高知工科大学

Kochi University of Technology

数 学 2

(2005年度版)

解答

＜ 1 ページ. 不定積分 (1) ＞

問の解答

(1) $\left(\dfrac{1}{\alpha+1}x^{\alpha+1}\right)' = x^\alpha \qquad \Rightarrow \qquad \displaystyle\int x^\alpha dx = \frac{1}{\alpha+1}x^{\alpha+1} + C$

(2) $\big(\log|x|\big)' = \dfrac{1}{x} \qquad \Rightarrow \qquad \displaystyle\int \frac{1}{x}dx = \log|x| + C$

(3) $(\sin x)' = \cos x \qquad \Rightarrow \qquad \displaystyle\int \cos x dx = \sin x + C$

(4) $(-\cos x)' = \sin x \qquad \Rightarrow \qquad \displaystyle\int \sin x dx = -\cos x + C$

(5) $(e^x)' = e^x \qquad \Rightarrow \qquad \displaystyle\int e^x dx = e^x + C$

< 2 ページ. 不定積分 (2) >

問の解答

(1) $\dfrac{1}{7}x^7 + C$

(2) $\dfrac{4}{5}x^{\frac{5}{4}} + C$

(3) $2x^{\frac{1}{2}} + C$

(4) $\displaystyle\int x^{-3}dx = \frac{1}{-2}x^{-2} + C = -\frac{1}{2x^2} + C$

(5) $\displaystyle\int x^{\frac{2}{3}}dx = \frac{3}{5}x^{\frac{5}{3}} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$

(6) $\displaystyle\int x^{-\frac{1}{4}}dx = \frac{4}{3}x^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{x^3} + C$

＜ 3ページ. 不定積分 (3) ＞

問の解答

(1) $\displaystyle\int\left(\frac{1}{x}-\frac{4}{x^2}+\frac{1}{x^3}\right)dx=\log|x|+\frac{4}{x}-\frac{1}{2x^2}+C$

(2) $\displaystyle\int\left(1-\frac{4}{x^2}+\frac{3}{x^4}\right)dx=x+\frac{4}{x}-\frac{1}{x^3}+C$

(3) $\displaystyle\int\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)dx=\frac{2}{3}x\sqrt{x}+4\sqrt{x}+C$

(4) $\displaystyle\int\left(1-\frac{2}{\sqrt{x}}+\frac{1}{x}\right)dx=x-4\sqrt{x}+\log|x|+C$

< 4ページ. 不定積分 (4) >

問 1 の解答

(1) $(\tan x)' = \dfrac{1}{\cos^2 x}$ $\Rightarrow$ $\displaystyle\int \frac{dx}{\cos^2 x} = \tan x + C$

(2) $\left(\dfrac{1}{\tan x}\right)' = -\dfrac{1}{\sin^2 x}$ $\Rightarrow$ $\displaystyle\int \frac{dx}{\sin^2 x} = -\frac{1}{\tan x} + C \quad (= -\cot x + C)$

(3) $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$ $\Rightarrow$ $\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

(4) $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$ $\Rightarrow$ $\displaystyle\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

問 2 の解答

(1) $-4\cos x - 3\sin x + C$

(2) $3\sin x - \tan x + C$

(3) $2\sin x + \cos x + C$

(4) $-\tan x + C$

(5) $-\cot x - x + C$

(6) $\tan x + C$

(7) $3\sin^{-1} x + C$

(8) $5\tan x + C$

＜ 5ページ. 積分記号 ＞

問の解答

(1) $10t - 4.9t^2 + C$

(2) $\frac{4}{3}\pi r^3 + C$

(3) $e^u + C$

(4) $\log|y| + C$

(5) $\sin u + C$

＜ 6 ページ. 置換積分法 (1) ＞

問の解答

(1) $\dfrac{1}{20}(5x+6)^4+C$

(2) $-\dfrac{1}{21(7x+5)^3}+C$

(3) $\dfrac{2}{15}(5x-3)\sqrt{5x-3}+C$

(4) $-\dfrac{1}{3}\cos(3x+2)+C$

(5) $-\dfrac{1}{3}e^{-3x+2}+C$

(6) $\dfrac{1}{4}\tan(4x+3)+C$

< 7ページ. 置換積分法 (2) >

問の解答

(1) $\frac{1}{2}e^{x^2+1} + C$

(2) $\frac{1}{4}e^{x^4} + C$

(3) $\frac{1}{3}\sin(x^3 + 2) + C$

(4) $-\frac{1}{2}\cos(x^3 + 3) + C$

(5) $\frac{1}{2}\log(x^2 + 3) + C$

(6) $\frac{1}{12}(x^2 + 1)^6 + C$

< 8 ページ. 不定積分の練習 (1) >

問の解答

(1) $\dfrac{1}{6}x^6 + \dfrac{1}{8}x^8 + C$

(2) $-x^{-1} + C \quad \left(= -\dfrac{1}{x} + C\right)$

(3) $\dfrac{3}{4}x^{\frac{4}{3}} + C \quad \left(= \dfrac{3}{4}x\sqrt[3]{x} + C\right)$

