高知工科大学

基礎数学ワークブック

(2004年度版)

初級編

No. 5

解答

＜１ページ. 実数＞

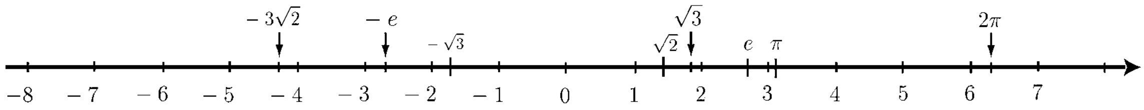
問１の解答

(1) $\dfrac{5}{8} = 0.625$

(2) $\dfrac{1}{15} = 0.0666\cdots = 0.0\dot{6}$

(3) $\dfrac{2}{11} = 0.18181818\cdots = 0.\dot{1}\dot{8}$

問２の解答



< 2ページ.虚数の導入 (1) >

問の解答

(1) $x = \pm 4i$

(2) $x = \pm \dfrac{5}{2}i$

(3) $x = \pm \dfrac{\sqrt{3}}{3}i$

＜ 3 ページ. 虚数の導入 (2) ＞

問の解答

(1) $x = \dfrac{1}{2} \pm \sqrt{3}\,i$

(2) $x = \dfrac{6 \pm \sqrt{36-52}}{2} = \dfrac{6 \pm \sqrt{16}\,i}{2} = 3 \pm 2i$

(3) $x = a \pm \dfrac{c}{b} i$

＜ 4ページ. 複素数の定義 ＞

問の解答

(1) $a = \dfrac{1}{2}$, $b = \dfrac{3}{2}$

(2) $a = 0$, $b = \dfrac{1-\sqrt{2}}{3}$

＜ 5 ページ. 複素数の四則演算 (1) ＞

問 1 の解答

(1) $(2+i)+(3-i)$

$=5$

(2) $(4-i)-(5-3i)$

$=-1+2i$

(3) $\left(0.13+\frac{1}{2}i\right)+\left(\frac{3}{4}-1.5i\right)$

$=0.13+0.75+\left(\frac{1}{2}-\frac{3}{2}\right)i=0.88-i$

(4) $\left(\frac{1}{4}-\frac{1}{3}i\right)-\left(\frac{1}{8}-\frac{1}{3}i\right)$

$=\frac{1}{8}$

(5) $\left(\sqrt{3}-i\right)+\left(\sqrt{1}-2i\right)$

$=\sqrt{3}+1-3i$

(6) $\left(\frac{1}{4}-\sqrt{2}i\right)-\left(\frac{1}{3}+\sqrt{3}i\right)$

$=-\frac{1}{12}-(\sqrt{2}+\sqrt{3})i$

問 2 の解答

(1) $3(4+i)$

$=12+3i$

(2) $6\left(\frac{1}{4}-\frac{1}{2}i\right)$

$=\frac{3}{2}-3i$

(3) $3(6-2i)-4(2-i)$

$=18-6i-8+4i$

$=10-2i$

(4) $\sqrt{3}\left(\frac{1}{\sqrt{3}}-\sqrt{3}i\right)+\left(\frac{1}{3}-2i\right)$

$=1-3i+\frac{1}{3}-2i$

$=\frac{4}{3}-5i$

< 6 ページ. 複素数の四則演算 (2) >

問の解答

(1) $i^3 = -i$ (2) $i^4 = 1$ (3) $i^5 = i$

(4) $i^6 = -1$ (5) $i^7 = -i$ (6) $i^8 = 1$

(7) $(1+i)(1-i) = 1 - i^2 = 2$

(8) $(2+\sqrt{3}i)(2-\sqrt{3}i) = 4 - 3i^2 = 7$

(9) $\left(\dfrac{\sqrt{3}+i}{2}\right)\left(\dfrac{\sqrt{3}-i}{2}\right) = \dfrac{3-i^2}{4} = 1$

(10) $(-1+i)^2 = 1 - 2i + i^2 = -2i$

(11) $(-1-i)^2 = 1 + 2i + i^2 = 2i$

(12) $(4+2i)(2-3i) = 8 - 12i + 4i - 6i^2$
$= 14 - 8i$

(13) $(3-2i)(1-3i) = 3 - 9i - 2i + 6i^2 = -3 - 11i$

(14) $(3-i)^3 = 3^3 - 3 \times 3^2 i + 3 \times 3 \times i^2 - i^3 = 27 - 27i - 9 - (-i)$
$= 18 - 26i$

＜7ページ. 複素数の四則演算 (3) ＞

問の解答

(1) $\dfrac{-1}{1+i} = \dfrac{-1(1-i)}{1^2-i^2} = \dfrac{i-1}{2}$

(2) $\dfrac{-1}{1-i} = \dfrac{-(1+i)}{1^2-i^2} = \dfrac{-1-i}{2}$

(3) $\dfrac{-i}{1-i} = \dfrac{-i(1+i)}{1^2-i^2} = \dfrac{-i+1}{2}$

$= \dfrac{1-i}{2}$

(4) $\dfrac{3}{\sqrt{5}-i} = \dfrac{3(\sqrt{5}+i)}{5-i^2} = \dfrac{3(\sqrt{5}+i)}{6}$

$= \dfrac{\sqrt{5}+i}{2}$

(5) $\dfrac{7}{3+\sqrt{5}i} = \dfrac{7(3-\sqrt{5}i)}{3^2-5i^2}$

$= \dfrac{7(3-\sqrt{5}i)}{14} = \dfrac{3-\sqrt{5}i}{2}$

(6) $\dfrac{-i}{1+i} = \dfrac{-i(1-i)}{1^2-i^2} = \dfrac{-i+i^2}{2}$

$= \dfrac{-1-i}{2}$

(7) $\dfrac{1}{\sqrt{3}i(\sqrt{3}+i)} = \dfrac{1}{3i-\sqrt{3}}$

$= \dfrac{3i+\sqrt{3}}{(3i)^2-3} = \dfrac{3i+\sqrt{3}}{-9-3} = -\dfrac{\sqrt{3}+3i}{12}$

(8) $\dfrac{\sqrt{2}}{\sqrt{2}-i} = \dfrac{\sqrt{2}(\sqrt{2}+i)}{2-i^2} = \dfrac{2+\sqrt{2}i}{3}$

(9) $\dfrac{1}{(\sqrt{2}-i)^2} = \dfrac{1}{2-2\sqrt{2}i+i^2}$

$= \dfrac{1}{1-2\sqrt{2}i} = \dfrac{1+2\sqrt{2}i}{1^2-(2\sqrt{2}i)^2}$

$= \dfrac{1+2\sqrt{2}i}{9}$

(10) $\dfrac{i}{(1+i)^4} = \dfrac{i}{1+4i+6i^2+4i^3+i^4}$

$= \dfrac{i}{1+4i-6-4i+1}$

$= -\dfrac{i}{4}$

＜ 8ページ. 負の数の平方根 ＞

問の解答

(1) $\sqrt{(-3)\times(-4)\times(-5)}$

$= \sqrt{-60} = \sqrt{60}i = 2\sqrt{15}i$

(2) $\sqrt{-3}\times\sqrt{-4}\times\sqrt{-5} = \sqrt{3}i\times 2i\times\sqrt{5}i$

$= -2\sqrt{15}i$

(3) $\dfrac{\sqrt{12}}{\sqrt{-4}} = \dfrac{2\sqrt{3}}{2i} = \dfrac{\sqrt{3}i}{i^2} = -\sqrt{3}i$

(4) $\sqrt{\dfrac{12}{-4}} = \sqrt{-3} = \sqrt{3}i$

＜ 9 ページ.2 次方程式 ＞

問の解答

(1) $x^2 + x + 2 = 0$ $\qquad x = \dfrac{-1 \pm \sqrt{1-8}}{2} = \dfrac{-1 \pm \sqrt{7}i}{2}$

(2) $x^2 + 3x + 9 = 1$

$x^2 + 3x + 8 = 0$ $\qquad x = \dfrac{-3 \pm \sqrt{9-32}}{2} = \dfrac{-3 \pm \sqrt{23}i}{2}$

