfrac{d}{dt}\left(\dfrac{1}{a-bi}e^{(a-bi)t}\right) = e^{(a-bi)t}$

＜ 28 ページ. 複素数値関数の積分 (1) ＞

(1) $\displaystyle\int (t^3 + t^5 i)dt = \frac{1}{4}t^4 + \frac{1}{6}t^6 i + C$

(2) $\displaystyle\int (\cos t + i\sin t)dt = \sin t - i\cos t + C$

(3) $\displaystyle\int (e^{2t} + i\cos(3t))dt = \frac{1}{2}e^{2t} + \frac{i}{3}\sin(3t) + C$

(4) $\displaystyle\int e^{bit}dt = \frac{1}{bi}e^{bit} + C$

(5) $\displaystyle\int e^{(2+3i)t}dt = \frac{1}{2+3i}e^{(2+3i)t} + C$

(6) $\displaystyle\int e^{(a-bi)t}dt = \frac{1}{a-bi}e^{(a-bi)t} + C$

＜ 29 ページ. 複素数値関数の積分 (2) ＞

問 1 の解答

$$\int e^{at}\cos(bt)dt + i\int e^{at}\sin(bt)dt = \int e^{at}\left(\cos(bt) + i\sin(bt)\right)dt$$

$$= \int e^{(a+bi)t}dt = \frac{1}{a+bi}e^{(a+bi)t} + C = \frac{a-bi}{a^2+b^2}e^{at}\big(\cos(bt) + i\sin(bt)\big) + C$$

$$= \frac{e^{at}}{a^2+b^2}\big(a\cos(bt) + b\sin(bt)\big) + i\frac{e^{at}}{a^2+b^2}\big(-b\cos(bt) + a\sin(bt)\big) + C$$

問 2 の解答

(1) $\displaystyle\int e^{at}\cos(bt)dt$

$$= \frac{e^{at}}{a^2+b^2}\big(a\cos(bt) + b\sin(bt)\big) + C$$

(2) $\displaystyle\int e^{at}\sin(bt)dt$

$$= \frac{e^{at}}{a^2+b^2}\big(-b\cos(bt) + a\sin(bt)\big) + C$$

問 3 の解答

(1) $\displaystyle\int e^{3t}\cos(4t)dt$

$$= \frac{e^{3t}}{25}\big(3\cos(4t) + 4\sin(4t)\big) + C$$

(2) $\displaystyle\int e^{3t}\sin(4t)dt$

$$= \frac{e^{3t}}{25}\big(-4\cos(4t) + 3\sin(4t)\big) + C$$

< 30 ページ. 練習問題 >

問 1 の解答

(1) $(\sqrt{3}+i)^5 = (2e^{\frac{\pi i}{6}})^5$

$= 2^5 e^{\frac{5\pi}{6}i} = -16\sqrt{3}+16i$

(2) $\left(\dfrac{-2+2i}{1+\sqrt{3}\,i}\right)^4 = \left(\dfrac{2\sqrt{2}e^{\frac{3\pi}{4}i}}{2e^{\frac{\pi}{3}i}}\right)^4$

$= (\sqrt{2})^4 e^{\frac{5\pi}{3}i} = 2-2\sqrt{3}i$

問 2 の解答

(1) $e^{\frac{\pi}{3}i} = \dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

(2) $e^{\frac{5\pi}{6}i} = -\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$

(3) $e^{1+\frac{\pi}{6}i} = \dfrac{\sqrt{3}}{2}e+\dfrac{e}{2}i$

(4) $e^{\frac{2-\pi i}{4}} = \dfrac{\sqrt{2e}}{2}-\dfrac{\sqrt{2e}}{2}i$

(5) $e^{\frac{2\pi}{3}i}\times e^{\frac{\pi}{2}i} = e^{\frac{7\pi}{6}i} = -\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i$

(6) $e^{\frac{4\pi}{3}i}\div e^{\frac{\pi}{2}i} = e^{\frac{5\pi}{6}i} = -\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$

問 3 の解答

(1) $\dfrac{-\sqrt{3}+i}{2e} = e^{-1+\frac{5\pi}{6}i}$

(2) $\dfrac{\sqrt{2e}-\sqrt{2e}\,i}{2} = e^{\frac{1}{2}-\frac{\pi}{4}i}$ $\left(= e^{\frac{1}{2}+\frac{7\pi}{4}i}\right)$

