高知工科大学

基礎数学ワークブック

(2004年度版)

初級編

$No.\,3$

解答

＜１ページ. 微分の復習 (1) ＞

問の解答

(1) $(x^3 + x^2)' = 3x^2 + 2x$

(2) $(3x^4 - 2x + 1)' = 12x^3 - 2$

(3) $(\sqrt[3]{x})' = \left(x^{\frac{1}{3}}\right)' = \dfrac{1}{3}x^{-\frac{2}{3}} = \dfrac{1}{3\sqrt[3]{x^2}}$

(4) $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$

(5) $\left(\dfrac{1}{\sqrt{x}}\right)' = \left(x^{-\frac{1}{2}}\right)' = -\dfrac{1}{2}x^{-\frac{3}{2}} = -\dfrac{1}{2x\sqrt{x}}$

(6) $\left(\sqrt{x^3}\right)' = \left(x^{\frac{3}{2}}\right)' = \dfrac{3}{2}x^{\frac{1}{2}} = \dfrac{3}{2}\sqrt{x}$

(7) $\left(\dfrac{2}{x^3}\right)' = \left(2x^{-3}\right)' = -6x^{-4} = -\dfrac{6}{x^4}$

(8) $\left(\dfrac{x^3 - 2x^2 - 1}{x^2}\right)' = \left(x - 2 - \dfrac{1}{x^2}\right)' = 1 + \dfrac{2}{x^3}$

(9) $\left(\dfrac{x^2 - x}{\sqrt{x}}\right)' = \left(x^{\frac{3}{2}} - x^{\frac{1}{2}}\right)' = \dfrac{3}{2}x^{\frac{1}{2}} - \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{3}{2}\sqrt{x} - \dfrac{1}{2\sqrt{x}}$

＜ 2 ページ. 微分の復習 (2) ＞

問の解答

(1) $(x\cos x)' = \cos x - x\sin x$

(2) $(e^x \sin x)' = e^x \sin x + e^x \cos x$

(3) $(e^x \cos x)' = e^x \cos x - e^x \sin x$

(4) $(x + x\log x)' = 1 + \log x + 1 = 2 + \log x$

(5) $$\left(\frac{1}{1-x}\right)' = -\frac{(1-x)'}{(1-x)^2} = -\frac{-1}{(1-x^2)} = \frac{1}{(1-x)^2}$$

(6) $$\left(\frac{x}{1-x}\right)' = \frac{1(1-x)-x(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\left(\text{または } \left(\frac{x}{1-x}\right)' = \left(\frac{-(1-x)+1}{1-x}\right)' = \left(-1+\frac{1}{1-x}\right)' = \left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2}\right)$$

(7) $$\left(\frac{2}{\sin x}\right)' = -\frac{2\times(\sin x)'}{(\sin x)^2} = -\frac{2\cos x}{\sin^2 x}$$

(8) $$\left(\frac{1}{e^x}\right)' = -\frac{e^x}{(e^x)^2} = -\frac{1}{e^x}$$

$$\left(\text{または } \left(\frac{1}{e^x}\right)' = \left(e^{-x}\right)' = -e^{-x} = -\frac{1}{e^x}\right)$$

(9) $$\left(\frac{\cos x}{\sin x}\right)' = \frac{(\cos x)'\sin x - \cos x(\sin x)'}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

＜ 3ページ. 微分の復習 (3) ＞

問の解答

(1) $\left(\sin(1-x)\right)'$

$= -\cos(1-x)$

(2) $\left(\cos(x^2+2)\right)'$

$= -2x\sin(x^2+2)$

(3) $\left(e^{x^4}\right)'$

$= 4x^3e^{x^4}$

(4) $\left((2x+1)^6\right)'$

$= 12(2x+1)^5$

(5) $\left(\sqrt{2x-5}\right)'$

$= \dfrac{1}{\sqrt{2x-5}}$

(6) $\left(\cos(2x)+e^{x-1}\right)'$

$= -2\sin(2x)+e^{x-1}$

(7) $\left(\dfrac{1}{\sqrt{3x+1}}\right)'$

$= -\dfrac{3}{2(3x+1)\sqrt{3x+1}}$

(8) $\left(\log(4x+1)\right)'$

$= \dfrac{4}{4x+1}$

(9) $\left(\log(x^2+3x)\right)'$

$= \dfrac{2x+3}{x^2+3x}$

(10) $\left(e^x\sin(2x)\right)'$

$= e^x\sin(2x)+2e^x\cos(2x)$

(11) $\left(e^{2x}\cos x\right)'$

$= 2e^{2x}\cos x - e^{2x}\sin x$

(12) $\left(\sin x\cos(2x)\right)'$

$= \cos x\cos(2x) - 2\sin x\sin(2x)$

＜ 4ページ. 微分の復習 (4) ＞

問１の解答

(1) $\left(\log|2x|\right)' = \frac{2}{2x} = \frac{1}{x}$

(2) $\left(\log|\cos x|\right)' = \frac{-\sin x}{\cos x} = -\tan x$

(3) $\left(\sin^{-1}(4x)\right)'$

$= \frac{4}{\sqrt{1-(4x)^2}} = \frac{4}{\sqrt{1-16x^2}}$

(4) $\left(\tan^{-1}(5x)\right)'$

$= \frac{5}{1+(5x)^2} = \frac{5}{1+25x^2}$

問２の解答

(1) $\left(\mathrm{cosec}x\right)' = \left(\frac{1}{\sin x}\right)' = -\frac{\cos x}{\sin^2 x}$