(4) $-\dfrac{1}{2x^2} + C$

(5) $\dfrac{2}{3}x\sqrt{x} + C$

(6) $\dfrac{4}{5}x\sqrt[4]{x} + C$

(7) $2\sqrt{x} + C$

(8) $\dfrac{x^2}{2} - 2x + \log|x| + C$

(9) $\log|x| + \dfrac{2}{x} - \dfrac{1}{2x^2} + C$

(10) $\dfrac{4}{3}x\sqrt{x} - 2\sqrt{x} + C$

(11) $x + 12\sqrt{x} + 9\log|x| + C$

(12) $2\sin x + 3\cos x + C$

(13) $-2\cos x + 3\sin x + C$

(14) $\tan x + 2x + C$

< 9 ページ. 不定積分の練習 (2) >

問の解答

(1) $\tan x + C$

(2) $-\cot x + C$

(3) $\tan x - x + C$

(4) $-\cot x - x + C$

(5) $5\sin^{-1} x + C$

(6) $4\tan x + C$

(7) $3e^x + C$

(8) $\dfrac{t^3}{3} - 3t^2 + 5t + C$

(9) $\dfrac{u^5}{5} - u^3 + C$

(10) $-\cos t + C$

(11) $\sin u + C$

(12) $e^u + C$

(13) $\log |u| + C$

(14) $\tan\theta + C$

＜ 10 ページ. 不定積分の練習 (3) ＞

問の解答

(1) $\frac{1}{2}e^{2x-3}+C$

(2) $\frac{1}{4}\sin(4x-2)+C$

(3) $-\frac{1}{3}\cos(3x+5)+C$

(4) $\frac{1}{5}\tan(5x+6)+C$

(5) $\frac{1}{4}\log|4x+3|+C$

(6) $\frac{1}{28}(7x-5)^4+C$

(7) $-\frac{1}{12(6x+1)^2}+C$

(8) $\frac{2}{15}(5x+1)\sqrt{5x+1}+C$

(9) $\frac{2}{7}\sqrt{7x-6}+C$

(10) $\frac{3}{20}(5x+1)\sqrt[3]{5x+1}+C$

(11) $-\frac{1}{2}e^{-x^2}+C$

(12) $-\frac{1}{3}e^{-x^3}+C$

(13) $\frac{1}{3}\sin(x^3+4)+C$

(14) $-\frac{1}{4}\cos(x^4)+C$

(15) $\frac{3}{2}\log(1+x^2)+C$

(16) $\frac{4}{3}\log|x^3+2|+C$

$<$ 11 ページ. 分数関数の積分 (1) $>$

問の解答

(1) $\log|x+1|+C$

(2) $\dfrac{1}{2}\log|2x+3|+C$

(3) $\dfrac{1}{a}\log|ax+b|+C$

(4) $-\dfrac{1}{x-3}+C$

(5) $-\dfrac{1}{3(3x-4)}+C$

(6) $-\dfrac{1}{a(ax+b)}+C$

(7) $-\dfrac{1}{8(4x-5)^2}+C$

(8) $-\dfrac{1}{2a(ax+b)^2}+C$

(9) $\tan^{-1}(x)+C$

(10) $\dfrac{1}{5}\tan^{-1}(\dfrac{x}{5})+C$

(11) $\dfrac{1}{6}\tan^{-1}\left(\dfrac{2x+1}{3}\right)+C$

(12) $\dfrac{1}{ar}\tan^{-1}\left(\dfrac{ax+b}{r}\right)+C$

< 12 ページ. 分数関数の積分 (2) >

問の解答

(1) $\log\left|\dfrac{x}{x+1}\right| + C$

(2) $\dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right| + C$

(3) $\log\left|\dfrac{x-3}{x-2}\right| + C$

(4) $\dfrac{1}{7}\log\left|\dfrac{x-3}{x+4}\right| + C$

(5) $\dfrac{1}{5}\log\left|\dfrac{2x+1}{3x+4}\right| + C$

$<$ 13ページ. 部分積分法 (1) $>$

問の解答

(1) $-x\cos x + \sin x + C$

(2) $xe^x - e^x + C$

(3) $\dfrac{x}{2}\sin(2x) + \dfrac{1}{4}\cos(2x) + C$

(4) $-\dfrac{x}{2}\cos(2x) + \dfrac{1}{4}\sin(2x) + C$

(5) $\dfrac{x}{3}e^{3x} - \dfrac{1}{9}e^{3x} + C$

< 14 ページ. 部分積分法 (2) >

問 1 の解答

(1) $\dfrac{x^2}{2}\log x - \dfrac{x^2}{4} + C$

(2) $\dfrac{x^3}{3}\log x - \dfrac{x^3}{9} + C$

問 2 の解答

(1) $-x^2\cos x + 2x\sin x + 2\cos x + C$

(2) $x^2e^x - 2xe^x + 2e^x + C$

＜ 15ページ. 三角関数の不定積分 ＞

問の解答

(1) $\dfrac{1}{2}x - \dfrac{1}{4}\sin(2x) + C$

(2) $\dfrac{1}{10}\sin(5x) + \dfrac{1}{2}\sin x + C$

(3) $\dfrac{1}{6}\sin(3x) - \dfrac{1}{10}\sin(5x) + C$

(4) $-\dfrac{1}{14}\cos(7x) - \dfrac{1}{2}\cos x + C$

(5) $\dfrac{1}{2}x + \dfrac{1}{12}\sin(6x) + C$

(6) $\dfrac{1}{2}x - \dfrac{1}{16}\sin(8x) + C$

＜ 16 ページ. 不定積分の検証 ＞

問の解答

(1) $\left\{ \dfrac{1}{4}(x^4-1)^4 \right\}' = 4x^3(x^4-1)^3$ より正しくない (×)

(2) $\left(\dfrac{1}{2}\log|x^2-1|\right)' = \dfrac{x}{x^2-1}$ より正しい (○)

(3) $(x^2e^x - 2xe^x + 2e^x)' = x^2e^x$ より正しい (○)

< 17 ページ. 不定積分の練習 (4) >

問の解答

(1) $\log|x+2|+C$

(2) $\dfrac{1}{2}\log|2x-3|+C$

(3) $\dfrac{4}{3}\log|3x+5|+C$

(4) $-\dfrac{1}{3(3x+4)}+C$

(5) $-\dfrac{3}{4(4x-5)}+C$

(6) $-\dfrac{1}{5(5x+4)^2}+C$

(7) $\log\left|\dfrac{x-1}{x+1}\right|+C$

(8) $\dfrac{1}{3}\log\left|\dfrac{x-1}{x+2}\right|+C$

(9) $\dfrac{1}{2}\log\left|\dfrac{x+1}{x+3}\right|+C$

(10) $\log\left|\dfrac{3x-2}{2x-1}\right|+C$

< 18ページ. 不定積分の練習 (5) >

問 1 の解答

(1) $2\tan^{-1}x + C$

(2) $\dfrac{1}{3}\tan^{-1}\left(\dfrac{x}{3}\right) + C$

(3) $\dfrac{1}{2}x + \dfrac{1}{4}\sin(2x) + C$

(4) $\dfrac{1}{2}x - \dfrac{1}{4}\sin(2x) + C$

(5) $xe^x - e^x + C$

(6) $x\sin x + \cos x + C$

(7) $x\log x - x + C$

(8) $\dfrac{x^4}{4}\log x - \dfrac{x^4}{16} + C$

(9) $x^2e^x - 2xe^x + 2e^x + C$

(10) $-\dfrac{1}{4}\cos(2x) + C$

問 2 の解答

(1) $\left(-\dfrac{1}{8(2x+3)^4}\right)' = \dfrac{1}{(2x+3)^5}$ より正しい

(2) $(x^2\sin x + 2x\cos x - 2\sin x)' = x^2\cos x$ より正しい

(3) $\left(\tan^{-1}\dfrac{x}{2}\right)' = \dfrac{\frac{1}{2}}{1+(\frac{x}{2})^2} = \dfrac{2}{4+x^2}$ より正しくない

(4) $$\left(\frac{1}{2}\log\left|\frac{x-2}{x+2}\right|\right)' = \left(\frac{1}{2}\log|x-2|\right)' - \left(\frac{1}{2}\log|x+2|\right)'$$
$$= \frac{1}{2}\times\frac{1}{x-2} - \frac{1}{2}\times\frac{1}{x+2} = \frac{(x+2)-(x-2)}{2(x-2)(x+2)}$$
$$= \frac{2}{x^2-4}$$

より正しくない

< 19 ページ. 定積分 (1) >

問の解答

(1) $\left[\dfrac{x^{n+1}}{n+1}\right]_a^b = \dfrac{b^{n+1}}{n+1} - \dfrac{a^{n+1}}{n+1}$

(2) $\Big[\log|x|\Big]_a^b = \log\left|\dfrac{b}{a}\right|$

(3) $\Big[x\Big]_a^b = b - a$

(4) $\Big[e^x\Big]_a^b = e^b - e^a$

(5) $\Big[\sin x\Big]_a^b = \sin b - \sin a$

(6) $\Big[-\cos x\Big]_a^b = -\cos b + \cos a$

(7) $\Big[\tan x\Big]_a^b = \tan b - \tan a$

(8) $\Big[\tan^{-1} x\Big]_a^b = \tan^{-1} b - \tan^{-1} a$

(9) $\Big[x\Big]_4^{10} = 6$

(10) $\left[-\dfrac{1}{x}\right]_1^5 = \dfrac{4}{5}$

(11) $\left[\dfrac{2}{3}x\sqrt{x}\right]_1^4 = \dfrac{14}{3}$

(12) $\left[\dfrac{3}{2}\sqrt[3]{x^2}\right]_1^8 = \dfrac{9}{2}$

(13) $\Big[\log x\Big]_1^e = 1$

(14) $\Big[e^x\Big]_0^2 = e^2 - 1$

(15) $\Big[\sin x\Big]_0^{\frac{\pi}{2}} = 1$

(16) $\Big[-\cos x\Big]_0^{\pi} = 2$

(17) $\Big[\tan x\Big]_0^{\frac{\pi}{4}} = 1$

(18) $\Big[\tan^{-1} x\Big]_0^1 = \dfrac{\pi}{4}$

< 20 ページ. 定積分 (2) >

問の解答

(1) $\left[2x - 3\log|x| - \dfrac{1}{x}\right]_1^2 = \dfrac{5}{2} - 3\log 2$

(2) $\displaystyle\int_{-1}^{1}(x^3 + x^4)dx = \left[\frac{x^4}{4} + \frac{x^5}{5}\right]_{-1}^{1} = \frac{2}{5}$