問 4 の解答

(1) $3-3i = 3\sqrt{2}e^{-\frac{\pi}{4}i}$

$(= 3\sqrt{2}e^{\frac{7}{4}\pi i})$

(2) $-1-\sqrt{3}i = 2e^{\frac{4\pi}{3}i}$

$(= 2e^{-\frac{2\pi}{3}i})$

問 5 の解答

(1) $\dfrac{d}{dt}e^{t^3+t^2} = (3t^2+2t)e^{t^3+t^2}$

(2) $\dfrac{d}{dt}\cos(4t) = -4\sin(4t)$

(3) $\dfrac{d}{dt}e^t\sin(2t) = e^t\sin(2t)+2e^t\cos(2t)$

(4) $\dfrac{d}{dt}e^{2t}\cos(3t) = 2e^{2t}\cos(3t)-3e^{2t}\sin(3t)$

(5) $\dfrac{d}{dt}e^{4t+5ti} = (4+5i)e^{(4+5i)t}$

(6) $\dfrac{d}{dt}e^{(3-4i)t} = (3-4i)e^{(3-4i)t}$

問 5 の解答

(1) $\displaystyle\int (3\sin(2t)+4\cos(3t))\,dt$

$= -\dfrac{3}{2}\cos(2t)+\dfrac{4}{3}\sin(3t)+C$

(2) $\displaystyle\int e^{3t-4}dt = \frac{1}{3}e^{3t-4}+C$

(3) $\displaystyle\int e^{4t+5ti}dt = \frac{1}{4+5i}4^{(4+5i)t}+C$

(4) $\displaystyle\int e^{4t}\cos(3t)dt$

$= \dfrac{e^{4t}}{25}\left\{4\cos(3t)+\sin(3t)\right\}+C$

< 31 ページ. 微分方程式 >

問の解答

(1) 1 階微分方程式

(2) 2 階微分方程式

(3) 3 階微分方程式

< 32ページ. 微分方程式の解 (1) >

問の解答

$y = 3e^t$, $y = -e^t$ など

< 33 ページ. 微分方程式の解 (2) >

問の解答

(1) $C = 3$, $\underline{y = 3e^t}$

(2) $C = -2$, $\underline{y = -2e^t}$

(3) $C = 0$, $\underline{y = 0}$

＜ 34 ページ. 微分方程式の解 (3) ＞

問 1 の解答

$t=0$ のとき $y=2$

問 2 の解答

$y=3e^{-t}$, $y=-e^{-t}$ など

問 3 の解答

$y=Ce^{-t}$

< 35 ページ. 求積法 >

問の解答

(1) $y = \dfrac{3}{2}t^2 + 3t + C$

(2) $y = \dfrac{1}{8}t^4 + t^5 + C$

(3) $y = \dfrac{2}{t} + \log|t| + C$

(4) $y = -4\cos t - 5\sin t + C$

< 36 ページ.1 階微分方程式の原理 >

問の解答

証明略

< 37 ページ. 変数分離形 (1) >

問の解答

$y = Ce^{2t}$ (C は任意定数)

< 38 ページ. 変数分離形 (2) >

問の解答

(1) $y = Ce^{3t^2+5t}$ (C は任意定数)

(2) $y = Ce^{t^3+4t}$ (C は任意定数)

< 39 ページ. 変数分離形 (3) >

問の解答

(1) $\displaystyle\int \frac{1}{y}dy = \int \frac{2t}{t^2+1}dt \Longrightarrow \log|y| = \log|t^2+1| + C_0$

$\displaystyle\Longrightarrow \log\left|\frac{y}{t^2+1}\right| = C_0 \Longrightarrow \frac{y}{t^2+1} = \pm e^{C_0} = C$ とおく。

$C = 0$ のとき $y = 0$ も解なので　(答) $y = C(t^2+1)$　(C は任意定数)

(2) $\displaystyle\int \frac{1}{y^2+1}dy = \int t dt \Longrightarrow \tan^{-1} y = \frac{t^2}{2} + C$

(答) $\displaystyle y = \tan\left(\frac{t^2}{2} + C\right)$　(C は任意定数)

< 40 ページ.1 階線形微分方程式 (1) >

問 1 の解答

(1) $y = Ce^{-at}$ (C は任意定数)

(2) $y = Ce^{5t^2}$ (C は任意定数)

(3) $y = Ce^{-2t^3-t}$ (C は任意定数)

問 2 の解答

$y = Ce^{-\int p(t)dt}$ (C は任意定数)