(3) $3x^2 - 5x + 4 = 0$ $\qquad x = \dfrac{5 \pm \sqrt{25-48}}{6} = \dfrac{5 \pm \sqrt{23}i}{6}$

＜ 10 ページ.2 次式の因数分解 ＞

問の解答

(1) $x^2-2x+5=(x-1)^2+4=(x-1-2i)(x-1+2i)$

(2) $-5x^2+4x-3=-5\left(x-\dfrac{2-\sqrt{11}i}{5}\right)\left(x-\dfrac{2+\sqrt{11}i}{5}\right)$

$=-5\left(x-\dfrac{2}{5}+\dfrac{\sqrt{11}}{5}i\right)\left(x-\dfrac{2}{5}-\dfrac{\sqrt{11}}{5}i\right)$

(3) $3x^2-3x+3=3\left(x^2-x+1\right)=3\left(x-\dfrac{1+\sqrt{3}i}{2}\right)\left(x-\dfrac{1-\sqrt{3}i}{2}\right)$

$=3\left(x-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)\left(x-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)$

< 11 ページ. 高次式の因数分解 >

問の解答

(1) $x^3 - 1 = (x-1)(x^2+x+1) = (x-1)\left(x - \frac{-1+\sqrt{3}i}{2}\right)\left(x - \frac{-1-\sqrt{3}i}{2}\right)$

$= (x-1)\left(x + \frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\left(x + \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

(2) $x^3 + 8 = (x+2)(x^2-2x+4) = (x+2)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)$

(3) $x^4 - 1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x-i)(x+i)$

＜ 12 ページ. 高次方程式 ＞

問の解答

(1) $x^3 - 1 = 0 \quad (x-1)\left(x - \frac{-1+\sqrt{3}i}{2}\right)\left(x - \frac{-1-\sqrt{3}i}{2}\right) = 0$

(答) $x = 1\ ,\ \frac{-1+\sqrt{3}i}{2}\ ,\ \frac{-1-\sqrt{3}i}{2}$

(2) $x^3 + 27 = 0 \quad (x+3)(x^2 - 3x + 9) = 0$

(答) $x = -3\ ,\ \frac{3 \pm 3\sqrt{3}i}{2}$

(3) $x^4 - 1 = 0 \quad (x-1)(x+1)(x-i)(x+i) = 0$

(答) $x = \pm 1\ ,\ \pm i$

< 13 ページ. 共役複素数 >

問 1 の解答

(1) $z = 1$, $\overline{z} = 1$

(2) $z = i$, $\overline{z} = -i$

(3) $z = 1 - i$, $\overline{z} = 1 + i$

(4) $z = \dfrac{1+i}{2}$, $\overline{z} = \dfrac{1-i}{2}$

問 2 の解答

(1) $\dfrac{1}{2}(z + \overline{z})$

$= 4$

(2) $\dfrac{1}{2i}(z - \overline{z})$

$= \dfrac{1}{2i}\left(4 + 3i - (4 - 3i)\right)$

$= \dfrac{1}{2i} \times 6i = 3$

(3) $z\overline{z}$

$= 4^2 - 3^2 i^2 = 25$

問 3 の解答

(1) $\dfrac{1}{2}(z + \overline{z})$

$= a$

(2) $\dfrac{1}{2i}(z - \overline{z})$

$= b$

(3) $z\overline{z}$

$= a^2 + b^2$

＜ 14 ページ. 絶対値 ＞

問 1 の解答

(1) $z = -1$　　(2) $z = 7i$　　(3) $z = 3 + 4i$　　(4) $z = \dfrac{1+i}{2}$

$|z| = 1$　　$|z| = 7$　　$|z| = 5$　　$|z| = \sqrt{\dfrac{1}{4} + \dfrac{1}{4}} = \dfrac{\sqrt{2}}{2}$

問 2 の解答

(1) $z = 4 - 3i$

$|z|^2 = 4^2 + 3^2 = 25$

$z^2 = (4-3i)^2 = 16 - 24i + 9i^2$

$= 7 - 24i$

$|z^2| = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$

(2) $z = 1 + i$

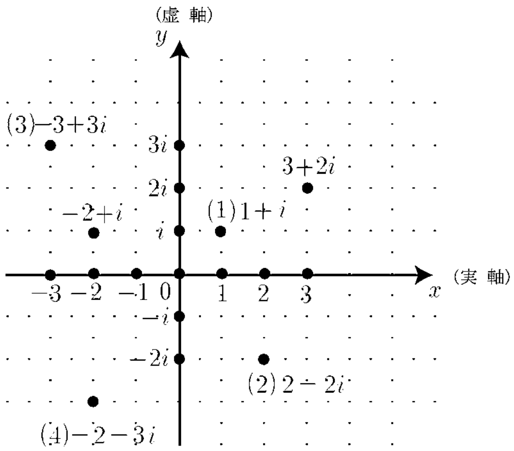
$|z|^2 = 1^2 + 1^2 = 2$

$z^2 = (1+i)^2 = 2i$

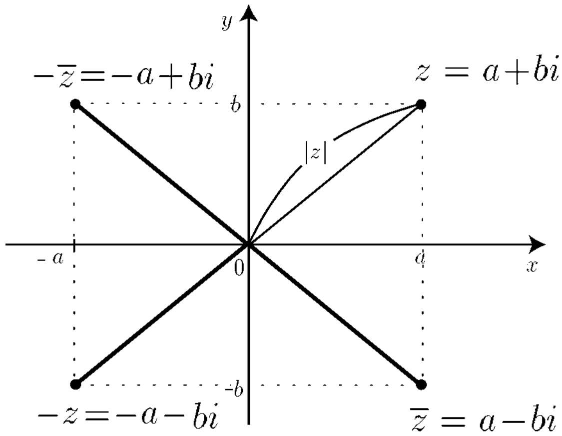
$|z^2| = \sqrt{2^2} = 2$

＜ 15 ページ. 複素平面 (1) ＞

問 1 の解答



問 2 の解答



＜ 16 ページ. 複素平面 (2) ＞

問の解答

① $z_1 + z_2 = (3+i) + (1+3i)$
$= 4 + 4i$

② $z_1 - z_2 = (3+i) - (1+3i)$
$= 2 - 2i$

③ $-\dfrac{3}{2}z_1 = -\dfrac{3}{2}(3+i)$

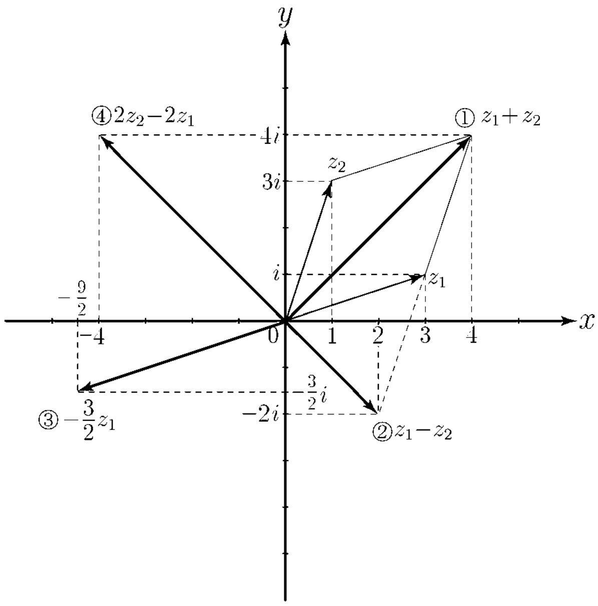
$= -\dfrac{9}{2} - \dfrac{3}{2}i$

④ $2z_2 - 2z_1 = 2(z_2 - z_1)$

$= 2\{(1+3i)-(3+i)\}$

$= 2(-2+2i)$

$= -4+4i$



＜ 17 ページ. 複素数の i 倍 ＞

問の解答

(1) $z = 1 + i$

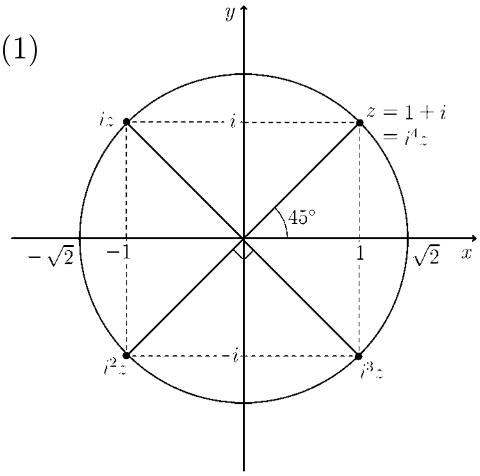
$$iz = (i+1) = i - 1 = -1 + i$$

$$i^2 z = i(i-1) = -1 - i$$

$$i^3 z = i(-1-i) = -i + 1 = 1 - i$$

$$i^4 z = i(-i+1) = 1 + i$$

(1)