(3) $\left[\log\left|\dfrac{x}{x+1}\right|\right]_1^2 = \log\left(\dfrac{4}{3}\right)$

(4) $\left[\dfrac{x}{2} + \dfrac{1}{4}\sin(2x)\right]_0^{\pi} = \dfrac{\pi}{2}$

(5) $\left[-\dfrac{1}{12}\cos(6x) - \dfrac{1}{4}\cos(2x)\right]_0^{\frac{\pi}{2}} = \dfrac{2}{3}$

< 21 ページ. 定積分の積分変数 >

問の解答

(1) $\Big[4t - 5t^2\Big]_1^3 = -32$

(2) $\Big[\pi r^2\Big]_0^R = \pi R^2$

(3) $\Big[-\cos\theta\Big]_0^\pi = 2$

(4) $\left[\dfrac{u^{n+1}}{n+1}\right]_a^b = \dfrac{b^{n+1}}{n+1} - \dfrac{a^{n+1}}{n+1}$

(5) $\left[\dfrac{2}{3}u^{\frac{3}{2}}\right]_1^9 = \dfrac{52}{3}$

$<$ 22 ページ. 定積分の置換積分法 (1) $>$

問の解答

(1) $\displaystyle\int_{-1}^{1} 3x^2(x^3+1)^4\,dx = \int_0^2 u^4\,du = \left[\frac{u^5}{5}\right]_0^2 = \frac{32}{5}$

$(u = x^3 + 1$ とおく$)$

(2) $\displaystyle\int_0^{\sqrt{3}} 2x\sqrt{x^2+1}\,dx = \int_1^4 \sqrt{u}\,du = \left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^4 = \frac{14}{3}$

$(u = x^2 + 1$ とおく$)$

(3) $\displaystyle\int_0^1 \frac{4x^3}{(x^4+1)^2}\,dx = \int_1^2 \frac{1}{u^2}\,du = \left[-\frac{1}{u}\right]_1^2 = \frac{1}{2}$

$(u = x^4 + 1$ とおく$)$

＜ 23 ページ. 定積分の置換積分法 (2) ＞

問の解答

(1) $\displaystyle\int_0^1 x(x^2+2)^3\,dx = \int_2^3 \frac{1}{2}u^3\,du = \left[\frac{1}{8}u^4\right]_2^3 = \frac{65}{8}$

($u = x^2+2$ とおく)

(2) $\displaystyle\int_0^3 xe^{x^2}\,dx = \int_0^9 \frac{1}{2}e^u\,du = \left[\frac{1}{2}e^u\right]_0^9 = \frac{1}{2}e^9 - \frac{1}{2}$

($u = x^2$ とおく)

(3) $\displaystyle\int_{-1}^2 \frac{x^2}{x^3+2}\,dx = \int_1^{10} \frac{1}{u}\times\frac{1}{3}\,du = \left[\frac{1}{3}\log|u|\right]_1^{10} = \frac{1}{3}\log 10$

($u = x^3+2$ とおく)

(4) $\displaystyle\int_0^2 \frac{x}{(x^2+1)^3}\,dx = \int_1^5 \frac{1}{u^3}\times\frac{1}{2}\,du = \left[-\frac{1}{4u^2}\right]_1^5 = \frac{6}{25}$

($u = x^2+1$ とおく)

< 24 ページ. 定積分の置換積分法 (3) >

問の解答

(1) $\displaystyle\int_0^1 x(1-x)^7\,dx = \int_1^0 (1-u)u^7(-1)\,du$

$(u = 1 - x$ とおく$)$ $\displaystyle = \int_1^0 (u^8 - u^7)\,du$

$\displaystyle = \left[\frac{u^9}{9} - \frac{u^8}{8}\right]_1^0$

$\displaystyle = \frac{1}{72}$

(2) $\displaystyle\int_2^5 x\sqrt{x-1}\,dx = \int_1^4 (u+1)\sqrt{u}\,du$

$(u = x - 1$ とおく$)$ $\displaystyle = \int_1^4 (u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du$

$\displaystyle = \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_1^4$

$\displaystyle = \frac{256}{15}$

< 25 ページ. 定積分の置換積分法 (4) >

問の解答

(1) $\displaystyle\int_0^a \sqrt{a^2-x^2}dx = \int_0^{\frac{\pi}{2}} \sqrt{a^2-a^2\sin^2\theta}\ a\cos\theta\ d\theta = \int_0^{\frac{\pi}{2}} a^2\cos^2\theta\ d\theta$

$(x = a\sin\theta$ とおく$)$ $\displaystyle = \int_0^{\frac{\pi}{2}} \left\{ \frac{a^2}{2} + \frac{a^2}{2}\cos(2\theta) \right\} d\theta = \left[\frac{a^2}{2}\theta + \frac{a^2}{4}\sin(2\theta)\right]_0^{\frac{\pi}{2}}$

$\displaystyle = \frac{\pi}{4}a^2$

(2) $\displaystyle\int_0^{\sqrt{3}} \sqrt{4-x^2}dx = \int_0^{\frac{\pi}{3}} \sqrt{4-4\sin^2\theta}\ 2\cos\theta\ d\theta = \int_0^{\frac{\pi}{3}} 4\cos^2\theta\ d\theta$

$(x = 2\sin\theta$ とおく$)$ $\displaystyle = \int_0^{\frac{\pi}{3}} \left\{ 2 + 2\cos(2\theta) \right\} d\theta = \left[2\theta + \sin(2\theta)\right]_0^{\frac{\pi}{3}}$

$\displaystyle = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

< 26 ページ. 定積分の部分積分法 (1) >

問の解答

(1) $\displaystyle\int_0^1 x(x-1)^3\ dx = \left[x\frac{(x-1)^4}{4}\right]_0^1 - \int_0^1 \frac{(x-1)^4}{4}\ dx = 0-0-\left[\frac{(x-1)^5}{20}\right]_0^1 = \frac{1}{20}$

(2) $\displaystyle\int_0^{\pi} x\cos x\ dx = \Big[x\sin x\Big]_0^{\pi} - \int_0^{\pi} \sin x\ dx = 0-0+\Big[\cos x\Big]_0^{\pi} = -2$

(3) $\displaystyle\int_0^{\frac{\pi}{2}} x\sin x\ dx = \Big[x(-\cos x)\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (-\cos x)\ dx = 0-0+\Big[\sin x\Big]_0^{\frac{\pi}{2}} = 1$

(4) $\displaystyle\int_0^1 xe^x\ dx = \Big[xe^x\Big]_0^1 - \int_0^1 e^x\ dx = e-0-\Big[e^x\Big]_0^1 = 1$

< 27 ページ. 定積分の部分積分法 (2) >

問の解答

(1) $$\int_1^e x\log x\ dx = \left[\frac{x^2}{2}\log x\right]_1^e - \int_1^e \frac{x}{2}\ dx$$
$$= \frac{e^2}{2} - 0 - \left[\frac{x^2}{4}\right]_1^e$$
$$= \frac{e^2}{4} + \frac{1}{4}$$

(2) $$\int_1^e x^2\log x\ dx = \left[\frac{x^3}{3}\log x\right]_1^e - \int_1^e \frac{x^2}{3}\ dx$$
$$= \frac{e^3}{3} - 0 - \left[\frac{x^3}{9}\right]_1^e$$
$$= \frac{2}{3}e^3 + \frac{1}{9}$$

(3) $$\int_1^{\sqrt{e}} x^3\log x\ dx = \left[\frac{x^4}{4}\log x\right]_1^{\sqrt{e}} - \int_1^{\sqrt{e}} \frac{x^3}{4}\ dx$$
$$= \frac{e^2}{4}\log\sqrt{e} - 0 - \left[\frac{x^4}{16}\right]_1^{\sqrt{e}}$$
$$= \frac{e^2}{16} + \frac{1}{16}$$