< 42 ページ.1 階線形微分方程式 (3) >

問の解答

(1) $y = \dfrac{5}{3} + Ce^{-3t}$ (C は任意定数)

(2) $y = \dfrac{b}{a} + Ce^{-at}$ (C は任意定数)

< 43 ページ.1 階線形微分方程式 (4) >

問の解答

(1) $y = C(t)e^{-4t}$ とおくと

$$\frac{dy}{dt} + 4y = C'(t)e^{-4t} = e^{5t}$$

$$C'(t) = e^{9t}$$

$$C(t) = \frac{1}{9}e^{9t} + C$$

$$y = \left(\frac{1}{9}e^{9t} + C\right)e^{-4t}$$

(答) $y = \frac{1}{9}e^{5t} + Ce^{-4t}$

(2) $y = C(t)e^{4t}$ とおくと

$$\frac{dy}{dt} - 4y = C'(t)e^{4t} = e^{5t}$$

$$C'(t) = e^{t}$$

$$C(t) = e^{t} + C$$

$$y = (e^{t} + C)e^{4t}$$

(答) $y = e^{5t} + Ce^{4t}$

< 44 ページ.1 階線形微分方程式 (5) >

問の解答

(1) $y = C(t)e^{2t}$ とおくと

$\dfrac{dy}{dt} - 2y = C'(t)e^{2t} = e^{2t}$ より

$C'(t) = 1$, $C(t) = t + C$

(答) $y = te^{2t} + Ce^{2t}$ (C は任意定数)

(2) $y = C(t)e^{-3t}$ とおくと

$\dfrac{dy}{dt} + 3y = C'(t)e^{-3t} = e^{-3t}$ より

$C'(t) = 1$, $C(t) = t + C$

(答) $y = te^{-3t} + Ce^{-3t}$ (C は任意定数)

(3) $y = C(t)e^{at}$ とおくと

$\dfrac{dy}{dt} - ay = C'(t)e^{at} = e^{at}$ より

$C'(t) = 1$, $C(t) = t + C$

(答) $y = te^{at} + Ce^{at}$ (C は任意定数)

＜ 45 ページ.1 階線形微分方程式 (6) ＞

問の解答

(1) $y = C(t)e^{at}$ とおくと

$$\frac{dy}{dt} - ay = C'(t)e^{at} + aC(t)e^{at} - aC(t)e^{at} = C'(t)e^{at} = e^{bt}$$ より

$$C'(t) = e^{(b-a)t} \Longrightarrow \quad C(t) = \frac{1}{b-a}e^{(b-a)t} + C$$

$$y = \left(\frac{1}{b-a}e^{(b-a)t} + C\right)e^{at}$$

(答) $y = \frac{1}{b-a}e^{bt} + Ce^{at}$ (C は任意定数)

(2) $y = C(t)e^{at}$ とおくと

$$\frac{dy}{dt} - ay = C'(t)e^{at} = Ke^{bt}$$ より

$$C'(t) = Ke^{(b-a)t} \Longrightarrow \quad C(t) = \frac{K}{b-a}e^{(b-a)t} + C$$

$$y = \left(\frac{K}{b-a}e^{(b-a)t} + C\right)e^{at}$$

(答) $y = \frac{K}{b-a}e^{bt} + Ce^{at}$ (C は任意定数)

(3) $y = C(t)e^{at}$ とおくと

$$\frac{dy}{dt} - ay = C'(t)e^{at} = Ke^{at}$$ より

$$C'(t) = K \Longrightarrow \quad C(t) = Kt + C$$

$$y = (Kt + C)e^{at}$$

(答) $y = Kte^{at} + Ce^{at}$ (C は任意定数)

< 46 ページ.1 階線形微分方程式 (7) >

問 1 の解答

(1) $z = C(t)e^{-at}$ とおくと $\dfrac{dz}{dt} + az = C'(t)e^{-at} = e^{bit}$ より

$$C'(t) = e^{(a+bi)t} \Longrightarrow \quad C(t) = \frac{1}{a+bi}e^{(a+bi)t} + C \Longrightarrow z = \left(\frac{1}{a+bi}e^{(a+bi)t} + C\right)e^{-at}$$

(答) $z = \dfrac{1}{a+bi}e^{bit} + Ce^{-at}$ (C は任意定数)