(2) $\left(\sec x\right)' = \left(\frac{1}{\cos x}\right)' = -\frac{-\sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$

(3) $\left(\cot x\right)' = \left(\frac{1}{\tan x}\right)' = -\frac{\frac{1}{\cos^2 x}}{\tan^2 x} = -\frac{1}{\sin^2 x}$

問３の解答

(1) $\left(\left(f(x)\right)^n\right)' = n\left(f(x)\right)^{n-1} \times f'(x)$

(2) $\left(\sin\left(f(x)\right)\right)' = \cos\left(f(x)\right) \times f'(x)$

(3) $\left(\cos\left(f(x)\right)\right)' = -\sin\left(f(x)\right) \times f'(x)$

(4) $\left(\tan\left(f(x)\right)\right)' = \frac{f'(x)}{\cos^2\left(f(x)\right)}$

(5) $\left(e^{f(x)}\right)' = e^{f(x)} \times f'(x)$

(6) $\left(\log\left|f(x)\right|\right)' = \frac{f'(x)}{f(x)}$

問３の解答

(1) $\left(x\sqrt{x}\right)'$

$= \frac{3}{2}\sqrt{x}$

(2) $\left(\frac{1}{x\sqrt{x}}\right)'$

$= -\frac{3}{2x^2\sqrt{x}}$

(3) $\left(\sqrt[3]{x^4}\right)'$

$= \frac{4}{3}\sqrt[3]{x}$

(4) $\left(\sin(4x-3)\right)'$

$= 4\cos(4x-3)$

(5) $\left(e^{-\frac{x^2}{2}}\right)'$

$= -xe^{-\frac{x^2}{2}}$

(6) $\left(x\log|x| - x\right)'$

$= \log|x|$

(7) $\left(\sin(2x)\cos(3x)\right)'$

$= 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$

(8) $\left(e^{2x}\sin(3x)\right)'$

$= 2e^{2x}\sin(3x) + 3e^{2x}\cos(3x)$

(9) $\left(e^{3x}\cos(4x)\right)'$

$= 3e^{3x}\cos(4x) - 4e^{3x}\sin(4x)$

＜ 5 ページ. 高階導関数 ＞

問１の解答

(1) $f'(x) = 3x^2 - 3$　(2) $f'(x) = \cos x$　(3) $f'(x) = \dfrac{1}{x}$

$f''(x) = 6x$　$f''(x) = -\sin x$　$f''(x) = -\dfrac{1}{x^2}$

問２の解答

(1) $f'(x) = 5x^4 - 3x^2 + 1$

$f''(x) = 20x^3 - 6x$

$f'''(x) = 60x^2 - 6$

(2) $f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

(3) $f'(x) = \log x + 1$

$f''(x) = \dfrac{1}{x}$

$f'''(x) = -\dfrac{1}{x^2}$

(4) $f'(x) = 2e^{2x}$

$f''(x) = 4e^{2x}$

$f'''(x) = 8e^{2x}$

＜ 6 ページ. 関数の極限 ＞

問の解答

(1) $\displaystyle\lim_{x\to 1}\frac{x^2-16}{x-4}$

$\displaystyle=\frac{1-16}{1-4}=5$

(2) $\displaystyle\lim_{x\to 4}\frac{x^2-16}{x-4}$

$\displaystyle=\lim_{x\to 4}\frac{(x-4)(x+4)}{x-4}=\lim_{x\to 4}(x+4)=8$

(3) $\displaystyle\lim_{x\to 2}\frac{x^3-27}{x-3}$

$\displaystyle=\frac{8-27}{2-3}=19$

(4) $\displaystyle\lim_{x\to 3}\frac{x^3-27}{x-3}$

$\displaystyle=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{x-3}=\lim_{x\to 3}(x^2+3x+9)=27$

(5) $\displaystyle\lim_{x\to 2}\frac{x^4-16}{x-2}$

$\displaystyle=\lim_{x\to 2}\frac{(x-2)(x^3+2x^2+4x+8)}{(x-2)}=\lim_{x\to 2}(x^3+2x^2+4x+8)=32$