(2) $z = 1 + \sqrt{3}i$

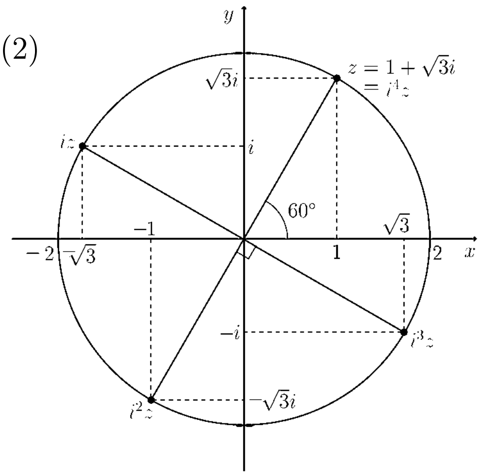
$$iz = i(1+\sqrt{3}i) = i - \sqrt{3} = -\sqrt{3} + i$$

$$i^2 z = i(i - \sqrt{3}) = -1 - \sqrt{3}i$$

$$i^3 z = i(-1 - \sqrt{3}i) = -i + \sqrt{3} = \sqrt{3} - i$$

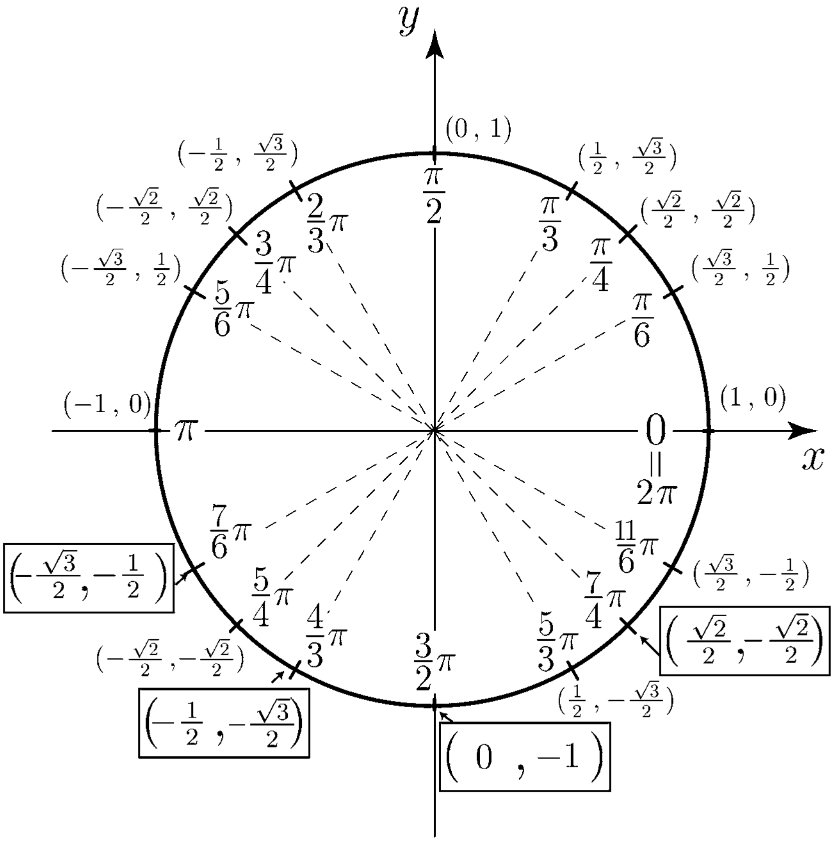
$$i^4 z = i(-i + \sqrt{3}) = 1 + \sqrt{3}i$$

(2)



< 18 ページ. 極座標表示 (1) >

問 1 の解答



問 2 の解答

(1) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \left(\cos\frac{2}{3}\pi\ ,\ \sin\frac{2}{3}\pi\right)$

(2) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(\cos\frac{\pi}{4}\ ,\ \sin\frac{\pi}{4}\right)$

(3) $\left(1, 0\right) = (\cos 0\ ,\ \sin 0)$

(4) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) = \left(\cos\frac{7}{6}\pi\ ,\ \sin\frac{7}{6}\pi\right)$

(5) $\left(0, -1\right) = \left(\cos\frac{3}{2}\pi\ ,\ \sin\frac{3}{2}\pi\right)$

(6) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = \left(\cos\frac{5}{4}\pi\ ,\ \sin\frac{5}{4}\pi\right)$

$<$ 19 ページ. 極座標表示 (2) $>$

問の解答

(1) $\left(\ 3\ ,\ 3\ \right) = \left(\ 3\sqrt{2}\cos\dfrac{\pi}{4}\ ,\ 3\sqrt{2}\sin\dfrac{\pi}{4}\ \right)$

(2) $\left(1,\ -\sqrt{3}\right) = \left(2\cos\left(-\frac{\pi}{3}\right),\ 2\sin\left(-\frac{\pi}{3}\right)\right)$

$\qquad = \left(2\cos\left(\frac{5\pi}{3}\right),\ 2\sin\left(\frac{5\pi}{3}\right)\right)$

(3) $\left(-3,\ \sqrt{3}\right) = \left(2\sqrt{3}\cos\dfrac{5\pi}{6},\ 2\sqrt{3}\sin\dfrac{5\pi}{6}\right)$

(4) $\left(\ -2\ ,\ -2\ \right) = \left(2\sqrt{2}\cos\left(\frac{5}{4}\pi\right),\ 2\sqrt{2}\sin\left(\frac{5}{4}\pi\right)\right)$

< 20 ページ. 複素数の練習 (1) >

問 1 の解答

(1) $i + i^4 + i^7 = 1$

(2) $(i+1)(i^2 - i + 1) = 1 - i$

(3) $\left(\dfrac{1+i^3}{2}\right)\left(\dfrac{1-i^3}{2}\right) = \dfrac{1}{2}$

(4) $\dfrac{1-i}{1+i} = -i$

(5) $\dfrac{2}{i-\sqrt{3}} = -\dfrac{\sqrt{3}}{2} - \dfrac{i}{2}$

(6) $\sqrt{-10} \times \sqrt{-6} \div \sqrt{-105} \times \sqrt{-7} = -2$

問 2 の解答

(1) $x = \dfrac{1 \pm \sqrt{23}i}{4}$

(2) $x = -1,\ x = \dfrac{1 \pm \sqrt{3}i}{2}$

(3) $x = 4,\ x = -2 \pm 2\sqrt{3}i$

(4) $x = \pm 2,\ x = \pm 2i$

問 3 の解答

(1) $\overline{z} = 3 - 2i$

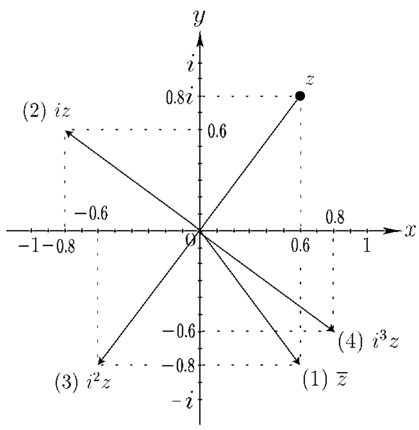
(2) $z\overline{z} = 13$

(3) $|z| = \sqrt{13}$

(4) $z^2 = 5 + 12i$

(5) $|z^2| = \sqrt{5^2 + 12^2} = 13$

問 4 の解答



問 5 の解答

(1) $\left(\ 2\ ,\ 2\sqrt{3}\ \right) = \left(4\cos\left(\dfrac{\pi}{3}\right),\ 4\sin\left(\dfrac{\pi}{3}\right)\right)$