(4) $$\int_1^e \log x\ dx = \int_1^e 1\times\log x\ dx$$
$$= \Big[x\log x\Big]_1^e - \int_1^e 1\ dx$$
$$= e - 0 - \Big[x\Big]_1^e = 1$$

＜ 28 ページ. 定積分の部分積分法 (3) ＞

問の解答

(1) $$\int_0^{\pi} x^2 \sin\pi \ dx = \Big[x^2(-\cos x)\Big]_0^{\pi} - \int_0^{\pi} 2x(-\cos x)\ dx$$
$$= -\pi^2\cos\pi - 0 + \Big[2x\sin x\Big]_0^{\pi} - \int_0^{\pi} 2\sin x\ dx$$
$$= \pi^2 + 2\pi\sin\pi - 0 + \Big[2\cos x\Big]_0^{\pi} = \pi^2 - 4$$

(2) $$\int_0^{\frac{\pi}{2}} x^2\cos x\ dx = \Big[x^2\sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x\sin x\ dx$$
$$= \frac{\pi^2}{4} - 0 + \Big[2x\cos x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2\cos x\ dx$$
$$= \frac{\pi^2}{4} + \pi\cos(\frac{\pi}{2}) - 0 + \Big[2\sin x\Big]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4} - 2$$

(3) $$\int_{-\pi}^{\pi} x^2\sin(2\pi)\ dx = \Big[x^2\Big(-\frac{\cos(2x)}{2}\Big)\Big]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} 2x\times\Big(-\frac{\cos(2x)}{2}\Big)\ dx$$
$$= -\frac{\pi^2}{2}\cos(2\pi) + \frac{\pi}{2}\cos(-2\pi) + \int_{-\pi}^{\pi} x\cos(2x)\ dx$$
$$= \Big[x\times\frac{\sin(2x)}{2}\Big]_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{\sin(2x)}{2}\ dx$$
$$= \frac{\pi}{2}\sin(2\pi) + \frac{\pi}{2}\sin(-2\pi) + \Big[\frac{\cos(2x)}{4}\Big]_{-\pi}^{\pi}$$
$$= \frac{\cos(2\pi)}{4} - \frac{\cos(-2\pi)}{4} = 0$$

< 29 ページ. 定積分の練習 (1) >

問の解答

(1) $\Big[x\Big]_{-1}^{3} = 4$

(2) $\Big[\log|x|\Big]_{1}^{\sqrt{e}} = \dfrac{1}{2}$

(3) $\Big[\dfrac{3}{4}x^{\frac{4}{3}}\Big]_{0}^{1} = \dfrac{3}{4}$

(4) $\Big[-3\cos x - 4\sin x\Big]_{0}^{\pi} = 6$

(5) $\Big[3x - 4\log|x| - \dfrac{1}{x}\Big]_{1}^{2} = \dfrac{7}{2} - 4\log 2$

(6) $\Big[2\sqrt{x}\Big]_{1}^{9} = 4$

(7) $\Big[\tan x\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{4}} = 1 + \sqrt{3}$

(8) $\Big[\dfrac{1}{3}\log|3x+1|\Big]_{0}^{2} = \dfrac{1}{3}\log 7$

(9) $\Big[\dfrac{1}{2}\log\Big|\dfrac{x-1}{x+1}\Big|\Big]_{2}^{3} = \dfrac{1}{2}\log\big(\dfrac{3}{2}\big)$

(10) $\displaystyle\int_0^{\pi} \frac{1}{2}\left\{\ \sin(3x) + \sin x\ \right\} dx = \Big[-\frac{1}{6}\cos(3x) - \frac{1}{2}\cos x\Big]_{0}^{\pi} = \frac{4}{3}$

(11) $\displaystyle\int_0^{\frac{\pi}{2}} \left\{\ \frac{1}{2} - \frac{1}{2}\cos(2x)\ \right\} dx = \Big[\frac{1}{2}x - \frac{1}{4}\sin(2x)\Big]_{0}^{\frac{\pi}{2}} = \frac{\pi}{4}$

(12) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left\{\ \frac{1}{2} + \frac{1}{2}\cos(4x)\ \right\} dx = \Big[\frac{1}{2}x + \frac{1}{8}\sin(4x)\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{4}$

(13) $\Big[\dfrac{1}{3}e^{3x-1}\Big]_{-2}^{2} = \dfrac{1}{3}e^{5} - \dfrac{1}{3}e^{-7}$

(14) $\Big[-\dfrac{1}{2}e^{-x^2}\Big]_{-1}^{1} = 0$

＜ 30 ページ. 定積分の練習 (2) ＞

問の解答

(1) $\displaystyle\int_1^4 \frac{1}{u^5}\times\frac{1}{3}\ du = \left[-\frac{1}{12}u^{-4}\right]_1^4 = \frac{85}{1024}$

($u = 3x+1$ とおく)

(2) $\displaystyle\int_4^{49} \sqrt{u}\times\frac{1}{5}\ du = \left[\frac{2}{15}u^{\frac{3}{2}}\right]_4^{49} = \frac{134}{3}$

($u = 5x-1$ とおく)

(3) $\displaystyle\int_1^2 \frac{1}{u^4}\times\frac{1}{3}\ du = \left[-\frac{1}{9}u^{-3}\right]_1^2 = \frac{7}{72}$

($u = x^3+1$ とおく)

(4) $\displaystyle\int_1^2 \frac{3}{u}\times\frac{1}{2}\ du = \left[\frac{3}{2}\log|u|\right]_1^2 = \frac{3}{2}\log 2$

($u = x^2+1$ とおく)

(5) $\displaystyle\left[x\times\frac{(x+1)^5}{5}\right]_0^1 - \int_0^1 \frac{(x+1)^5}{5}\ dx = \frac{2^5}{5} - 0 - \left[\frac{(x+1)^6}{30}\right]_0^1 = \frac{43}{10}$

(6) $\displaystyle\Big[x\sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\ dx = \frac{\pi}{2}\sin\frac{\pi}{2} - 0 + \Big[\cos x\Big]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$

(7) $\displaystyle\Big[x(-\cos x)\Big]_0^{\pi} - \int_0^{\pi} (-\cos x)\ dx = \pi\cos\pi - 0 + \Big[\sin x\Big]_0^{\pi} = \pi$

(8) $\displaystyle\Big[x\log x\Big]_e^{e^2} - \int_e^{e^2} 1\ dx = e^2\log e^2 - e\log e - \Big[x\Big]_e^{e^2} = e^2$

(9) $\displaystyle\Big[xe^x\Big]_{-1}^{1} - \int_{-1}^{1} e^x\ dx = e^1 - e^{-1} - \Big[e^x\Big]_{-1}^{1} = 2e^{-1} = \frac{2}{e}$

(10) $$\begin{aligned}\Big[x^2\sin x\Big]_0^{\pi} - \int_0^{\pi} 2x\sin x\ dx &= \pi^2\sin\pi - 0 + \Big[2x\cos x\Big]_0^{\pi} - \int_0^{\pi} 2\cos x\ dx \\ &= 2\pi\cos\pi - 0 - \Big[2\sin x\Big]_0^{\pi} \\ &= -2\pi\end{aligned}$$

＜ 31 ページ. 和の記号 Σ ＞

問 1 の解答

(1) $\displaystyle\sum_{k=1}^{n} k^3$

(2) $\displaystyle\sum_{k=1}^{n-1} k^2$

(3) $\displaystyle\sum_{k=1}^{n} k$

(4) $\displaystyle\sum_{k=1}^{n} \frac{k^2}{n^2}$

問 2 の解答

(1) $\displaystyle\sum_{k=1}^{1000} k = \frac{1000 \times 1001}{2} = 500500$

(2) $\displaystyle\sum_{k=1}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$