(6) $\displaystyle\lim_{x\to 1}\frac{x^5-1}{x-1}$

$\displaystyle=\lim_{x\to 1}\frac{(x-1)(x^4+x^3+x^2+x+1)}{(x-1)}=\lim_{x\to 1}(x^4+x^3+x^2+x+1)=5$

＜ 7 ページ. ロピタルの定理 (1) ＞

問の解答

(1) $\displaystyle\lim_{x\to 2}\frac{x^4-16}{x-2} = \lim_{x\to 2}\frac{4x^3-0}{1-0} = 4\times 2^3 = 32$

(2) $\displaystyle\lim_{x\to 1}\frac{e^x-e}{x-1} = \lim_{x\to 1}\frac{e^x-0}{1-0} = e^1 = e$

(3) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\cos x}{1} = \cos 0 = 1$

＜ 8ページ. ロピタルの定理 (2) ＞

問の解答

(1) $\displaystyle \lim_{x\to1}\frac{x^5-1-5(x-1)}{(x-1)^2}=\lim_{x\to1}\frac{5x^4-5}{2(x-1)}=\lim_{x\to1}\frac{20x^3}{2}=10$

(2) $\displaystyle \lim_{x\to2}\frac{x^5-2^5-5\times2^4(x-2)}{(x-2)^2}=\lim_{x\to2}\frac{5x^4-80}{2(x-2)}=\lim_{x\to2}\frac{20x^3}{2}=80$

(3) $\displaystyle \lim_{x\to1}\frac{x^4-1-4(x-1)-6(x-1)^2}{(x-1)^3}=\lim_{x\to1}\frac{4x^3-4-12(x-1)}{3(x-1)^2}$

$\displaystyle =\lim_{x\to1}\frac{12x^2-12}{6(x-1)}=\lim_{x\to1}\frac{24x}{6}=4$

(4) $\displaystyle \lim_{x\to1}\frac{x^5-1-5(x-1)-10(x-1)^2-10(x-1)^3}{(x-1)^4}$

$\displaystyle =\lim_{x\to1}\frac{5x^4-5-20(x-1)-30(x-1)^2}{4(x-1)^3}$

$\displaystyle =\lim_{x\to1}\frac{20x^3-20-60(x-1)}{12(x-1)^2}=\lim_{x\to1}\frac{60x^2-60}{24(x-1)}=\lim_{x\to1}\frac{120x}{24}=5$

< 9ページ. 微分記号 (1) >

問の解答

(1) 0　　(2) $b^4 + 2c^5x$

(3) 0　　(4) $(a-b)^2$

(5) a^4　　(6) $2a^3(x+c)$

(7) $4a(ax+b)^3$　　(8) $5(x-a)^4$

(9) $3a^3(x-a)^2$　　(10) $16a^3(x-b)^3$

(11) $2x-2a$

(12) $3x^2 - 3a^2 - 12a(x-a)$
$(= 3x^2 - 12ax + 9a^2)$

＜ 10 ページ. 微分記号 (2) ＞

問１の解答

(1) $12t^2 + 10t - 2$　　(2) $30y^5 - 21y^2 + 32y^3$

(3) $5(t+4)^4$　　(4) $18(3y+1)^5$

(5) $60(t-5)^5$　　(6) $120(y-4)^7$

問２の解答

(1) $(a-b)^2$　　(2) a^4

(3) $2a(at+b)$　　(4) $3a(ay-b)^2$

(5) $2a^4(t-1)$　　(6) $3a^5(y-b)^2$

(7) $4a^5(t-a)^3$　　(8) $15a^2(y+a)^4$

＜ 11 ページ. ロピタルの定理 (3) ＞

問の解答

(1) $\displaystyle\lim_{x\to a}\frac{x^2-a^2-2a(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{2x-2a}{2(x-a)}=1$

(2) $\displaystyle\lim_{x\to a}\frac{x^4-a^4-4a^3(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{4x^3-4a^3}{2(x-a)}=\lim_{x\to a}\frac{12x^2}{2}=6a^2$

(3) $\displaystyle\lim_{x\to a}\frac{x^5-a^5-5a^4(x-a)}{(x-a)^2}=\lim_{x\to a}\frac{5x^4-5a^4}{2(x-a)}=\lim_{x\to a}\frac{20x^3}{2}=10a^3$

＜ 12 ページ. ロピタルの定理 (4) ＞

問の解答

(1) $\displaystyle \lim_{y\to a}\frac{\log y-\log a}{y-a}=\lim_{y\to a}\frac{\frac{1}{y}}{1}=\frac{1}{a}$

(2) $\displaystyle \lim_{t\to b}\frac{\cos t-\cos b}{t-b}=\lim_{t\to b}\frac{-\sin t}{1}=-\sin b$

(3) $\displaystyle \lim_{\beta\to a}\frac{a\sin(b\beta)-\beta\sin(ab)}{a^3-a\beta^2}=\lim_{\beta\to a}\frac{ab\cos(b\beta)-\sin(ab)}{-2a\beta}$

$$=\frac{ab\cos(ab)-\sin(ab)}{-2a^2}=\frac{\sin(ab)}{2a^2}-\frac{b\cos(ab)}{2a}$$