(2) $z(t) = \dfrac{1}{a+bi}e^{bit} + Ce^{-at} = \dfrac{a-bi}{a^2+b^2}(\cos(bt) + i\sin(bt)) + Ce^{-at}$

$$= \frac{1}{a^2+b^2}\{a\cos(bt) + b\sin(bt)\} + i\cdot\frac{1}{a^2+b^2}\{-b\cos(bt) + a\sin(bt)\} + Ce^{-at}$$

問 2 の解答

$x = C(t)e^{-at}$ とおくと $\dfrac{dx}{dt} + ax = C'(t)e^{-at} = \cos(bt)$ より

$$C'(t) = e^{at}\cos(bt) \Longrightarrow \quad C(t) = \frac{e^{at}}{a^2+b^2}\{a\cos(bt) + b\sin(bt)\} + C$$

$$x = \left[\frac{e^{at}}{a^2+b^2}\{a\cos(bt) + b\sin(bt)\} + C\right]e^{-at}$$

(答) $x = \dfrac{1}{a^2+b^2}\{a\cos(bt) + b\sin(bt)\} + Ce^{-at}$ (C は任意定数)

問 3 の解答

$y = C(t)e^{-at}$ とおくと $\dfrac{dy}{dt} + ay = C'(t)e^{-at} = \sin(bt)$ より

$$C'(t) = e^{at}\sin(bt) \Longrightarrow \quad C(t) = \frac{e^{at}}{a^2+b^2}\{-b\cos(bt) + a\sin(bt)\} + C$$

$$y = \left[\frac{e^{at}}{a^2+b^2}\{-b\cos(bt) + a\sin(bt)\} + C\right]e^{-at}$$

(答) $y = \dfrac{1}{a^2+b^2}\{-b\cos(bt) + a\sin(bt)\} + Ce^{-at}$ (C は任意定数)

< 47 ページ.1 階線形微分方程式 (8) >

問の解答

(1) $y = 2t^3 - 4t^2 + 5t + C \quad$ (C は任意定数)

(2) $y = \dfrac{1}{2}e^{2t} + \dfrac{5}{3}\cos(3t) + 3\sin(2t) + C \quad$ (C は任意定数)

(3) $y = Ce^{10t} \quad$ (C は任意定数)

(4) $y = Ce^{-9t} \quad$ (C は任意定数)

(5) $y = Ce^{t^2+t} \quad$ (C は任意定数)

(6) (解) $\displaystyle\int \frac{1}{y+4}dy = \int \frac{1}{t-1}dt \quad \Rightarrow \quad \log|y+4| = \log|t-1| + C_0$

$\Rightarrow \quad \dfrac{y+4}{t-1} = \pm e^{C_0} = C$ とおく

$y + 4 = C(t-1)$

$C = 0$ のとき $y = -4$ も解だから

(答) $y = C(t-1) - 4 \quad$ (C は任意定数)

(7) $\dfrac{3}{4} + Ce^{-4t} \quad$ (C は任意定数)

(8) $y = -\dfrac{5}{3} + Ce^{3t} \quad$ (C は任意定数)

(9) $y = -e^{2t} + Ce^{3t} \quad$ (C は任意定数)

(10) $y = te^{3t} + Ce^{3t} \quad$ (C は任意定数)

(11) $y = -\dfrac{1}{3}e^{-3t} + Ce^{3t} \quad$ (C は任意定数)

(12) $y = 4te^{-3t} + Ce^{-3t} \quad$ (C は任意定数)

(13) $\dfrac{1}{13}\Big(3\cos(2t) + 2\sin(2t)\Big) + Ce^{-3t} \quad$ (C は任意定数)