(2) $\left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right) = \left(\cos\left(\dfrac{4\pi}{3}\right),\ \sin\left(\dfrac{4\pi}{3}\right)\right) = \left(\cos\left(-\dfrac{2\pi}{3}\right),\ \sin\left(-\dfrac{2\pi}{3}\right)\right)$

(3) $\left(-3\,,\ 3\right) = \left(3\sqrt{2}\cos\left(\dfrac{3\pi}{4}\right),\ 3\sqrt{2}\sin\left(\dfrac{3\pi}{4}\right)\right)$

(4) $\left(\ 3\ ,\ -\sqrt{3}\ \right) = \left(2\sqrt{3}\cos\left(\dfrac{11\pi}{6}\right),\ 2\sqrt{3}\sin\left(\dfrac{11\pi}{6}\right)\right) = \left(2\sqrt{3}\cos\left(-\dfrac{\pi}{6}\right),\ 2\sqrt{3}\sin\left(-\dfrac{\pi}{6}\right)\right)$

＜ 21 ページ. 絶対値 1 の複素数 ＞

問の解答

(1) $\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, (2) $\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)=\frac{1}{2}+\frac{\sqrt{3}}{2}i$, (3) $\cos\left(\frac{2}{3}\pi\right)+i\sin\left(\frac{2}{3}\pi\right)=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

(4) $\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, (5) $\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$, (6) $\cos(\pi)+i\sin(\pi)=-1$

(7) $\cos\left(\frac{7}{6}\pi\right)+i\sin\left(\frac{7}{6}\pi\right)=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$, (8) $\cos\left(\frac{5}{4}\pi\right)+i\sin\left(\frac{5}{4}\pi\right)=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$, (9) $\cos\left(\frac{4}{3}\pi\right)+i\sin\left(\frac{4}{3}\pi\right)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

(10) $\cos\left(\frac{3}{2}\pi\right)+i\sin\left(\frac{3}{2}\pi\right)=-i$, (11) $\cos\left(\frac{5}{3}\pi\right)+i\sin\left(\frac{5}{3}\pi\right)=\frac{1}{2}-\frac{\sqrt{3}}{2}i$, (12) $\cos\left(\frac{7}{4}\pi\right)+i\sin\left(\frac{7}{4}\pi\right)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

< 22 ページ. 極形式 (1) >

問の解答

(1) $4i = 4\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

(2) $-2 = 2(\cos\pi + i\sin\pi)$

(3) $-\sqrt{2}i = \sqrt{2}\left(\cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right)\right)$

$\left(= \sqrt{2}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)\right)$

$<$ 23 ページ. 極形式 (2) $>$

問の解答

(1) $z = 1 + i = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

(2) $z = -1 + \sqrt{3}i = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

(3) $z = -\sqrt{6} - \sqrt{6}i = 2\sqrt{3}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

(4) $z = -3 - \sqrt{3}i = 2\sqrt{3}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2\sqrt{3}\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right)$

$$= 2\sqrt{3}\left(\cos\left(-\frac{5}{6}\pi\right) + i\sin\left(-\frac{5}{6}\pi\right)\right)$$

(5) $z = \sqrt{6} - \sqrt{2}i = 2\sqrt{2}\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

$$= 2\sqrt{2}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$$

< 24 ページ. 複素数の積 >

問の解答

(1) $\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)z=\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)r(\cos\theta+i\sin\theta)$

$=r\left(\cos\left(\theta+\dfrac{\pi}{3}\right)+i\sin\left(\theta+\dfrac{\pi}{3}\right)\right)$

原点を中心として反時計まわりに $\dfrac{\pi}{3}(=60^\circ)$ 回転する

(2) $\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}i\right)z=\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)r(\cos\theta+i\sin\theta)$

$=r\left(\cos\left(\theta+\dfrac{\pi}{4}\right)+i\sin\left(\theta+\dfrac{\pi}{4}\right)\right)$

原点を中心として反時計まわりに $\dfrac{\pi}{4}(=45^\circ)$ 回転する

(3) $i^2z=-z=(\cos\pi+i\sin\pi)\,r\,(\cos\theta+i\sin\theta)$

$=r\Bigl\{\cos(\theta+\pi)+i\sin(\theta+\pi)\Bigr\}$

原点を中心として $\pi(=180^\circ)$ 回転する。

< 25 ページ. 複素数の商 >

問の解答

(1) $$\frac{1+\sqrt{3}i}{\sqrt{3}+i} = \frac{2(\frac{1}{2}+\frac{\sqrt{3}}{2}i)}{2(\frac{\sqrt{3}}{2}+\frac{1}{2}i)} = \frac{\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)} = \cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)$$

(2) $$\frac{2-2i}{-1+i} = \frac{2\sqrt{2}(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i)}{\sqrt{2}(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i)} = \frac{2\left(\cos\left(\frac{7}{4}\pi\right)+i\sin\left(\frac{7}{4}\pi\right)\right)}{\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)} = 2\left(\cos\pi+i\sin\pi\right)$$

$$\left(=2\big(\cos(-\pi)+i\sin(-\pi)\big)\right)$$

(3) $$\frac{-1-i}{-\sqrt{3}+i} = \frac{\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)}{2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)} = \frac{\sqrt{2}}{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$$

< 26 ページ. ド・モアブルの定理 >

問の解答

(1) $$(-\sqrt{3}+i)^3 = \left(2\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\right)^3 = 2^3\left(\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)\right)^3$$
$$= 8\left(\cos\left(\frac{5}{2}\pi\right)+i\sin\left(\frac{5}{2}\pi\right)\right) = 8i$$

(2) $$\left(\frac{-1+\sqrt{3}i}{2}\right)^6 = \left(\cos\left(\frac{2}{3}\pi\right)+i\sin\left(\frac{2}{3}\pi\right)\right)^6$$
$$= \cos(4\pi)+i\sin(4\pi) = 1$$

(3) $$(1-i)^4 = \left\{\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)\right\}^4$$
$$= (\sqrt{2})^4\left\{\cos(-\pi)+i\sin(-\pi)\right\} = -4$$

(4) $$\left(\frac{-1+i}{\sqrt{3}+i}\right)^{12} = \left(\frac{\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)}{2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}\right)^{12} = \left(\frac{\sqrt{2}}{2}\right)^{12} \times \left(\frac{\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)}{\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)}\right)^{12}$$
$$= \left(\frac{1}{\sqrt{2}}\right)^{12} \times \left(\cos\left(\frac{7}{12}\pi\right)+i\sin\left(\frac{7}{12}\pi\right)\right)^{12}$$
$$= \frac{1}{2^6} \times (\cos(7\pi)+i\sin(7\pi)) = -\frac{1}{64}$$

＜ 27 ページ.1 の累乗根 ＞

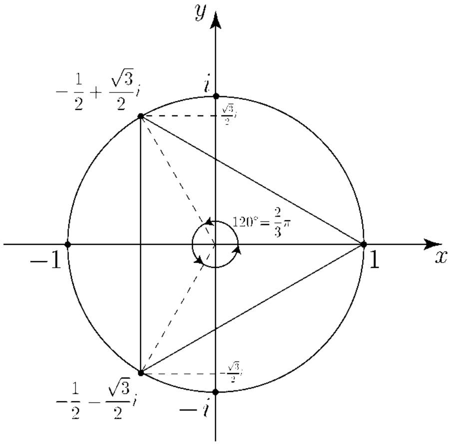
問の解答

(1) $z^3 = 1$

$\cos(3\theta) + i\sin(3\theta) = 1$

$\theta = 0 \ , \ \dfrac{2}{3}\pi \ , \ \dfrac{4}{3}\pi$

$\underline{z \ = \ 1 \ , \ -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i \ , \ -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i}$