＜ 13 ページ. ロピタルの定理 (5) ＞

問の解答

(1) $\displaystyle \lim_{x\to a} \frac{x^4 - a^4 - 4a^3(x-a) - 6a^2(x-a)^2}{(x-a)^3}$

$\displaystyle = \lim_{x\to a} \frac{4x^3 - 4a^3 - 12a^2(x-a)}{3(x-a)^2} = \lim_{x\to a} \frac{12x^2 - 12a^2}{6(x-a)} = \lim_{x\to a} \frac{24x}{6} = 4a$

(2) $\displaystyle \lim_{x\to a} \frac{x^6 - a^6 - 6a^5(x-a) - 15a^4(x-a)^2}{(x-a)^3}$

$\displaystyle = \lim_{x\to a} \frac{6x^5 - 6a^5 - 30a^4(x-a)}{3(x-a)^2} = \lim_{x\to a} \frac{30x^4 - 30a^4}{6(x-a)} = \lim_{x\to a} \frac{120x^3}{6} = 20a^3$

(3) $\displaystyle \lim_{x\to a} \frac{x^7 - a^7 - 7a^6(x-a) - 21a^5(x-a)^2}{(x-a)^3}$

$\displaystyle = \lim_{x\to a} \frac{7x^6 - 7a^6 - 42a^5(x-a)}{3(x-a)^2} = \lim_{x\to a} \frac{42x^5 - 42a^5}{6(x-a)} = \lim_{x\to a} \frac{210x^4}{6} = 35a^4$

＜ 14 ページ. ロピタルの定理 (6) ＞

問の解答

(1) $\displaystyle \lim_{x\to a}\frac{x^5-a^5-5a^4(x-a)-10a^3(x-a)^2-10a^2(x-a)^3}{(x-a)^4}$

$\displaystyle =\lim_{x\to a}\frac{5x^4-5a^4-20a^3(x-a)-30a^2(x-a)^2}{4(x-a)^3}$

$\displaystyle =\lim_{x\to a}\frac{20x^3-20a^3-60a^2(x-a)}{12(x-a)^2}=\lim_{x\to a}\frac{60x^2-60a^2}{24(x-a)}$

$\displaystyle =\lim_{x\to a}\frac{120x}{24}=5a$

(2) $\displaystyle \lim_{x\to a}\frac{x^7-a^7-7a^6(x-a)-21a^5(x-a)^2-35a^4(x-a)^3}{(x-a)^4}$

$\displaystyle =\lim_{x\to a}\frac{7x^6-7a^6-42a^5(x-a)-105a^4(x-a)^2}{4(x-a)^3}$

$\displaystyle =\lim_{x\to a}\frac{42x^5-42a^5-210a^4(x-a)}{12(x-a)^2}=\lim_{x\to a}\frac{210x^4-210a^4}{24(x-a)}$

$\displaystyle =\lim_{x\to a}\frac{840x^3}{24}=35a^3$

＜ 15 ページ. ロピタルの定理 (7) ＞

問の解答

(1) $\displaystyle \lim_{x\to 3}\frac{x^4-1}{x-1}=\frac{81-1}{3-1}=40$

(2) $\displaystyle \lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to 1}\frac{4x^3}{1}=4$

(3) $\displaystyle \lim_{x\to 0}\frac{\cos x}{x-\frac{\pi}{2}}=\frac{\cos 0}{0-\frac{\pi}{2}}=-\frac{2}{\pi}$

(4) $\displaystyle \lim_{x\to \frac{\pi}{2}}\frac{\cos x}{x-\frac{\pi}{2}}=\lim_{x\to \frac{\pi}{2}}\frac{-\sin x}{1}=-\sin\frac{\pi}{2}=-1$

(5) $\displaystyle \lim_{x\to 1}\frac{e^x-e^2}{x-2}=\frac{e^1-e^2}{1-2}=e^2-e$

(6) $\displaystyle \lim_{x\to 1}\frac{\log x}{x-1}=\lim_{x\to 1}\frac{\frac{1}{x}}{1}=1$

(7) $$\lim_{x\to 1}\frac{e^x-e-e(x-1)}{(x-1)^2}=\lim_{x\to 1}\frac{e^x-e}{2(x-1)}=\lim_{x\to 1}\frac{e^x}{2}=\frac{e}{2}$$

(8) $$\lim_{x\to 2}\frac{\log x-\log 2-\frac{1}{2}(x-2)}{(x-2)^2}=\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{2(x-2)}=\lim_{x\to 2}\frac{-\frac{1}{x^2}}{2}=-\frac{1}{8}$$