＜ 48 ページ.1 階微分方程式の初期値問題 ＞

問の解答

(1) $y = -4.9t^2 + 10t + 6$

(2) $y = 4e^{-5t}$

(3) $y = \dfrac{9.8}{k} + Ce^{-kt}$

$t = 0$ のとき $y = \dfrac{9.8}{k} + C = 0 \;\Rightarrow\; C = -\dfrac{9.8}{k}$

(答) $y = \dfrac{9.8}{k}(1 - e^{-kt})$

(4) $y = -\dfrac{g}{k} + Ce^{-kt}$

$t = 0$ のとき $y = -\dfrac{g}{k} + C = 4 \;\Rightarrow\; C = 4 + \dfrac{g}{k}$

(答) $y = -\dfrac{g}{k} + (4 + \dfrac{g}{k})e^{-kt}$

< 49 ページ.2 階線形微分方程式 (1) >

問の解答

(1) $y'(t) = 8t + C_1$

$y(t) = 4t^2 + C_1 t + C_2$

$y(0) = C_2 = 7$

$y'(0) = C_1 = 6$

<u>(答) $y = 4t^2 + 6t + 7$</u>

(2) $y'(t) = 3t^2 + 2t + C_1$

$y(t) = t^3 + t^2 + C_1 t + C_2$

$y(0) = C_2 = 8$

$y'(0) = C_1 = 9$

<u>(答) $y = t^3 + t^2 + 9t + 8$</u>

< 50 ページ.2 階線形微分方程式 (2) >

問の解答

(1) $y(t) = C_1 e^{3t} + C_2 e^{-3t}$

$y'(t) = 3C_1 e^{3t} - 3C_2 e^{-3t}$

$y(0) = C_1 + C_2 = 8 \quad \cdots$ ①

$y'(0) = 3C_1 - 3C_2 = 6 \ \Rightarrow \ C_1 - C_2 = 2 \quad \cdots$ ②

①，②より $C_1 = 5$, $C_2 = 3$

(答) $y(t) = 5e^{3t} + 3e^{-3t}$

(2) $y(0) = C_1 + C_2 = \alpha \quad \cdots$ ①

$y'(0) = 3C_1 - 3C_2 = \beta \quad \cdots$ ②

①+②/3 $\quad 2C_1 = \alpha + \dfrac{\beta}{3} \ \Rightarrow \ C_1 = \dfrac{\alpha}{2} + \dfrac{\beta}{6}$

$C_2 = \alpha - C_1 = \dfrac{\alpha}{2} - \dfrac{\beta}{6}$

(答) $y(t) = \left(\dfrac{\alpha}{2} + \dfrac{\beta}{6}\right) e^{3t} + \left(\dfrac{\alpha}{2} - \dfrac{\beta}{6}\right) e^{-3t}$

< 51 ページ.2 階線形同次微分方程式 (1) >

問の解答

もう 1 つの基本解は $y = e^{2t}$ であり、一般解は

<u>$y = C_1 e^{2t} + C_2 e^{-2t}$ ($= C_1$，C_2 は任意定数)</u>

< 52 ページ.2 階線形同次微分方程式 (2) >

問の解答

もう 1 つの基本解は $y = e^t$ であり、一般解は

$y = C_1e^t + C_2e^{3t}$ 　(C_1，C_2 は任意定数)

< 53 ページ. 定数係数 2 階線形同次微分方程式 (1) >

問の解答

(1) $y = C_1e^{2t} + C_2e^{3t}$ (C_1, C_2 は任意定数)

(2) $y = C_1e^{-t} + C_2e^{-3t}$ (C_1, C_2 は任意定数)

(3) $y = C_1e^{4t} + C_2e^{-t}$ (C_1, C_2 は任意定数)

(4) $y = C_1e^{4t} + C_2e^{-4t}$ (C_1, C_2 は任意定数)

< 54 ページ. 定数係数 2 階線形同次微分方程式 (2) >

問の解答

(1) $y = C_1 \cos t + C_2 \sin t$　　(C_1，C_2 は任意定数)

(2) $y = C_1 \cos(2t) + C_2 \sin(2t)$　(C_1，C_2 は任意定数)

(3) $y = C_1 \cos(\omega t) + C_2 \sin(\omega t)$　　(C_1，C_2 は任意定数)

< 55 ページ. 定数係数 2 階線形同次微分方程式 (3) >

問 1 の解答

$$\frac{dy}{dt} = 2e^{2t}\sin(3t) + 3e^{2t}\cos(3t)$$

$$\begin{aligned}\frac{d^2y}{dt^2} &= 4e^{2t}\sin(3t) + 6e^{2t}\cos(3t) + 6e^{2t}\cos(3t) - 9e^{2t}\cos(3t) \\ &= -5e^{2t}\sin(3t) + 12e^{2t}\cos(3t)\end{aligned}$$

$$\begin{aligned}\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y &= -5e^{2t}\sin(3t) + 12e^{2t}\cos(3t) \\ &\quad -4(2e^{2t}\sin(3t) + 3e^{2t}\cos(3t)) + 13e^{2t}\sin(3t) \\ &= 0\end{aligned}$$

問 2 の解答

(1) $\lambda^2 - 2\lambda + 5 = 0 \quad \Rightarrow \quad \lambda = 1 \pm 2i$ より

(答)$y = C_1e^t\cos(2t) + C_2e^t\sin(2t)$ 　(C_1，C_2 は任意定数)