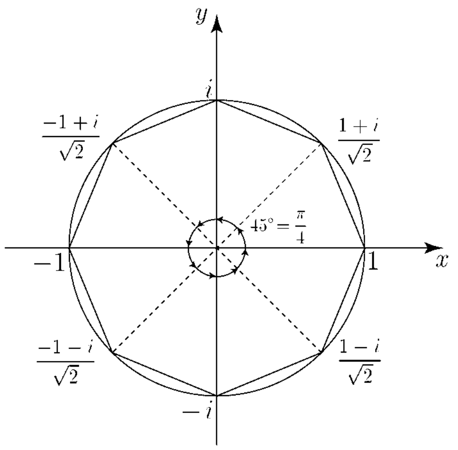


(2) $z^8 = 1$

$\cos(8\theta) + i\sin(8\theta) = 1$

$\theta = 0 \ , \ \dfrac{\pi}{4} \ , \ \dfrac{\pi}{2} \ , \ \dfrac{3}{4}\pi \ , \ \pi \ , \ \dfrac{5}{4}\pi \ , \ \dfrac{3}{2}\pi \ , \ \dfrac{7}{4}\pi$

$\underline{z \ = \ 1 \ , \ \dfrac{1+i}{\sqrt{2}} \ , \ i \ , \ \dfrac{-1+i}{\sqrt{2}} \ , \ -1 \ , \ \dfrac{-1-i}{\sqrt{2}} \ , \ -i \ , \ \dfrac{1-i}{\sqrt{2}}}$



< 28 ページ. オイラーの公式 (1) >

問の解答

(1) $e^{2\pi i} = 1$

(2) $e^{-\frac{\pi}{2}i} = -i$

(3) $e^{\frac{3}{4}\pi i} = -\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$

(4) $e^{\frac{5}{3}\pi i} = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

(5) $e^{-\frac{3}{4}\pi i} = -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i$

(6) $e^{-\frac{\pi}{6}i} = \dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$

< 29 ページ. オイラーの公式 (2) >

問の解答

(1) $e^{2-2\pi i} = e^2$

(2) $e^{0+\frac{\pi}{3}i} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

(3) $e^{2+\frac{3}{4}\pi i} = e^2\left(-\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)$

(4) $e^{\frac{1}{2}-\frac{3}{2}\pi i} = \sqrt{e}\,i$

(5) $e^{\log 2+\frac{5}{4}\pi i} = 2\left(\cos\left(\dfrac{5}{4}\pi\right) + i\sin\left(\dfrac{5}{4}\pi\right)\right) = -\sqrt{2} - \sqrt{2}i$

(6) $e^{\frac{1}{3}\log 8+\frac{\pi}{6}i} = 2\left(\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\pi\right)\right) = \sqrt{3} + i$

< 30 ページ. 複素数の指数表示 >

問 1 の解答

$e^{i\theta_1} \times e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$

問 2 の解答

(1) $e^{\frac{3}{2}\pi i} \times e^{\frac{\pi}{2}i} = e^{2\pi i} = 1$

(2) $e^{\frac{4}{3}\pi i} \div e^{\frac{\pi}{6}i} = e^{\frac{7}{6}\pi i} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

(3) $\left(e^{\frac{\pi}{8}i}\right)^4 = e^{\frac{\pi}{2}i} = i$

(4) $\left(e^{\frac{\pi}{48}i}\right)^{12} = e^{\frac{\pi}{4}i} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

＜ 31 ページ. 指数法則 ＞

問 1 の解答

(2) $\dfrac{e^{z_1}}{e^{z_2}} = e^{\boxed{z_1 - z_2}}$　　(3) $\left(e^{z}\right)^n = e^{\boxed{nz}}$

問 2 の解答

(1) $e^{5+\pi i} \times e^{-1+\pi i} = e^{4+2\pi i} = e^4$　　(2) $e^{2+\frac{\pi}{4}i} \div e^{6+\frac{\pi}{4}i} = e^{-4} = \dfrac{1}{e^4}$

(3) $\left(e^{\frac{3}{4}-\frac{3}{8}\pi i}\right)^4 = e^{3-\frac{3}{2}\pi i} = e^3 i$

問 3 の解答

$$\frac{\left(e^{\frac{\pi}{6}}i\right)^8 \times (e^{\frac{\pi}{12}}i)^4}{\left(e^{\frac{\pi}{4}}i\right)^{10}} = \frac{e^{\frac{4\pi}{3}i} \times e^{\frac{\pi}{3}i}}{e^{\frac{5\pi}{2}i}} = e^{\frac{4\pi}{3}i + \frac{\pi i}{3} - \frac{5\pi}{2}i}$$

$$= e^{\left(\frac{10-15}{6}\right)\pi i} = e^{-\frac{5}{6}\pi i} = \cos\left(-\frac{5}{6}\pi\right) + i\sin\left(-\frac{5}{6}\pi\right) = -\frac{\sqrt{3}}{2} - \frac{i}{2}$$

< 32 ページ. 複素数の簡易表示 >

問 1 の解答

(1) $z_1 = \sqrt{3} + i$

$$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= 2e^{\frac{\pi}{6}i}$$

(2) $z_2 = -1 + i$

$$= \sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$

$$= \sqrt{2}e^{\frac{3}{4}\pi i}$$

(3) $z_3 = -\sqrt{3} - 3i$

$$= 2\sqrt{3}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

または $\left(2\sqrt{3}e^{\frac{4}{3}\pi i}\right)$

問 2 の解答

(1) $z_1 z_2$

$$= 2e^{\frac{\pi}{6}i} \times \sqrt{2}e^{\frac{3}{4}\pi i}$$

$$= 2\sqrt{2}e^{\frac{11}{12}\pi i}$$

(2) $z_2 z_3$

$$= \sqrt{2}e^{\frac{3}{4}\pi i} \times 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

$$= 2\sqrt{6}e^{\frac{1}{12}\pi i}$$

$$\left(= 2\sqrt{6}e^{\frac{25}{12}\pi i}\right)$$

(3) $\dfrac{z_3}{z_1} = \dfrac{2\sqrt{3}e^{-\frac{2}{3}\pi i}}{2e^{\frac{\pi}{6}i}}$

$$= \sqrt{3}e^{\left(-\frac{2}{3} - \frac{1}{6}\right)\pi i}$$

$$= \sqrt{3}e^{-\frac{5}{6}\pi i}$$

$$\left(= \sqrt{3}e^{\frac{7}{6}\pi i}\right)$$

＜ 33 ページ. 複素数の練習 (2) ＞

問 1 の解答

(1) $3-\sqrt{3}i = 2\sqrt{3}\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = 2\sqrt{3}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

(2) $-2+2i = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

問 2 の解答

(1) $\left(\frac{\sqrt{3}+i}{2}\right)^{12} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{12} = \cos 2\pi + i\sin 2\pi = 1$

(2) $(1-i)^8 = \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right)^8 = \left(\sqrt{2}\right)^8\left(\cos(-2\pi) + i\sin(-2\pi)\right) = 16$

問 3 の解答

(1) $e^{-\frac{2\pi}{3}i} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

(2) $e^{3+\frac{\pi}{4}i} = \frac{e^3}{\sqrt{2}} + \frac{e^3}{\sqrt{2}}i$

(3) $e^{\frac{\pi}{3}i} \div e^{\frac{\pi}{2}i} = e^{-\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{i}{2}$

問 4 の解答

(1) $\frac{1-\sqrt{3}i}{2} = e^{-\frac{\pi}{3}i}$

(2) $-\frac{\sqrt{2}e}{2} + \frac{\sqrt{2}e}{2}i = e^{1+\frac{3\pi}{4}i}$

問 5 の解答

(1) $1+\sqrt{3}i = 2e^{\frac{\pi}{3}i}$

(2) $-3+\sqrt{3}i = 2\sqrt{3}\ e^{\frac{5\pi}{6}i}$

問 6 の解答

(1) $e^{\frac{5\pi}{3}i} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

(2) $e^{\frac{2+3\pi i}{4}} = -\sqrt{\frac{e}{2}} + \sqrt{\frac{e}{2}}\ i$

(3) $(e^{\frac{\pi}{6}i})^7 \div e^{\frac{4\pi}{3}i} = e^{-\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{i}{2}$

(4) $\frac{1-i}{\sqrt{2}}e^{\frac{\pi}{6}i} + \frac{1+i}{\sqrt{2}}e^{-\frac{\pi}{6}i} = e^{-\frac{\pi}{4}i+\frac{\pi}{6}i} + e^{\frac{\pi}{4}i-\frac{\pi}{6}i}$