(9) $$\lim_{x\to 4}\frac{\sqrt{x}-2-\frac{1}{4}(x-4)}{(x-4)^2}=\lim_{x\to 4}\frac{\frac{1}{2\sqrt{x}}-\frac{1}{4}}{2(x-4)}=\lim_{x\to 4}\frac{-\frac{1}{4x\sqrt{x}}}{2}=-\frac{1}{64}$$

(10) $$\lim_{x\to a}\frac{e^x-e^a-e^a(x-a)-\frac{e^a}{2}(x-a)^2}{(x-a)^3}=\lim_{x\to a}\frac{e^x-e^a-e^a(x-a)}{3(x-a)^2}$$

$$=\lim_{x\to a}\frac{e^x-e^a}{6(x-a)}=\lim_{x\to a}\frac{e^x}{6}=\frac{e^a}{6}$$

(11) $$\lim_{x\to a}\frac{\sin x-\sin a-(\cos a)(x-a)+(\frac{1}{2}\sin a)(x-a)^2}{(x-a)^3}=\lim_{x\to a}\frac{\cos x-\cos a+(\sin a)(x-a)}{3(x-a)^2}$$

$$=\lim_{x\to a}\frac{-\sin x+\sin a}{6(x-a)}=\lim_{x\to a}\frac{-\cos x}{6}=-\frac{\cos a}{6}$$

(12) $$\lim_{x\to a}\frac{\cos x-\cos a+(\sin a)(x-a)+(\frac{1}{2}\cos a)(x-a)^2-(\frac{1}{6}\sin a)(x-a)^3}{(x-a)^4}$$

$$=\lim_{x\to a}\frac{-\sin x+\sin a+(\cos a)(x-a)-(\frac{1}{2}\sin a)(x-a)^2}{4(x-a)^3}$$

$$=\lim_{x\to a}\frac{-\cos x+\cos a-(\sin a)(x-a)}{12(x-a)^2}=\lim_{x\to a}\frac{\sin x-\sin a}{24(x-a)}=\lim_{x\to a}\frac{\cos x}{24}=\frac{\cos a}{24}$$

＜ 16 ページ. 接線の傾き ＞

問の解答

(1) $y = e^x$ の $x = 1$ における接線の傾き $= e^1 = e$

(2) $y = e^x$ の $x = -1$ における接線の傾き $= e^{-1} = \dfrac{1}{e}$

(3) $f(x) = \sin x$ の $x = 0$ における接線の傾き $= \cos 0 = 1$

(4) $f(x) = \sin x$ の $x = \pi$ における接線の傾き $= \cos \pi = -1$

(5) $f(x) = \cos x$ の $x = 0$ における接線の傾き $= -\sin 0 = 0$

(6) $f(x) = \cos x$ の $x = \dfrac{\pi}{2}$ における接線の傾き $= -\sin\left(\dfrac{\pi}{2}\right) = -1$

(7) $f(x) = \log x$ の $x = 1$ における接線の傾き $= \dfrac{1}{1} = 1$

(8) $f(x) = \log x$ の $x = 2$ における接線の傾き $= \dfrac{1}{2}$

＜ 17 ページ. 接線の方程式 ＞

問の解答

(1) $y' = e^x = e^0 = 1$ 　　 $y = 1(x-0) + e^0$ 　　 (答) $y = x + 1$

(2) $y' = \frac{1}{x} = \frac{1}{1} = 1$ 　　 $y = 1(x-1) + \log 1$ 　　 (答) $y = x - 1$

(3) $y' = -\sin x = -\sin\frac{\pi}{2} = -1$ 　　 $y = -1(x - \frac{\pi}{2}) + \cos\frac{\pi}{2}$ 　　 (答) $y = -x + \frac{\pi}{2}$

(4) $y' = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{4}}$ 　　 $y = \frac{1}{4}(x-4) + \sqrt{4}$ 　　 (答) $y = \frac{1}{4}x + 1$

(5) $y = e^a(x-a) + e^a$ 　　 (答) $y = e^a x + (1-a)e^a$

(6) $y = \cos a(x-a) + \sin a$ 　　 (答) $y = (\cos a)x - a\cos a + \sin a$

(7) $y = \frac{1}{a}(x-a) + \log a$ 　　 (答) $y = \frac{1}{a}x - 1 + \log a$

(8) $y = \frac{1}{2\sqrt{a}}(x-a) + \sqrt{a}$ 　　 (答) $y = \frac{1}{2\sqrt{a}}x + \frac{1}{2}\sqrt{a}$

(9) $y = \frac{1}{3\sqrt[3]{a^2}}(x-a) + \sqrt[3]{a}$ 　　 (答) $y = \frac{1}{3\sqrt[3]{a^2}}x + \frac{2}{3}\sqrt[3]{a}$