(2) $\lambda^2 + 6\lambda + 10 = 0 \quad \Rightarrow \quad \lambda = -3 \pm i$ より

(答)$y = C_1e^{-3t}\cos t + C_2e^{-3t}\sin t$ 　(C_1，C_2 は任意定数)

< 56 ページ. 定数係数 2 階線形同次微分方程式 (4) >

問 1 の解答

(解) $y = C(t)e^{3t}$ とおくと

$\dfrac{dy}{dt} - 3y = C'(t)e^{3t} = C_1 e^{3t}$ より

$C'(t) = C_1 \quad \Rightarrow \quad C(t) = C_1 t + C$

$\underline{(\text{答})y = C_1 te^{3t} + Ce^{3t}}$ (C は任意定数)

問 2 の解答

(解) $y'' - 2ay' + a^2 y = 0$

$y'' - ay' = ay' - a^2 y$

$(y' - ay)' = a(y' - ay)$

$\Downarrow$

$z = y' - ay$ とおくと

$z' = az$

$z = C_1 e^{at}$

$\Downarrow$

$y' - ay = C_1 e^{at}$

$\Downarrow$

$y = C_1 te^{at} + C_2 e^{at}$

$\underline{(\text{答})y = C_1 te^{at} + C_2 e^{at}}$ (C_1, C_2 は任意定数)

< 57 ページ. 定数係数 2 階線形同次微分方程式 (5) >

問の解答

(1) $\lambda^2 - 5\lambda - 6 = 0 \ \Rightarrow \ \lambda = 6, -1$

(答)$y = C_1 e^{6t} + C_2 e^{-t}$ （C_1，C_2 は任意定数)

(2) $\lambda^2 + 16 = 0 \ \Rightarrow \ \lambda = \pm 4i$

(答)$y = C_1 \cos(4t) + C_2 \sin(4t)$ （C_1，C_2 は任意定数)

(3) $\lambda^2 - 8\lambda + 20 = 0 \ \Rightarrow \ \lambda = 4 \pm 2i$

(答)$y = C_1 e^{4t} \cos(2t) + C_2 e^{4t} \sin(2t)$ （C_1，C_2 は任意定数)

(4) $\lambda^2 + 8\lambda + 16 = 0 \ \Rightarrow \ \lambda = -4$

(答)$y = C_1 t e^{-4t} + C_2 e^{-4t}$ （C_1，C_2 は任意定数)

＜ 58 ページ. 定数係数 2 階線形非同次微分方程式 (1) ＞

問の解答

(1) $\lambda^2 + \lambda - 2 = 0 \ \Rightarrow \ \lambda = 1, -2$

特解は $\dfrac{6}{-2} = 3$ より

(答)$y = C_1 e^t + C_2 e^{-2t} - 3$ （C_1，C_2 は任意定数)

(2) $\lambda^2 + 16 = 0 \ \Rightarrow \ \lambda = \pm 4i$

特解は $\dfrac{20}{16} = \dfrac{5}{4}$ より

(答)$y = C_1 \cos(4t) + C_2 \sin(4t) + \dfrac{5}{4}$ （C_1，C_2 は任意定数)

(3) $\lambda^2 - 4\lambda + 5 = 0 \ \Rightarrow \ \lambda = 2 \pm i$

特解は $\dfrac{7}{5}$ より

(答)$y = C_1 e^{2t} \cos t + C_2 e^{2t} \sin t + \dfrac{7}{5}$ （C_1，C_2 は任意定数)

(4) $\lambda^2 + 4\lambda + 4 = 0 \ \Rightarrow \ \lambda = -2$

特解は $\dfrac{6}{4} = \dfrac{3}{2}$ より

(答)$y = C_1 t e^{-2t} + C_2 e^{-2t} + \dfrac{3}{2}$ （C_1，C_2 は任意定数)

< 59 ページ. 定数係数 2 階線形非同次微分方程式 (2) >

問の解答

(解) $a = 0$, $b = \omega^2$, $\alpha = 0$, $\beta = \omega$, $A = 0$, $B = 0$ より

表の⑦の場合であるから特解は $-\dfrac{r}{2\beta}te^{\alpha t}\cos(\beta t) = -\dfrac{r}{2\omega}t\cos(\omega t)$

(答)$y = C_1\cos(\omega t) + C_2\sin(\omega t) - \dfrac{r}{2\omega}t\cos(\omega t)$ (C_1, C_2 は任意定数)