(3) $\displaystyle\sum_{k=1}^{n} \frac{k^2}{n^3} = \left(\sum_{k=1}^{n} k^2\right) \times \frac{1}{n^3} = \frac{n(n+1)(2n+1)}{6} \times \frac{1}{n^3} = \frac{(n+1)(2n+1)}{6n^2}$

(4) $\displaystyle\left(\sum_{k=1}^{n-1} k^2\right) \times \frac{1}{n^3} = \frac{(n-1)n(2n-1)}{6} \times \frac{1}{n^3} = \frac{(n-1)(2n-1)}{6n^2}$

(5) $\displaystyle\left(\sum_{k=1}^{n} k^3\right)\frac{1}{n^4} = \left\{ \frac{n(n+1)}{2} \right\}^2 \times \frac{1}{n^4} = \frac{(n+1)^2}{4n^2}$

(6) $\displaystyle\left(\sum_{k=1}^{n-1} k^2\right)\frac{x^3}{n^3} = \frac{(n-1)n(2n-1)}{6} \times \frac{x^3}{n^3} = \frac{(n-1)(2n-1)}{6n^2} \times x^3$

< 32 ページ. 和の極限値 >

問の解答

(1) $\displaystyle \lim_{n\to\infty}\left\{\frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+\cdots+\frac{(n-1)^2}{n^3}\right\}$

$$=\lim_{n\to\infty}\left(\sum_{k=1}^{n-1}k^2\right)\times\frac{1}{n^3}=\lim_{n\to\infty}\frac{(n-1)(2n-1)}{6n^2}=\lim_{n\to\infty}\frac{\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}=\frac{1}{3}$$

(2) $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n-1}\left(\frac{k}{n}\right)^3\times\frac{1}{n}$

$$=\lim_{n\to\infty}\left(\sum_{k=1}^{n-1}k^3\right)\times\frac{1}{n^4}=\lim_{n\to\infty}\left\{\frac{(n-1)n}{2}\right\}^2\times\frac{1}{n^4}=\lim_{n\to\infty}\frac{(1-\frac{1}{n})^2}{4}=\frac{1}{4}$$

(3) $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n-1}\left(\frac{kx}{n}\right)^2\times\frac{x}{n}$

$$=\lim_{n\to\infty}\left(\sum_{k=1}^{n-1}k^2\right)\times\frac{x^3}{n^3}=\lim_{n\to\infty}\frac{(n-1)(2n-1)}{6n^2}\times x^3=\frac{1}{3}x^3$$

(4) $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{kx}{n}\right)^3\times\frac{x}{n}$

$$=\lim_{n\to\infty}\left(\sum_{k=1}^{n}k^3\right)\times\frac{x^4}{n^4}=\lim_{n\to\infty}\frac{n^2(n+1)^2}{4}\times\frac{x^4}{n^4}=\frac{x^4}{4}$$

< 34 ページ. 区分求積法 (2) >

問 1 の解答

$$S_n = \sum_{k=1}^{n-1}(x_k)^2 \times \frac{1}{n} = \sum_{k=1}^{n-1}\left(\frac{k}{n}\right)^2 \times \frac{1}{n}$$

$$= \left(\sum_{k=1}^{n-1} k^2\right) \times \frac{1}{n^3} = \frac{(n-1)n(2n-1)}{6} \times \frac{1}{n^3}$$

$$= \frac{(n-1)(2n-1)}{6n^2}$$

問 2 の解答

$$S_1 = 0$$

$$S_2 = \frac{1}{8}$$

$$S_3 = \frac{5}{27}$$

問 3 の解答

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6} = \frac{1}{3}$$

< 35 ページ. 区分求積法 (3) >

問の解答

(1) $S_n^* = (x_1)^2 \times \dfrac{1}{n} + (x_2)^2 \times \dfrac{1}{n} + \cdots + (x_n)^2 \times \dfrac{1}{n}$

$$= \sum_{k=1}^{n} (x_k)^2 \times \frac{1}{n} = \sum_{k=1}^{n} \left(\frac{k}{n}\right)^2 \times \frac{1}{n}$$

$$= \left(\sum_{k=1}^{n} k^2\right) \times \frac{1}{n^3} = \frac{n(n+1)(2n+1)}{6} \times \frac{1}{n^3}$$

$$= \frac{(n+1)(2n+1)}{6n^2}$$

(2) $S_1^* = 1$ $\quad S_2^* = \dfrac{5}{8}$ $\quad S_3^* = \dfrac{14}{27}$

(3) $\displaystyle \lim_{n\to\infty} S_n^* = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6} = \frac{1}{3}$

(4) $S = \dfrac{1}{3}$

< 36 ページ. 区分求積法 (4) >

問 1 の解答

(1) $$S_n = \sum_{k=1}^{n-1}(x_k)^3 \times \frac{1}{n} = \sum_{k=1}^{n-1}\left(\frac{k}{n}\right)^3 \times \frac{1}{n}$$

$$= \left(\sum_{k=1}^{n-1} k^3\right) \times \frac{1}{n^4} = \left\{\frac{n(n-1)}{2}\right\}^2 \times \frac{1}{n^4}$$

$$= \frac{(n-1)^2}{4n^2}$$

(2) $$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{(n-1)^2}{4n^2} = \lim_{n\to\infty} \frac{\left(1-\frac{1}{n}\right)^2}{4} = \frac{1}{4}$$

問 2 の解答

$$S_n^* = \sum_{k=1}^{n}(x_k)^3 \times \frac{1}{n} = \left(\sum_{k=1}^{n} k^3\right) \times \frac{1}{n^4} = \frac{(n+1)^2}{4n^2}$$

$$\lim_{n\to\infty} S_n^* = \lim_{n\to\infty} \frac{(n+1)^2}{4n^2} = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^2}{4} = \frac{1}{4}$$

問 3 の解答

$$S = \frac{1}{4}$$

< 37 ページ. 区分求積法 (5) >

問 1 の解答

(1) $$S_n(x) = \sum_{k=1}^{n-1}(x_k)^2 \times \left(\frac{x}{n}\right) = \sum_{k=1}^{n-1}\left(\frac{kx}{n}\right)^2 \times \frac{x}{n}$$

$$= \left(\sum_{k=1}^{n-1} k^2\right) \times \frac{x^3}{n^3} = \frac{(n-1)n(2n-1)}{6} \times \frac{x^3}{n^3}$$

$$= \frac{(n-1)(2n-1)}{6n^2} \times x^3$$

(2) $$\lim_{n\to\infty} S_n(x) = \lim_{n\to\infty}\left(\frac{(n-1)(2n-1)}{6n^2}\right)x^3 = \frac{1}{3}x^3$$

問 2 の解答

(1) $$S_n^*(x) = \sum_{k=1}^{n}(x_k)^2 \times \left(\frac{x}{n}\right) = \left(\sum_{k=1}^{n} k^2\right) \times \frac{x^3}{n^3} = \frac{(n+1)(2n+1)}{6n^2}x^3$$

(2) $$\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty}\frac{(n+1)(2n+1)}{6n^2}x^3 = \frac{1}{3}x^3$$

問 3 の解答

$$S(x) = \frac{1}{3}x^3$$

< 38 ページ. 区分求積法 (6) >

問 1 の解答

(1) $$S_n(x) = \sum_{k=1}^{n-1}(x_k)^3 \times \frac{x}{n}$$
$$= \sum_{k=1}^{n-1}\left(\frac{kx}{n}\right)^3 \times \frac{x}{n} = \left(\sum_{k=1}^{n-1}k^3\right) \times \frac{x^4}{n^4}$$
$$= \left\{\frac{(n-1)n}{2}\right\}^2 \times \frac{x^4}{n^4} = \frac{(n-1)^2}{4n^2}x^4$$