$= e^{-\frac{\pi}{12}i} + e^{\frac{\pi}{12}i} = 2\cos\left(\frac{\pi}{12}\right)$

問 7 の解答

(1) $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

(2) $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}\left(= \frac{\left(e^{-i\theta}-e^{i\theta}\right)i}{2}\right)$

< 34 ページ. 微分の復習 (1) >

問の解答

(1) $(4x^5 - 6x^7 + 3\sqrt{x})' = 20x^4 - 42x^6 + \dfrac{3}{2\sqrt{x}}$

(2) $\left(\dfrac{x^3 + 4x + \sqrt{x^3}}{x^2}\right)' = 1 - \dfrac{4}{x^2} - \dfrac{1}{2x\sqrt{x}}$

(3) $\left(\dfrac{4}{x^3} - 3\log|x|\right)' = -\dfrac{12}{x^4} - \dfrac{3}{x}$

(4) $(4e^x - 5\sin x)' = 4e^x - 5\cos x$

(5) $(4\cos x + 3\tan x)' = -4\sin x + \dfrac{3}{\cos^2 x}$

(6) $\left(\sqrt[3]{x^4}\right)' = \frac{4}{3}\sqrt[3]{x}$

< 35 ページ. 微分の復習 (2) >

問 次の関数を微分せよ。

(1) $y = \sin(4 - x)$

$$\frac{dy}{dx} = -\cos(4 - x)$$

(2) $y = \cos(3x + 2)$

$$\frac{dy}{dx} = -3\sin(3x + 2)$$

(3) $y = e^{5x+1}$

$$\frac{dy}{dx} = 5e^{5x+1}$$

(4) $y = (3x - 1)^6$

$$\frac{dy}{dx} = 18(3x - 1)^5$$

(5) $y = \log|6x - 3|$

$$\frac{dy}{dx} = \frac{6}{6x - 3} = \frac{2}{2x - 1}$$

(6) $y = \sqrt{4x + 3}$

$$\frac{dy}{dx} = \frac{2}{\sqrt{4x + 3}}$$

(7) $y = \dfrac{1}{(5x + 7)^3}$

$$\frac{dy}{dx} = -\frac{15}{(5x + 7)^4}$$

(8) $y = \sin(x + x^2)$

$$\frac{dy}{dx} = (1 + 2x)\cos(x + x^2)$$

(9) $y = e^{-x^2+x}$

$$\frac{dy}{dx} = (-2x + 1)e^{-x^2+x}$$

(10) $y = \log|\cos x|$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan x$$

$<$ 36 ページ. 微分の復習 (3) $>$

問の解答

(1) $(x^2\cos x)' = 2x\cos x - x^2\sin x$

(2) $(e^x\sin x)' = e^x\sin x + e^x\cos x$

(3) $(e^x\cos x)' = e^x\cos x - e^x\sin x$

(4) $(e^{2x}\sin x)' = 2e^{2x}\sin x + e^{2x}\cos x$

(5) $(e^x\sin(2x))' = e^x\sin(2x) + 2e^x\cos(2x)$

(6) $(e^{3x}\cos x)' = 3e^{3x}\cos x - e^{3x}\sin x$

(7) $(e^{2x}\sin(3x))' = 2e^{2x}\sin(3x) + 3e^{2x}\cos(3x)$

(8) $(e^{3x}\cos(4x))' = 3e^{3x}\cos(4x) - 4e^{3x}\sin(4x)$

(9) $(xe^x)' = e^x + xe^x$

(10) $(xe^{2x})' = e^{2x} + 2xe^{2x}$

(11) $(xe^{3x})' = e^{3x} + 3xe^{3x}$

(12) $(x^2e^{3x})' = 2xe^{3x} + 3x^2e^{3x}$

(13) $(-x + x\log x)' = \log x$

(14) $\left(\dfrac{1}{1-x}\right)' = -\dfrac{-1}{(1-x)^2} = \dfrac{1}{(1-x)^2}$

(15) $\left(\dfrac{1}{e^x}\right)' = -\dfrac{1}{e^x}$

(16) $\left(\dfrac{\cos x}{\sin x}\right)' = -\dfrac{1}{\sin^2 x}$

< 37 ページ. 微分の復習 (4) >

問の解答

(1) $f(x) = \log|x|$ 　 $f'(x) = \dfrac{1}{x}$ 　 $f''(x) = -\dfrac{1}{x^2}$

(2) $f(x) = e^{3x}$ 　 $f'(x) = 3e^{3x}$ 　 $f''(x) = 9e^{3x}$

(3) $f(x) = \cos(4x)$ 　 $f'(x) = -4\sin(4x)$ 　 $f''(x) = -16\cos(4x)$

(4) $y = xe^{2x}$ 　 $\dfrac{dy}{dx} = e^{2x} + 2xe^{2x}$ 　 $\dfrac{d^2y}{dx^2} = 2e^{2x} + 2e^{2x} + 4xe^{2x} = (4+4x)e^{2x}$

(5) $y = xe^{-3x}$ 　 $\dfrac{dy}{dx} = e^{-3x} - 3xe^{-3x}$ 　 $\dfrac{d^2y}{dx^2} = (-6+9x)e^{-3x}$

(6) $y = e^x \sin(2x)$ 　 $\dfrac{dy}{dx} = e^x\Big(\sin(2x) + 2\cos(2x)\Big)$ 　 $\dfrac{d^2y}{dx^2} = e^x\Big(-3\sin(2x) + 4\cos(2x)\Big)$

(7) $y = e^{2x}\cos x$ 　 $\dfrac{dy}{dx} = e^{2x}\Big(2\cos x - \sin x\Big)$ 　 $\dfrac{d^2y}{dx^2} = e^{2x}(3\cos x - 4\sin x)$

(8) $y = e^{2x}\sin(3x)$ 　 $\dfrac{dy}{dx} = e^{2x}\Big(2\sin(3x) + 3\cos(3x)\Big)$ 　 $\dfrac{d^2y}{dx^2} = e^{2x}\Big(-5\sin(3x) + 12\cos(3x)\Big)$

(9) $y = e^{-x}\cos(4x)$ 　 $\dfrac{dy}{dx} = e^{-x}\Big(-\cos(4x) - 4\sin(4x)\Big)$ 　 $\dfrac{d^2y}{dx^2} = e^{-x}\Big(-15\cos(4x) + 8\sin(4x)\Big)$

< 38 ページ. 時間変数 t による微分 (1) >

問の解答

(1) $\dfrac{d}{dt}\left(9-6t^2+3t^3\right)=-12t+9t^2$

(2) $\dfrac{d}{dt}\left(-t^8+3t^4+2t^2+6e^t\right)=-8t^7+12t^3+4t+6e^t$

(3) $\dfrac{d}{dt}\left(2t^5-6\cos t+\dfrac{1}{2}\log t\right)=10t^4+6\sin t+\dfrac{1}{2t}$

(4) $\dfrac{d}{dt}\left(\dfrac{5}{t}+\dfrac{4}{\sqrt{t^3}}\right)=-\dfrac{5}{t^2}-\dfrac{6}{t^2\sqrt{t}}$

< 39 ページ. 時間変数 t による微分 (2) >

問 1 の解答

(1) $\dfrac{d}{dt}\sin(5t+4) = 5\cos(5t+4)$

(2) $\dfrac{d}{dt}e^{3t+2} = 3e^{3t+2}$

(3) $\dfrac{d}{dt}\cos\left(-2t+\dfrac{1}{2}\right) = 2\sin\left(-2t+\dfrac{1}{2}\right) \quad \left(\text{または} = -2\sin\left(2t-\dfrac{1}{2}\right)\right)$