＜ 60 ページ. 定数係数 2 階線形非同次微分方程式 (3) ＞

問の解答

(1) (解) $v = \dfrac{dy}{dt}$ とおくと

$$\frac{dv}{dt} + 2v = 0 \quad \Rightarrow \quad v = C_1 e^{-2t}$$

$$y = \int v dt = \int C_1 e^{-2t} dt = -\frac{C_1}{2} e^{-2t} + C_2$$

初期条件より

$$v(0) = y'(0) = C_1 = 6$$

$$y(0) = -\frac{C_1}{2} + C_2 = -3 + C_2 = 10$$

$$\Rightarrow \quad C_2 = 13$$

(答)$y = -3e^{-2t} + 13$

(2) (解) $v = \dfrac{dy}{dt}$ とおくと

$$\frac{dv}{dt} + 2v = 6 \quad \Rightarrow \quad v = 3 + C_1 e^{-2t}$$

$$y = \int v dt = 3t - \frac{C_1}{2} e^{-2t} + C_2$$

$$y'(0) = v(0) = 3 + C_1 = 8 \quad \Rightarrow \quad C_1 = 5$$

$$y(0) = -\frac{C_1}{2} + C_2 = -\frac{5}{2} + C_2 = 10$$

$$\Rightarrow \quad C_2 = \frac{25}{2}$$

(答)$y = 3t - \dfrac{5}{2} e^{-2t} + \dfrac{25}{2}$

< 61 ページ. 微分方程式の応用 (1) >

問の解答

(解) $\dfrac{dv}{dt} = -9.8 \quad \Rightarrow \quad v(t) = -9.8t + C_1$

$v(0) = 7$ より $\quad C_1 = 7 \quad \Rightarrow \quad v(t) = -9.8t + 7$

$y(t) = \displaystyle\int v dt = -4.9t^2 + 7t + C_2$

$y(0) = C_2 = 10$

<u>(答) $v(t) = -9.8t + 7 \ \ \mathrm{(m/s)}$</u>

<u>(答) $y(t) = -4.9t^2 + 7t + 10 \ \ \mathrm{(m)}$</u>

< 62 ページ. 微分方程式の応用 (2) >

問 1 の解答

(解) $v(t) = -\dfrac{9.8}{\gamma} + Ce^{-\gamma t}$

$v(0) = -\dfrac{9.8}{\gamma} + C = 0$ より

$C = \dfrac{9.8}{\gamma}$

(答) $v(t) = -\dfrac{9.8}{\gamma} + \dfrac{9.8}{\gamma}e^{-\gamma t}$

問 2 の解答

(解) $v(t) = -9.8 - \gamma v \quad \Rightarrow \quad v(t) = -\dfrac{9.8}{\gamma} + Ce^{-\gamma t}$

初期条件より

$v(0) = -\dfrac{9.8}{\gamma} + C = 5 \quad \Rightarrow \quad C = 5 + \dfrac{9.8}{\gamma}$

(答) $v(t) = -\dfrac{9.8}{\gamma} + \left(5 + \dfrac{9.8}{\gamma}\right)e^{-\gamma t}$

$<$ 63ページ. 微分方程式の応用 (3) $>$

問 1 の解答

(解) $\dfrac{dv}{dt} + \gamma v = -9.8 \quad \Rightarrow \quad v(t) = -\dfrac{9.8}{\gamma} + C_1 e^{-\gamma t}$

$$y = \int v dt = -\frac{9.8}{\gamma}t - \frac{C_1}{\gamma}e^{-\gamma t} + C_2$$

問題文より $v(0) = 5 \quad \Rightarrow \quad C_1 = 5 + \dfrac{9.8}{\gamma}$

$$y(0) = 10 \quad \Rightarrow \quad C_2 = \frac{C_1}{\gamma} + 10 = \frac{1}{\gamma}\left(5 + \frac{9.8}{\gamma}\right) + 10$$

(答) $v(t) = -\dfrac{9.8}{\gamma} + \left(5 + \dfrac{9.8}{\gamma}\right)e^{-\gamma t}$ (m/s)

$y(t) = -\dfrac{9.8}{\gamma}t - \dfrac{1}{\gamma}\left(5 + \dfrac{9.8}{\gamma}\right)e^{-\gamma t} + \dfrac{1}{\gamma}\left(5 + \dfrac{9.8}{\gamma}\right) + 10$ (m)