(2) $$\lim_{n\to\infty} S_n(x) = \lim_{n\to\infty}\frac{(n-1)^2}{4n^2}x^4 = \frac{1}{4}x^4$$

問 2 の解答

(1) $$S_n^*(x) = \sum_{k=1}^{n}(x_k)^3 \times \frac{x}{n} = \left(\sum_{k=1}^{n}k^3\right)\frac{x^4}{n^4} = \frac{(n+1)^2}{4n^2}x^4$$

(2) $$\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty}\frac{(n+1)^2}{4n^2}x^4 = \frac{1}{4}x^4$$

問 3 の解答

$$S(x) = \frac{1}{4}x^4$$

< 39 ページ. 面積関数 (1) >

問 1 の解答

(1) $S(x) = x$

(2) $S(x) = \frac{1}{2}x^2$

問 2 の解答

(1) $S(x) = \frac{1}{3}x^3$

(2) $S(x) = \frac{1}{4}x^4$

問 3 の解答

(1) $S(x) = \frac{1}{5}x^5$

(2) $S(x) = \frac{1}{n+1}x^{n+1}$

問 4 の解答

$S'(x) = f(x) \qquad \left(\int f(x)dx = S(x)\right)$

< 40 ページ. 面積関数 (2) >

問の解答

(証明略)

＜ 42ページ. 面積 (1) ＞

問の解答

(1) $\displaystyle\int_0^1 e^x dx = \left[e^x\right]_0^1 = e - 1$

(2) $\displaystyle\int_1^9 \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^9 = \frac{52}{3}$

(3) $\displaystyle\int_1^2 \frac{1}{x^2}dx = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

(4) $\displaystyle\int_1^2 \frac{1}{x}dx = \Big[\log|x|\Big]_1^2 = \log 2$

$<$ 43ページ. 面積 (2) $>$

問 1 の解答

$$S=\int_a^b\{0-g(x)\}dx=-\int_a^b g(x)dx$$

問 2 の解答

$$S=\int_{\frac{\pi}{4}}^{\frac{5}{4}\pi}(\sin x-\cos x)dx=\Big[-\cos x-\sin x\Big]_{\frac{\pi}{4}}^{\frac{5}{4}\pi}$$

$$=2\sqrt{2}$$

問 3 の解答

(1) $\displaystyle\int_0^1\big(\sqrt{x}-x^2\big)dx=\left[\frac{2}{3}x^{\frac{3}{2}}-\frac{x^3}{3}\right]_0^1=\frac{1}{3}$

(2) $\displaystyle\int_1^4\left(-\frac{1}{4}x+\frac{5}{4}-\frac{1}{x}\right)dx=\left[-\frac{x^2}{8}+\frac{5}{4}x-\log|x|\right]_1^4$

$$==\frac{15}{8}-\log 4$$

< 44 ページ. 面積 (3) >

問 1 の解答

求める面積を S とおくと

$$S/4 = \int_0^a \sqrt{a^2 - x^2}\ dx = \int_0^{\frac{\pi}{2}} a^2 \cos^2\theta\ d\theta \qquad (x = a\sin\theta \text{ とおく})$$

$$= \int_0^{\frac{\pi}{2}} \left\{ \frac{a^2}{2} + \frac{a^2}{2}\cos(2\theta) \right\} d\theta = \left[\frac{a^2}{2}\theta + \frac{a^2}{4}\sin(2\theta) \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}a^2$$

よって $S = \left(\frac{\pi}{4}a^2\right) \times 4 = \pi a^2$ (答) $\underline{S = \pi a^2}$

問 2 の解答

$$S = \int_0^1 = \sqrt{4 - x^2}\ dx \qquad (x = 2\sin\theta \text{ とおく})$$

$$= \int_0^{\frac{\pi}{6}} \sqrt{4 - 4\sin^2\theta}\ 2\cos\theta\ d\theta = \int_0^{\frac{\pi}{6}} 4\cos^2\theta\ d\theta$$

$$= \int_0^{\frac{\pi}{6}} \left\{ 2 + 2\cos(2\theta) \right\} d\theta = \Big[2\theta + \sin(2\theta) \Big]_0^{\frac{\pi}{6}} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

< 45 ページ. 偶関数・奇関数の定積分 >

問の解答

(1) $\displaystyle\int_{-1}^{1}\left(x^3+x^4+x^5\right)dx = 2\int_0^1 x^4\ dx = 2\left[\frac{x^5}{5}\right]_0^1 = \frac{2}{5}$

(2) $\displaystyle\int_{-1}^{1}\left(x+x^3+x^6\right)dx = 2\int_0^1 x^6\ dx = 2\left[\frac{x^7}{7}\right]_0^1 = \frac{2}{7}$

(3) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin x+\cos x)\,dx = 2\int_0^{\frac{\pi}{2}}\cos x\ dx = 2\big[\sin x\big]_0^{\frac{\pi}{2}} = 2$

(4) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{1}{\cos^2 x}+\tan x\right)dx = 2\int_0^{\frac{\pi}{4}}\frac{1}{\cos^2 x}dx = 2\big[\tan x\big]_0^{\frac{\pi}{4}} = 2$

＜ 46 ページ. 定積分の応用問題 ＞

問 1 の解答

(1) $\displaystyle\sum_{k=1}^{100} k = \frac{100 \times 101}{2} = 5050$

(2) $\displaystyle\sum_{k=1}^{20} k^2 = \frac{20 \times 21 \times 41}{6} = 2870$

(3) $\displaystyle\sum_{k=1}^{10} k^3 = \left\{\frac{10 \times 11}{2}\right\}^2 = 3025$

問 2 の解答

(1) $\displaystyle\lim_{n\to\infty} \frac{n(n+1)}{2} \times \frac{1}{n^2} = \frac{1}{2}$

(2) $\displaystyle\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6} \times \frac{1}{n^3} = \frac{1}{3}$

(3) $\displaystyle\lim_{n\to\infty} \frac{n^2(n+1)^2}{4} \times \frac{1}{n^4} = \frac{1}{4}$

問 3 の解答

(1) $\displaystyle\int_a^b f(x)dx$ (2) $\displaystyle\int_0^1 f(x)dx$ (3) $\displaystyle\int_0^1 x^4dx$

問 4 の解答

(1) $\displaystyle 2\int_0^1 (x^2 + x^4)dx = 2\left[\frac{x^3}{3} + \frac{x^5}{5}\right]_0^1 = \frac{16}{15}$

(2) $\displaystyle 2\int_0^{\frac{\pi}{3}} \cos x \ dx = 2\Big[\sin x\Big]_0^{\frac{\pi}{3}} = \sqrt{3}$

問 5 の解答

(1) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}} \ dx = \Big[2\sqrt{x}\Big]_1^4 = 2$

(2) $\displaystyle\int_{-1}^2 \left\{(-x^2+3) - (x^2-2x-1)\right\} \ dx = \left[-\frac{2}{3}x^3 + x^2 + 4x\right]_{-1}^2 = 9$

(3) $\displaystyle\int_{-1}^0 \left\{x^3 - x\right\} dx + \int_0^1 (x - x^3)dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$

(4) $\displaystyle\int_1^e \log x dx = \Big[x \log x\Big]_1^e - \int_1^e 1dx = e\log e - 1\log 1 - [x]_1^e = 1$