＜ 18 ページ. 関数の 1 次近似 (1) ＞

問の解答

(1)　$f'(a) = 6a^5$ より x ; a のとき　　$x^6 ; a^6 + 6a^5(x-a) = 6a^5x - 5a^6$

(2)　$f'(a) = -\dfrac{1}{a^2}$ より x ; a のとき　　$\dfrac{1}{x} ; \dfrac{1}{a} - \dfrac{1}{a^2}(x-a) = -\dfrac{1}{a^2}x + \dfrac{2}{a}$

(3)　$f'(a) = \dfrac{1}{2\sqrt{a}}$ より x ; a のとき　　$\sqrt{x} ; \sqrt{a} + \dfrac{1}{2\sqrt{a}}(x-a) = \dfrac{1}{2\sqrt{a}}x + \dfrac{\sqrt{a}}{2}$

(4)　$f'(a) = \dfrac{1}{4\sqrt[4]{a^3}}$ より x ; a のとき　　$\sqrt[4]{x} ; \sqrt[4]{a} + \dfrac{1}{4\sqrt[4]{a^3}}(x-a)$

$$= \frac{1}{4\sqrt[4]{a^3}}x + \frac{3}{4}\sqrt[4]{a}$$

＜19 ページ. 関数の１次近似 (2) ＞

問１の解答

(1) x ; a のとき　$\cos x$; $\cos a - (\sin a)(x-a)$

(2) x ; a のとき　$\tan x$; $\tan a + \dfrac{1}{\cos^2 a}(x-a)$

(3) x ; a のとき　$\log x$; $\log a + \dfrac{1}{a}(x-a)$

(4) x ; a のとき　e^x ; $e^a + e^a(x-a)$

問２の解答

(1) x ; 0 のとき　$\sin x$; x

(2) x ; $\frac{\pi}{2}$ のとき　$\cos x$; $-(x-\frac{\pi}{2}) = -x + \frac{\pi}{2}$

(3) x ; 0 のとき　e^x ; $1 + x$

(4) x ; 1 のとき　$\log x$; $x - 1$

(5) x ; 1 のとき　$\sqrt{x}$; $1 + \dfrac{1}{2}(x-1) = \dfrac{1}{2}x + \dfrac{1}{2}$

(6) x ; 1 のとき　$\sqrt[3]{x}$; $1 + \dfrac{1}{3}(x-1) = \dfrac{1}{3}x + \dfrac{2}{3}$

＜ 20 ページ.1 次近似値 ＞

問の解答

(1) $\sqrt[3]{x}$; $\sqrt[3]{a} + \dfrac{1}{3\sqrt[3]{a^2}}(x-a)$　で $x = 1.1,\ a = 1$ とおくと

$\sqrt[3]{1.1}$; $\sqrt[3]{1} + \dfrac{1}{3\sqrt[3]{1^2}}(1.1-1) = 1 + \dfrac{1}{3} \times 0.1 = \underline{1 + \dfrac{1}{30}\ ;\ 1.03}$

(2) $\sqrt[4]{x}$; $\sqrt[4]{a} + \dfrac{1}{4\sqrt[4]{a^3}}(x-a)$　で $x = 1.1,\ a = 1$ とおくと

$\sqrt[4]{1.1}$; $\sqrt[4]{1} + \dfrac{1}{4\sqrt[4]{1^3}}(1.1-1) = 1 + \dfrac{1}{4} \times 0.1 = \underline{1 + \dfrac{1}{40} = 1.025}$

(3) $\log x$; $\log a + \dfrac{1}{a}(x-a)$　で $x = 1.1,\ a = 1$ とおくと

$\log 1.1$; $\log 1 + \dfrac{1}{1}(1.1-1) = 0 + 0.1 = \underline{0.1}$

(4) e^x ; $e^a + e^a(x-a)$　で $x = 0.1,\ a = 0$ とおくと

$e^{0.1}$; $e^0 + e^0(0.1-0) = 1 + 1 \times 0.1 = \underline{1.1}$

(5) $\sin x$; $\sin a + \cos a(x-a)$　で $x = 0.1,\ a = 0$ とおくと

$\sin 0.1$; $\sin 0 + \cos 0(0.1-0) = 0 + 1 \times 0.1 = \underline{0.1}$

(6) $\cos x$; $\cos a + \sin a(x-a)$　で $x = 0.1,\ a = 0$ とおくと

$\cos 0.1$; $\cos 0 + \sin 0(0.1-0) = 1 - 0 \times 0.1 = \underline{1}$

＜ 21 ページ. 関数の高次近似 (1) ＞

問の解答

(1) $f'(x) - f'(a)$

(2) $f''(x) - f''(a)$

(3) $f'(x) - f'(a) - f''(a)(x - a)$

(4) $f''(x) - f''(a) - f'''(a)(x - a)$

(5) $f'(x) - f'(a) - f''(a)(x - a) - \frac{1}{2}f'''(a)(x - a)^2$

＜ 22 ページ. 関数の高次近似 (2) ＞

問の解答

(1) $\displaystyle \lim_{x\to a}\frac{f(x)-f(a)-f'(a)(x-a)-\frac{1}{2}f''(a)(x-a)^2}{(x-a)^3}$