(4) $\dfrac{d}{dt}\log(9-2t) = \dfrac{-2}{9-2t} \left(= \dfrac{2}{2t-9}\right)$

問 2 の解答

(1) $\dfrac{d}{dt}\sin(2t^3-t) = (6t^2-1)\cos(2t^3-t)$

(2) $\dfrac{d}{dt}\left(e^{-t^3}\right) = -3t^2e^{-t^3}$

(3) $\dfrac{d}{dt}\cos(2+3t-4t^2) = (8t-3)\sin(2+3t-4t^2)$

$\left(\text{または} - (8t-3)\sin(4t^2-3t-2)\right)$

(4) $\dfrac{d}{dt}\log(t^5-2t^3+t) = \dfrac{5t^4-6t^2+1}{t^5-2t^3+t}$

< 40 ページ. 時間変数 t による微分 (3) >

問 1 の解答

(1) $\dfrac{d}{dt}(2te^t) = 2e^t + 2te^t$

(2) $\dfrac{d}{dt}(t^3 \cos t) = 3t^2 \cos t - t^3 \sin t$

(3) $\dfrac{d}{dt}\left(\dfrac{1}{2}e^t \sin t\right) = \dfrac{1}{2}e^t \sin t + \dfrac{1}{2}e^t \cos t$

(4) $\dfrac{d}{dt}(t^2 \log t) = 2t \log t + t$

問 2 の解答

(1) $\dfrac{d}{dt}\left(\dfrac{1}{2}e^t \sin(2t)\right) = \dfrac{1}{2}e^t \sin(2t) + e^t \cos(2t)$

(2) $\dfrac{d}{dt}\big(e^{3t} \cos(6t)\big) = 3e^{3t} \cos(6t) - 6e^{3t} \sin(6t)$

(3) $\dfrac{d}{dt}\big(4e^{\frac{t}{2}} \sin(-5t)\big) = 2e^{\frac{t}{2}} \sin(-5t) - 20e^{\frac{t}{2}} \cos(-5t)$

$\left(\text{または } -2e^{\frac{t}{2}} \sin(5t) - 20e^{\frac{t}{2}} \cos(5t)\right)$

(4) $\dfrac{d}{dt}\big(3e^{-2t} \cos(4t)\big) = -6e^{-2t} \cos(4t) - 12e^{-2t} \sin(4t)$

< 41 ページ. 時間変数 t による微分 (4) >

問 次の関数の 1 階および 2 階導関数を求めよ。

(1) $F(t) = e^{-2t}$, $F'(t) = -2e^{-2t}$, $F''(t) = 4e^{-2t}$

(2) $F(t) = te^{-t}$, $F'(t) = e^{-t} - te^{-t}$, $F''(t) = -2e^{-t} + te^{-t}$

(3) $F(t) = \sin(3t)$, $F'(t) = 3\cos(3t)$, $F''(t) = -9\sin(3t)$

(4) $y = e^t \sin(2t)$

$$\frac{dy}{dt} = e^t\left(\sin(2t) + 2\cos(2t)\right)$$

$$\frac{d^2y}{dt^2} = e^t\left(-3\sin(2t) + 4\cos(2t)\right)$$

(5) $y = e^{2t}\cos(t)$

$$\frac{dy}{dt} = e^{2t}\left(2\cos t - \sin t\right)$$

$$\frac{d^2y}{dt^2} = e^{2t}\left(3\cos t - 4\sin t\right)$$

(6) $y = e^{2t}\cos(3t)$

$$\frac{dy}{dt} = e^{2t}\left(2\cos(3t) - 3\sin(3t)\right)$$

$$\frac{d^2y}{dt^2} = e^{2t}\left(-5\cos(3t) - 12\sin(3t)\right)$$

＜ 42 ページ. 関数と導関数 ＞

問 1 の解答

(1) $y = 4e^{3t}$　　　$\dfrac{dy}{dt} = 3y$

(2) $y = 5e^{-2t}$　　　$\dfrac{dy}{dt} = -2y$

問 2 の解答

(1) $y = \cos(2t)$　　　$\dfrac{d^2y}{dt^2} = -4y$

(2) $y = \sin(3t)$　　　$\dfrac{d^2y}{dt^2} = -9y$

(3) $y = \cos(3t)$　　　$\dfrac{d^2y}{dt^2} = -9y$

(4) $y = \sin(2t) + \cos(2t)$　　　$\dfrac{d^2y}{dt^2} = -4y$

(5) $y = 4\sin(3t) - 5\cos(3t)$　　　$\dfrac{d^2y}{dt^2} = -9y$

< 43 ページ. 複素数値関数の微分 (1) >

問の解答

(1) $z(t) = 3t^4 + ie^{2t}$

$$\frac{dz}{dt} = 12t^3 + 2e^{2t}i$$

(2) $z(t) = \cos(bt) + i\sin(bt)$

$$\frac{dz}{dt} = -b\sin(bt) + b\cos(bt)i$$

(3) $z(t) = e^{(3+4i)t}$

$$\frac{dz}{dt} = \frac{d}{dt}\left\{e^{3t}\cos(4t) + e^{3t}\sin(4t)i\right\} = e^{3t}\Big(3\cos(4t) - 4\sin(4t)\Big) + ie^{3t}\Big(3\sin(4t) + 4\cos(4t)\Big)$$

$$= e^{3t}\Big\{(3+4i)\cos(4t) + (-4+3i)\sin(4t)\Big\}$$

(4) $z(t) = e^{(a+bi)t}$

$$\frac{dz}{dt} = \frac{d}{dt}\left\{e^{at}\cos(bt) + e^{at}\sin(bt)i\right\} = e^{at}\Big(a\cos(bt) - b\sin(bt)\Big) + ie^{at}\Big(a\sin(bt) + b\cos(bt)\Big)$$

$$= e^{at}\Big\{(a+bi)\cos(bt) + (-b+ai)\sin(bt)\Big\}$$

< 44ページ. 複素数値関数の微分 (2) >

問の解答

(1) $\dfrac{d}{dt}e^{3it} = 3ie^{3ti}$

(2) $\dfrac{d}{dt}e^{-2it} = -2ie^{-2ti}$

(3) $\dfrac{d}{dt}e^{bit} = bie^{bit}$

(4) $\dfrac{d}{dt}e^{(1+i)t} = (1+i)e^{(1+i)t}$

(5) $\dfrac{d}{dt}e^{(2-i)t} = (2-i)e^{(2-i)t}$

(6) $\dfrac{d}{dt}e^{(-3+2i)t} = (-3+2i)e^{(-3+2i)t}$

(7) $\dfrac{d}{dt}e^{(a-i)t} = (a-i)e^{(a-i)t}$

(8) $\dfrac{d}{dt}e^{(a-bi)t} = (a-bi)e^{(a-bi)t}$

(9) $\dfrac{d}{dt}\left(\dfrac{1}{a+bi}e^{(a+bi)t}\right) = e^{(a+bi)t}$

(10) $\dfrac{d}{dt}\left(\dfrac{1}{a-bi}e^{(a-bi)t}\right) = e^{(a-bi)t}$

< 45 ページ. 積分の復習 (1) >

問の解答

(1) $\displaystyle\int dt = t + C$

(2) $\displaystyle\int t^n dt = \frac{1}{n+1} t^{n+1} + C$

(3) $\displaystyle\int \frac{1}{y} dy = \log|y| + C$

(4) $\displaystyle\int e^u du = e^u + C$

(5) $\displaystyle\int \cos v dv = \sin v + C$

(6) $\displaystyle\int \sin t dt = -\cos t + C$

< 46 ページ. 積分の復習 (2) >

問の解答

(1) $\displaystyle\int \cos(3x+4)dx = \frac{1}{3}\sin(3x+4) + C$

(2) $\displaystyle\int \sin(5x-2)dx = -\frac{1}{5}\cos(5x-2) + C$

(3) $\displaystyle\int e^{3x+5}dx = \frac{1}{3}e^{3x+5} + C$

(4) $\displaystyle\int \frac{1}{5x-3}dx = \frac{1}{5}\log|5x-3| + C$

(5) $\displaystyle\int (8x+7)^5 = \frac{1}{48}(8x+7)^6 + C$

(6) $\displaystyle\int \cos(3t)dt = \frac{1}{3}\sin(3t) + C$

(7) $\displaystyle\int e^{2t-3}dt = \frac{1}{2}e^{2t-3} + C$

(8) $\displaystyle\int \cos(at+b)dt = \frac{1}{a}\sin(at+b) + C$

(9) $\displaystyle\int \sin(at+b)dt = -\frac{1}{a}\cos(at+b) + C$

(10) $\displaystyle\int e^{at+b}dt = \frac{1}{a}e^{at+b} + C$

< 47 ページ. 複素数値関数の積分 (1) >

(1) $\displaystyle\int (t^3 + t^5 i)dt = \frac{1}{4}t^4 + \frac{1}{6}t^6 i + C$

(2) $\displaystyle\int (\cos t + i\sin t)dt = \sin t - i\cos t + C$

(3) $\displaystyle\int (e^{2t} + i\cos(3t))dt = \frac{1}{2}e^{2t} + \frac{1}{3}\sin(3t)i + C$