< 47 ページ. 関数の極限 >

問の解答

(1) $\displaystyle\lim_{x\to1}\frac{x^2-16}{x-4}$

$\displaystyle=\frac{1-16}{1-4}=5$

(2) $\displaystyle\lim_{x\to4}\frac{x^2-16}{x-4}$

$\displaystyle=\lim_{x\to4}\frac{(x-4)(x+4)}{x-4}=\lim_{x\to4}(x+4)=8$

(3) $\displaystyle\lim_{x\to2}\frac{x^3-27}{x-3}$

$\displaystyle=\frac{8-27}{2-3}=19$

(4) $\displaystyle\lim_{x\to3}\frac{x^3-27}{x-3}$

$\displaystyle=\lim_{x\to3}\frac{(x-3)(x^2+3x+9)}{x-3}=\lim_{x\to3}(x^2+3x+9)=27$

(5) $\displaystyle\lim_{x\to2}\frac{x^4-16}{x-2}$

$\displaystyle=\lim_{x\to2}\frac{(x-2)(x^3+2x^2+4x+8)}{(x-2)}=\lim_{x\to2}(x^3+2x^2+4x+8)=32$

(6) $\displaystyle\lim_{x\to1}\frac{x^5-1}{x-1}$

$\displaystyle=\lim_{x\to1}\frac{(x-1)(x^4+x^3+x^2+x+1)}{(x-1)}=\lim_{x\to1}(x^4+x^3+x^2+x+1)=5$

$<$ 48 ページ. ロピタルの定理 (1) $>$

問の解答

(1) $\displaystyle\lim_{x\to 2}\frac{x^4-16}{x-2}=\lim_{x\to 2}\frac{4x^3-0}{1-0}=4\times 2^3=32$

(2) $\displaystyle\lim_{x\to 1}\frac{e^x-e}{x-1}=\lim_{x\to 1}\frac{e^x-0}{1-0}=e^1=e$

(3) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\cos x}{1}=\cos 0=1$

＜ 49 ページ. ロピタルの定理 (2) ＞

問の解答

(1) $$\lim_{x\to1}\frac{x^5-1-5(x-1)}{(x-1)^2}=\lim_{x\to1}\frac{5x^4-5}{2(x-1)}=\lim_{x\to1}\frac{20x^3}{2}=10$$

(2) $$\lim_{x\to2}\frac{x^5-32-80(x-2)}{(x-2)^2}=\lim_{x\to2}\frac{5x^4-80}{2(x-2)}=\lim_{x\to2}\frac{20x^3}{2}=80$$

(3) $$\lim_{x\to1}\frac{x^4-1-4(x-1)-6(x-1)^2}{(x-1)^3}=\lim_{x\to1}\frac{4x^3-4-12(x-1)}{3(x-1)^2}$$
$$=\lim_{x\to1}\frac{12x^2-12}{6(x-1)}=\lim_{x\to1}\frac{24x}{6}=4$$

(4) $$\lim_{x\to1}\frac{x^5-1-5(x-1)-10(x-1)^2-10(x-1)^3}{(x-1)^4}$$
$$=\lim_{x\to1}\frac{5x^4-5-20(x-1)-30(x-1)^2}{4(x-1)^3}$$
$$=\lim_{x\to1}\frac{20x^3-20-60(x-1)}{12(x-1)^2}=\lim_{x\to1}\frac{60x^2-60}{24(x-1)}=\lim_{x\to1}\frac{120x}{24}=5$$

＜ 50 ページ. 微分記号 ＞

問の解答

(1) 0

(2) $b^4 + 2c^5x$

(3) 0

(4) $(a-b)^2$

(5) a^4

(6) $2a^3(x+c)$

(7) $4a(ax+b)^3$

(8) $5(x-a)^4$

(9) $3a^3(x-a)^2$

(10) $16a^3(x-b)^3$

(11) $2x - 2a$

(12) $3x^2 - 3a^2 - 12a(x-a)$
$(= 3x^2 - 12ax + 9a^2)$

< 51 ページ. ロピタルの定理 (3) >

問の解答

(1) $\displaystyle\lim_{x\to a}\frac{x^2-a^2-2a(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{2x-2a}{2(x-a)}=1$

(2) $\displaystyle\lim_{x\to a}\frac{x^4-a^4-4a^3(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{4x^3-4a^3}{2(x-a)}=\lim_{x\to a}\frac{12x^2}{2}=6a^2$

(3) $\displaystyle\lim_{x\to a}\frac{x^5-a^5-5a^4(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{5x^4-5a^4}{2(x-a)}=\lim_{x\to a}\frac{20x^3}{2}=10a^3$

< 52 ページ. ロピタルの定理 (4) >

問の解答

(1) $$\lim_{x\to a}\frac{x^4-a^4-4a^3(x-a)-6a^2(x-a)^2}{(x-a)^3}$$

$$=\lim_{x\to a}\frac{4x^3-4a^3-12a^2(x-a)}{3(x-a)^2}=\lim_{x\to a}\frac{12x^2-12a^2}{6(x-a)}=\lim_{x\to a}\frac{24x}{6}=4a$$

(2) $$\lim_{x\to a}\frac{x^6-a^6-6a^5(x-a)-15a^4(x-a)^2}{(x-a)^3}$$

$$=\lim_{x\to a}\frac{6x^5-6a^5-30a^4(x-a)}{3(x-a)^2}=\lim_{x\to a}\frac{30x^4-30a^4}{6(x-a)}=\lim_{x\to a}\frac{120x^3}{6}=20a^3$$

(3) $$\lim_{x\to a}\frac{x^7-a^7-7a^6(x-a)-21a^5(x-a)^2-35a^4(x-a)^3}{(x-a)^4}$$

$$=\lim_{x\to a}\frac{7x^6-7a^6-42a^5(x-a)-105a^4(x-a)^2}{4(x-a)^3}$$

$$=\lim_{x\to a}\frac{42x^5-42a^5-210a^4(x-a)}{12(x-a)^2}=\lim_{x\to a}\frac{210x^4-210a^4}{24(x-a)}$$

$$=\lim_{x\to a}\frac{840x^5}{24}=35a^3$$

＜ 53 ページ. 関数の 1 次近似 ＞

問の解答

(1) $f'(x) = 6x^5$ より　(答) $x \fallingdotseq a$ のとき　$x^6 \fallingdotseq a^6 + 6a^5(x-a)$

(2) $f'(x) = \dfrac{1}{2\sqrt{x}}$ より　(答) $x \fallingdotseq a$ のとき　$\sqrt{x} \fallingdotseq \sqrt{a} + \dfrac{1}{2\sqrt{a}}(x-a)$

(3) $f'(x) = \dfrac{1}{x}$ より　(答) $x \fallingdotseq a$ のとき　$\log x \fallingdotseq \log a + \dfrac{1}{a}(x-a)$

(4) $f'(x) = \cos x$ より　(答) $x \fallingdotseq a$ のとき　$\sin x \fallingdotseq \sin a + (\cos a)(x-a)$

(5) $f'(x) = -\sin x$ より　(答) $x \fallingdotseq a$ のとき　$\cos x \fallingdotseq \cos a - (\sin a)(x-a)$

(6) $f'(x) = e^x$ より　(答) $x \fallingdotseq a$ のとき　$e^x \fallingdotseq e^a + e^a(x-a)$

< 54 ページ. 関数の高次近似 (1) >

問の解答

(1) $f'(x) - f'(a)$

(2) $f''(x) - f''(a)$

(3) $f'(x) - f'(a) - f''(a)(x-a)$

(4) $f''(x) - f''(a) - f'''(a)(x-a)$

(5) $f'(x) - f'(a) - f''(a)(x-a) - \dfrac{1}{2}f'''(a)(x-a)^2$

＜ 55 ページ. 関数の高次近似 (2) ＞

問の解答

(1) $$\lim_{x \to a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2}{(x-a)^3}$$

$$= \lim_{x \to a} \frac{f'(x) - f'(a) - f''(a)(x-a)}{3(x-a)^2}$$

$$= \lim_{x \to a} \frac{f''(x) - f''(a)}{6(x-a)} = \lim_{x \to a} \frac{f'''(x)}{6} = \frac{f'''(a)}{6}$$