$\displaystyle = \lim_{x\to a}\frac{f'(x)-f'(a)-f''(a)(x-a)}{3(x-a)^2}$

$\displaystyle = \lim_{x\to a}\frac{f''(x)-f''(a)}{6(x-a)} = \lim_{x\to a}\frac{f'''(x)}{6} = \frac{f'''(a)}{6}$

(2) $\displaystyle \lim_{x\to a}\frac{f(x)-f(a)-f'(a)(x-a)-\frac{1}{2}f''(a)(x-a)^2-\frac{1}{6}f'''(a)(x-a)^3}{(x-a)^4}$

$\displaystyle = \lim_{x\to a}\frac{f'(x)-f'(a)-f''(a)(x-a)-\frac{1}{2}f'''(a)(x-a)^2}{4(x-a)^3}$

$\displaystyle = \lim_{x\to a}\frac{f''(x)-f''(a)-f'''(a)(x-a)}{12(x-a)^2}$

$\displaystyle = \lim_{x\to a}\frac{f'''(x)-f'''(a)}{24(x-a)} = \lim_{x\to a}\frac{f''''(x)}{24} = \frac{f''''(a)}{24}$

< 23ページ. 関数の高次近似 (3) >

問の解答

$f(x)$ ； $f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3 + \frac{1}{24}f''''(a)(x-a)^4$

< 24 ページ. 高階微分係数 >

問の解答

(1) $f^{(4)}(x) = e^x$, $f^{(4)}(0) = e^0 = 1$

(2) $f^{(n)}(x) = e^x$, $f^{(n)}(0) = e^0 = 1$

(3) $f^{(1)}(x) = \cos x$, $f^{(2)}(x) = -\sin x$, $f^{(3)}(x) = -\cos x$, $f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x$, $f^{(6)}(x) = -\sin x$, $f^{(7)}(x) = -\cos x$, $f^{(8)}(x) = \sin x$

$f^{(1)}(0) = \cos 0 = 1$, $f^{(2)}(0) = -\sin 0 = 0$, $f^{(3)}(0) = -\cos 0 = -1$, $f^{(4)}(0) = \sin 0 = 0$

$f^{(5)}(0) = \cos 0 = 1$, $f^{(6)}(0) = -\sin 0 = 0$, $f^{(7)}(0) = -\cos 0 = -1$, $f^{(8)}(0) = \sin 0 = 0$

＜ 25 ページ. 関数の n 次近似 (1) ＞

問の解答

$$f(x) \fallingdotseq f(a) + \frac{1}{1!}f^{(1)}(a)(x-a) + \frac{1}{2!}f^{(2)}(a)(x-a)^2 + \frac{1}{3!}f^{(3)}(a)(x-a)^3 + \frac{1}{4!}f^{(4)}(a)(x-a)^4$$

＜ 26 ページ. 関数の n 次近似 (2) ＞

問の解答

(1) $e^x \fallingdotseq e^a + e^a(x-a) + \dfrac{e^a}{2!}(x-a)^2 + \dfrac{e^a}{3!}(x-a)^3 + \cdots + \dfrac{1}{n!}e^a(x-a)^n$

(2) $e^x \fallingdotseq e + e(x-1) + \dfrac{e}{2!}(x-1)^2 + \dfrac{e}{3!}(x-1)^3 + \cdots + \dfrac{e}{n!}(x-1)^n$

(3) $e^x \fallingdotseq 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!}$

＜ 27 ページ. テーラー展開 ＞

問１の解答

$$e^x = e^a + e^a(x-a) + \frac{1}{2!}e^a(x-a)^2 + \frac{1}{3!}e^a(x-a)^3 + \cdots + \frac{1}{n!}e^a(x-a)^n + \cdots$$

問２の解答

(1) $e^x = e + e(x-1) + \frac{1}{2!}e(x-1)^2 + \frac{1}{3!}e(x-1)^3 + \cdots + \frac{1}{n!}e(x-1)^n + \cdots$

(2) $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$

＜ 28 ページ. マクローリン展開 (1) ＞

問１の解答

$f(0) = \sin 0 = 0$, $f^{(1)}(0) = 1$, $f^{(2)}(0) = 0$, $f^{(3)}(0) = -1$, $f^{(4)}(0) = 0$, $f^{(5)}(0) = 1$, $\cdots$

より

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} + \cdots$$

問２の解答

(1) e^x のマクローリン展開より

$$\lim_{x\to 0}\frac{e^x - 1 - x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!}}{x^5}$$

$$= \lim_{x\to 0}\frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots\right) - 1 - x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!}}{x^5}$$

$$= \lim_{x\to 0}\frac{\frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots}{x^5} = \lim_{x\to 0}\left(\frac{1}{5!} + \frac{x}{6!} + \frac{x^2}{7!} + \cdots\right) = \frac{1}{5!}$$