(4) $\displaystyle\int e^{bit}dt = \frac{1}{bi}e^{bit} + C$

(5) $\displaystyle\int e^{(2+3i)t}dt = \frac{1}{2+3i}e^{(2+3i)t} + C$

(6) $\displaystyle\int e^{(a-bi)t}dt = \frac{1}{a-bi}e^{(a-bi)t} + C$

< 48ページ. 複素数値関数の積分 (2) >

問の解答

(1) $\displaystyle\int e^{3t}\cos(4t)dt$　　　　(2) $\displaystyle\int e^{3t}\sin(4t)dt$

(1), (2) の解答

$$\int e^{3t}\cos(4t)dt + i\int e^{3t}\sin(4t)dt = \int e^{3t}\Big\{\cos(4t)+i\sin(4t)\Big\}dt$$

$$\int e^{3t}\times e^{4ti}dt = \int e^{(3+4i)t}dt = \frac{1}{3+4i}e^{(3+4i)t}+C$$

$$= \frac{3-4i}{3^2+4^2}e^{3t}\Big\{\cos(4t)+i\sin(4t)\Big\}+C$$

$$= \frac{e^{3t}}{25}\Big\{3\cos(4t)+4\sin(4t)\Big\} + i\,\frac{e^{3t}}{25}\Big\{-4\cos(4t)+3\sin(4t)\Big\}+C$$

(1) の (答): $\displaystyle\int e^{3t}\cos(4t)dt = \frac{e^{3t}}{25}\Big\{3\cos(4t)+4\sin(4t)\Big\}+C$

(2) の (答): $\displaystyle\int e^{3t}\sin(4t)dt = \frac{e^{3t}}{25}\Big\{-4\cos(4t)+3\sin(4t)\Big\}+C$

(3) $\displaystyle\int e^{at}\cos(bt)dt$　　　　(4) $\displaystyle\int e^{at}\sin(bt)dt$

(3), (4) の解答

$$\int e^{at}\cos(bt)dt + i\int e^{at}\sin(bt)dt = \int e^{at}\Big\{\cos(bt)+i\sin(bt)\Big\}dt$$

$$= \int e^{at}e^{bti}dt = \int e^{(a+bi)t}dt = \frac{1}{a+bi}e^{(a+bi)t}+C$$

$$= \frac{a-bi}{a^2+b^2}e^{at}\Big\{\cos(bt)+i\sin(bt)\Big\}+C$$

$$= \frac{e^{at}}{a^2+b^2}\Big\{a\cos(bt)+b\sin(bt)\Big\} + i\,\frac{e^{at}}{a^2+b^2}\Big\{-b\cos(bt)+a\sin(bt)\Big\}+C$$

(3) の答: $\displaystyle\int e^{at}\cos(bt)dt = \frac{e^{at}}{a^2+b^2}\Big\{a\cos(bt)+b\sin(bt)\Big\}+C$

(4) の答: $\displaystyle\int e^{at}\sin(bt)dt = \frac{e^{at}}{a^2+b^2}\Big\{-b\cos(bt)+a\sin(bt)\Big\}+C$

< 49 ページ. まとめの問題 (1) >

問 1 の解答

(1) $i^3 - i^5 = -2i$

(2) $(3+i)^2 = 8+6i$

(3) $\left(\dfrac{\sqrt{3}-i}{2}\right)\left(\dfrac{\sqrt{3}+i}{2}\right) = 1$

(4) $\dfrac{2}{1+i} = 1-i$

(5) $\dfrac{\sqrt{24}}{\sqrt{-6}} = -2i$

(6) $\sqrt{-3}\times\sqrt{-6}\times\sqrt{-2} = -6i$

問 2 の解答

(1) $x^2 - 6x + 11 = 0$ $\quad \underline{x = 3 \pm \sqrt{2}\, i}$

(2) $x^3 - 8 = 0$ $\quad \underline{x = 2,\ x = -1 \pm \sqrt{3}i}$

問 3 の解答

$\bar{z} = \dfrac{2-i}{5}$ $\qquad |\bar{z}| = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

問 4 の解答

(1) $2i = 2\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

(2) $-3+\sqrt{3}i = 2\sqrt{3}\left(\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right)$

(3) $-1-\sqrt{3}i = 2\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)$

問 5 の解答

(1) $i \times r(\cos\theta + i\sin\theta) = r\left(\cos\left(\theta+\dfrac{\pi}{2}\right) + i\sin\left(\theta+\dfrac{\pi}{2}\right)\right)$

(2) $\dfrac{1+i}{1-i} = \dfrac{(1+i)^2}{1^2-i^2} = i = \cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}$

＜ 50 ページ. まとめの問題 (2) ＞

問 1 の解答

(1) $(1+i)^8 = \left(\sqrt{2}e^{\frac{\pi}{4}i}\right)^8 = 2^4 e^{2\pi i} = 16$

(2) $\left(\dfrac{\sqrt{3}-i}{2}\right)^6 = \left(e^{-\frac{\pi}{6}i}\right)^6 = e^{-\pi i} = -1$

問 2 の解答

(1) $e^{\frac{\pi}{3}i} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

(2) $e^{\frac{5\pi}{6}i} = -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$

(3) $e^{1+\frac{\pi}{6}i} = \dfrac{\sqrt{3}}{2}e + \dfrac{e}{2}i$

(4) $e^{\frac{2-\pi i}{4}} = \sqrt{\dfrac{e}{2}} - \sqrt{\dfrac{e}{2}}\ i$

(5) $e^{\frac{2\pi}{3}i} \times e^{\frac{\pi}{2}i} = e^{\frac{7\pi}{6}i} = -\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$

(6) $e^{\frac{4\pi}{3}i} \div e^{\frac{\pi}{2}i} = e^{\frac{5}{6}\pi i} = -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$

問 3 の解答

(1) $-\dfrac{\sqrt{3}}{2} + \dfrac{i}{2} = e^{\frac{5\pi}{6}i}$

(2) $\dfrac{\sqrt{2}e}{2} + \dfrac{\sqrt{2}e}{2}i = e^{1+\frac{\pi}{4}i}$

問 4 の解答

(1) $3-3i = 3\sqrt{2}e^{-\frac{\pi}{4}i}\left(= 3\sqrt{2}e^{\frac{7\pi}{4}i}\right)$

(2) $-1-\sqrt{3}i = 2e^{\frac{4\pi}{3}i}\left(= 2e^{-\frac{2\pi}{3}i}\right)$

問 5 の解答

(1) $\dfrac{d}{dt}e^{t^3+t^2} = (3t^2+2t)e^{t^3+t^2}$

(2) $\dfrac{d}{dt}\cos(4t) = -4\sin(4t)$

(3) $\dfrac{d}{dt}e^t\sin(2t) = e^t\Big(\sin(2t)+2\cos(2t)\Big)$

(4) $\dfrac{d}{dt}e^{2t}\cos(3t) = e^{2t}\Big(2\cos(3t)-3\sin(3t)\Big)$

(5) $\dfrac{d}{dt}e^{4t+5ti} = (4+5i)e^{4t+5ti}$

(6) $\dfrac{d}{dt}e^{(3-4i)t} = (3-4i)e^{(3-4i)t}$

問 6 の解答

(1) $\displaystyle\int\Big(3\sin(2t)+4\cos(3t)\Big)dt = -\frac{3}{2}\cos(2t) + \frac{4}{3}\sin(3t) + C$

(2) $\displaystyle\int e^{3t-4}dt = \frac{1}{3}e^{3t-4} + C$

(3) $\displaystyle\int e^{4t+5ti}dt = \frac{1}{4+5i}e^{(4+5i)t} + C$

(4) $\displaystyle\int e^{4t}\cos(3t)dt = \frac{e^{4t}}{25}\Big\{4\cos(3t)+3\sin(3t)\Big\} + C$