問 2 の解答

(解) $\dfrac{dI}{dt} + \dfrac{R}{L}I = \dfrac{E}{L}$, $I(0) = q'(0) = 0 \quad \Rightarrow \quad I(t) = \dfrac{\frac{E}{L}}{\frac{R}{L}} + C_1 e^{-\frac{R}{L}t}$

$$= \frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t}$$

$$q(t) = \int I(t)dt = \frac{E}{R}t + \frac{EL}{R^2}e^{-\frac{R}{L}t} + C_2 \quad , \quad q(0) = 0 \quad \Rightarrow \quad C_2 = -\frac{EL}{R^2}$$

(答) $I(t) = \dfrac{E}{R} - \dfrac{E}{R}e^{-\frac{R}{L}t}$, $q(t) = \dfrac{E}{R}t + \dfrac{EL}{R^2}e^{-\frac{R}{L}t} - \dfrac{EL}{R^2}$

< 64 ページ. 微分方程式の応用 (4) >

問 1 の解答

(解) $\lambda^2 + 4\lambda + 13 = 0 \quad \Rightarrow \quad \lambda = -2 \pm 3i$

$y(t) = C_1 e^{-2t} \cos(3t) + C_2 e^{-2t} \sin(3t)$

$y'(t) = -2C_1 e^{-2t} \cos(3t) - 3C_1 e^{-2t} \sin(3t) - 2C_2 e^{-2t} \sin(3t) + 3C_2 e^{-2t} \cos(3t)$

$y(0) = C_1 = L \quad , \quad y'(0) = -2C_1 + 3C_2 = 0 \quad \Rightarrow \quad C_2 = \dfrac{2}{3} L$

(答) $y = Le^{-2t} \cos(3t) + \dfrac{2}{3} Le^{-2t} \sin(3t)$

問 2 の解答

(解) $\lambda^2 + 6\lambda + 9 = 0 \quad \Rightarrow \quad \lambda = -3$

$y(t) = C_1 t e^{-3t} + C_2 e^{-3t}$

$y'(t) = C_1 e^{-3t} - 3C_1 t e^{-3t} - 3C_2 e^{-3t}$

$y(0) = C_2 = L \quad , \quad y'(0) = C_1 - 3C_2 = 0 \quad \Rightarrow \quad C_1 = 3L$

(答) $y = 3Lte^{-3t} + Le^{-3t}$

< 65 ページ. 微分方程式の練習 >

問 1 の解答

(1) $y = \dfrac{1}{2}\sin(2t) + C$ (C は任意定数)

(2) $y = Ce^{-2t^2}$ (C は任意定数)

(3) $y = C(t^3 + 1)$ (C は任意定数)

(4) $y = \dfrac{3}{2} + Ce^{-2t}$ (C は任意定数)

(5) $y = -e^t + Ce^{2t}$ (C は任意定数)

(6) $y = te^{3t} + Ce^{3t}$ (C は任意定数)

(7) $y = \dfrac{at^3}{6} + \dfrac{bt^2}{2} + C_1 t + C_2$ (C_1 , C_2 は任意定数)

(8) $y = C_1 e^{2t} + C_2 e^{3t}$ (C_1 , C_2 は任意定数)

(9) $y = C_1 e^{3t} + C_2 e^{-3t}$ (C_1 , C_2 は任意定数)

(10) $y = C_1 \cos(3t) + C_2 \sin(3t)$ (C_1 , C_2 は任意定数)

(11) $y = C_1 e^t \cos(2t) + C_2 e^t \sin(2t)$ (C_1 , C_2 は任意定数)

(12) $y = C_1 te^{3t} + C_2 e^{3t}$ (C_1 , C_2 は任意定数)

(13) $y = C_1 e^{3t} + C_2 e^{-2t} - \dfrac{2}{3}$ (C_1 , C_2 は任意定数)

(14) $y = C_1 \cos(2t) + C_2 \sin(2t) + \dfrac{5}{4}$ (C_1 , C_2 は任意定数)

問 2 の解答

(1) $y = 4e^{-3t}$

(2) $v = 2 + 3e^{-3t}$

(3) $y(t) = \dfrac{2}{3}e^t + \dfrac{4}{3}e^{4t}$

(4) $y = 5\cos(2t) + 3\sin(2t)$

(5) $y(t) = e^{-t}\cos t + e^{-t}\sin t$

(6) $y(t) = 2te^{-2t} + e^{-2t}$