(2) $$\lim_{x \to a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2 - \frac{1}{6}f'''(a)(x-a)^3}{(x-a)^4}$$

$$= \lim_{x \to a} \frac{f'(x) - f'(a) - f''(a)(x-a) - \frac{1}{2}f'''(a)(x-a)^2}{4(x-a)^3}$$

$$= \lim_{x \to a} \frac{f''(x) - f''(a) - f'''(a)(x-a)}{12(x-a)^2}$$

$$= \lim_{x \to a} \frac{f'''(x) - f'''(a)}{24(x-a)} = \lim_{x \to a} \frac{f''''(x)}{24} = \frac{f''''(a)}{24}$$

< 56 ページ. 関数の高次近似 (3) >

問の解答

$$f(x) \fallingdotseq f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3 + \frac{1}{24}f''''(a)(x-a)^4$$

< 57 ページ. 高階微分係数 >

問の解答

(1) $f^{(4)}(x) = e^x \quad , \quad f^{(4)}(0) = e^0 = 1$

(2) $f^{(n)}(x) = e^x \quad , \quad f^{(n)}(0) = e^0 = 1$

(3) $f^{(1)}(x) = \cos x$, $f^{(2)}(x) = -\sin x$, $f^{(3)}(x) = -\cos x$, $f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x$, $f^{(6)}(x) = -\sin x$, $f^{(7)}(x) = -\cos x$, $f^{(8)}(x) = \sin x$

$f^{(1)}(0) = \cos 0 = 1$, $f^{(2)}(0) = -\sin 0 = 0$, $f^{(3)}(0) = -\cos 0 = -1$, $f^{(4)}(0) = \sin 0 = 0$

$f^{(5)}(0) = \cos 0 = 1$, $f^{(6)}(0) = -\sin 0 = 0$, $f^{(7)}(0) = -\cos 0 = -1$, $f^{(8)}(0) = \sin 0 = 0$

< 58 ページ. 関数の n 次近似 >

問の解答

(1) $e^x \fallingdotseq e^a + e^a(x-a) + \dfrac{1}{2!}e^a(x-a)^2 + \dfrac{1}{3!}e^a(x-a)^3 + \cdots + \dfrac{1}{n!}e^a(x-a)^n$

(2) $e^x \fallingdotseq e + e(x-1) + \dfrac{1}{2!}e(x-1)^2 + \dfrac{1}{3!}e(x-1)^3 + \cdots + \dfrac{1}{n!}e(x-1)^n$

(3) $e^x \fallingdotseq 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!}$

< 59 ページ. テーラー展開 >

問 1 の解答

$$e^x = e^a + e^a(x-a) + \frac{1}{2!}e^a(x-a)^2 + \frac{1}{3!}e^a(x-a)^3 + \cdots + \frac{1}{n!}e^a(x-a)^n + \cdots$$

問 2 の解答

(1) $e^x = e + e(x-1) + \frac{1}{2!}e(x-1)^2 + \frac{1}{3!}e(x-1)^3 + \cdots + \frac{1}{n!}e(x-1)^n + \cdots$

(2) $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$

＜ 60 ページ. マクローリン展開 (1) ＞

問 1 の解答

$f(0) = \sin 0 = 0$, $f^{(1)}(0) = 1$, $f^{(2)}(0) = 0$, $f^{(3)}(0) = -1$, $f^{(4)}(0) = 0$, $f^{(5)}(0) = 1$, $\cdots$

より

$$\underline{\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} + \cdots}$$

問 2 の解答

(1) e^x のマクローリン展開より

$$\lim_{x\to 0}\frac{e^x - 1 - x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!}}{x^5}$$

$$= \lim_{x\to 0}\frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots\right) - 1 - x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!}}{x^5}$$

$$= \lim_{x\to 0}\frac{\frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots}{x^5} = \lim_{x\to 0}\left(\frac{1}{5!} + \frac{x}{6!} + \frac{x^2}{7!} + \cdots\right) = \frac{1}{5!}$$

(2) $\cos x$ のマクローリン展開より

$$\lim_{x\to 0}\frac{\cos x - 1 + \frac{x^2}{2!} - \frac{x^4}{4!}}{x^6}$$

$$= \lim_{x\to 0}\frac{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots\right) - 1 + \frac{x^2}{2!} - \frac{x^4}{4!}}{x^6}$$

$$= \lim_{x\to 0}\frac{-\frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots}{x^6} = \lim_{x\to 0}\left(-\frac{1}{6!} + \frac{x^2}{8!} - \frac{x^4}{10!} + \cdots\right) = -\frac{1}{6!}$$

(3) $\sin x$ のマクローリン展開より

$$\lim_{x\to 0}\frac{\sin x - x + \frac{x^3}{3!} - \frac{x^5}{5!}}{x^7}$$

$$= \lim_{x\to 0}\frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots\right) - x + \frac{x^3}{3!} - \frac{x^5}{5!}}{x^7}$$

$$= \lim_{x\to 0}\frac{-\frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots}{x^7} = \lim_{x\to 0}\left(-\frac{1}{7!} + \frac{x^2}{9!} - \frac{x^4}{11!} + \cdots\right) = -\frac{1}{7!}$$

< 62 ページ. マクローリン展開 (3) >

問の解答

$$
\begin{aligned}
e &\fallingdotseq 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} \\
&= \frac{65}{24} \\
&\fallingdotseq 2.70833
\end{aligned}
$$

＜ 63 ページ. 練習問題 ＞

問 1 の解答

$$\lim_{x \to 1} \frac{x^6 - 1}{x - 1} = \lim_{x \to 1} \frac{6x^5}{1} = 6$$

(2) $\displaystyle \lim_{x \to 0} \frac{\sin x - x}{x^3} = \lim_{x \to 0} \frac{\cos x - 1}{3x^2} = \lim_{x \to 0} \frac{-\sin x}{6x} = \lim_{x \to 0} \frac{-\cos x}{6} = -\frac{1}{6}$

(3) $\displaystyle \lim_{x \to 2} \frac{e^x - e^2 - e^2(x-2)}{(x-2)^2} = \lim_{x \to 2} \frac{e^x - e^2}{2(x-2)} = \lim_{x \to 2} \frac{e^x}{2} = \frac{e^2}{2}$

(4) $\displaystyle \lim_{x \to a} \frac{x^4 - a^4 - 4a^3(x-a) - 6a^2(x-a)^2}{(x-a)^3} = \lim_{x \to a} \frac{4x^3 - 4a^3 - 12a^2(x-a)}{3(x-a)^2}$

$\displaystyle = \lim_{x \to a} \frac{12x^2 - 12a^2}{6(x-a)} = \lim_{x \to a} \frac{24x}{6} = 4a$

問 2 の解答

(1) $x \fallingdotseq a$ のとき $\sin x \fallingdotseq \sin a + (\cos a)(x - a)$

(2) $x \fallingdotseq a$ のとき $e^x \fallingdotseq e^a + e^a(x - a)$

(3) $x \fallingdotseq a$ のとき $x^7 \fallingdotseq a^7 + 7a^6(x - a)$

(4) $x \fallingdotseq a$ のとき $f(x) \fallingdotseq f(a) + f'(a)(x-a) + \dfrac{1}{2}f''(a)(x-a)^2$

問 3 の解答

(1) $\displaystyle f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} \cdots$

(2) $\displaystyle f(x) = \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots$

(3) $\displaystyle f(x) = \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$

問 4 の解答

(1) $\displaystyle e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \cdots + \frac{1}{n!} \cdots$

(2) $\displaystyle \sin 1 = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} - \frac{1}{11!} + \cdots$

(3) $\displaystyle \cos 1 = 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} - \frac{1}{10!} + \cdots$