(2) $\cos x$ のマクローリン展開より

$$\lim_{x\to 0}\frac{\cos x - 1 + \frac{x^2}{2!} - \frac{x^4}{4!}}{x^6}$$

$$= \lim_{x\to 0}\frac{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots\right) - 1 + \frac{x^2}{2!} - \frac{x^4}{4!}}{x^6}$$

$$= \lim_{x\to 0}\frac{-\frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots}{x^6} = \lim_{x\to 0}\left(-\frac{1}{6!} + \frac{x^2}{8!} - \frac{x^4}{10!} + \cdots\right) = -\frac{1}{6!}$$

(3) $\sin x$ のマクローリン展開より

$$\lim_{x\to 0}\frac{\sin x - x + \frac{x^3}{3!} - \frac{x^5}{5!}}{x^7}$$

$$= \lim_{x\to 0}\frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots\right) - x + \frac{x^3}{3!} - \frac{x^5}{5!}}{x^7}$$

$$= \lim_{x\to 0}\frac{-\frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots}{x^7} = \lim_{x\to 0}\left(-\frac{1}{7!} + \frac{x^2}{9!} - \frac{x^4}{11!} + \cdots\right) = -\frac{1}{7!}$$

＜ 30 ページ. マクローリン展開 (3) ＞

問の解答

$$
\begin{aligned}
e &\fallingdotseq 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} \\
&= 2 + \frac{12 + 4 + 1}{24} \\
&= \frac{65}{24} \\
&\fallingdotseq 2.708
\end{aligned}
$$

＜ 31 ページ. 近似の練習 (1) ＞

問１の解答

(1) $x \fallingdotseq a$ のとき　$\cos x \fallingdotseq \cos a - (\sin a)(x-a) - \frac{1}{2}(\cos a)(x-a)^2$

(2) $x \fallingdotseq a$ のとき　$e^x \fallingdotseq e^a + e^a(x-a) + \frac{1}{2}e^a(x-a)^2$

(3) $x \fallingdotseq a$ のとき　$\log x \fallingdotseq \log a + \frac{1}{a}(x-a) - \frac{1}{2a^2}(x-a)^2$

(4) $x \fallingdotseq a$ のとき　$\sqrt{x} \fallingdotseq \sqrt{a} + \frac{1}{2\sqrt{a}}(x-a) - \frac{1}{8a\sqrt{a}}(x-a)^2$

(5) $x \fallingdotseq a$ のとき　$e^{-x} \fallingdotseq e^{-a} - e^{-a}(x-a) + \frac{1}{2}e^{-a}(x-a)^2$

問２の解答

(1) $x \fallingdotseq \pi$ のとき　$\cos x \fallingdotseq \cos \pi - (\sin \pi)(x-\pi) - \frac{1}{2}(\cos \pi)(x-\pi)^2$

$$= -1 + \frac{1}{2}(x-\pi)^2$$

(2) $x \fallingdotseq 1$ のとき　$e^x \fallingdotseq e^1 + e^1(x-1) + \frac{1}{2}e^1(x-1)^2$

$$= \frac{e}{2}x^2 + \frac{e}{2}$$

(3) $x \fallingdotseq 1$ のとき　$\log x \fallingdotseq \log 1 + \frac{1}{1}(x-1) - \frac{1}{2\times 1^2}(x-1)^2$

$$= -\frac{1}{2}x^2 + 2x - \frac{3}{2}$$

(4) $x \fallingdotseq 1$ のとき　$\sqrt{x} \fallingdotseq \sqrt{1} + \frac{1}{2\sqrt{1}}(x-1) - \frac{1}{8\times 1\sqrt{1}}(x-1)^2$

$$= -\frac{1}{8}x^2 + \frac{3}{4}x + \frac{3}{8}$$

＜ 32 ページ. 近似の練習 (2) No.1 ＞

問１の解答

(1) $x \fallingdotseq 0$ のとき

$$e^{-x} \fallingdotseq e^{-0} - e^{-0}(x-0) + \frac{1}{2}e^{-0}(x-0)^2 = 1 - x + \frac{1}{2}x^2$$

(2) $f'(x) = -2xe^{-x^2}$ ， $f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$,

$x \fallingdotseq 0$ のとき

$$e^{-x^2} \fallingdotseq e^{-0^2} - 2\times 0 \times e^{-0^2}(x-0) + \frac{-2e^{-0^2} + 4\times 0^2 \times e^{-0^2}}{2}(x-0)^2$$

$$= 1 - x^2$$

問２の解答

$$\sqrt{x} \fallingdotseq \sqrt{1} + \frac{1}{2\sqrt{1}}(x-1) - \frac{1}{8\times 1\sqrt{1}}(x-1)^2 = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$$

$x = 1.1$ とおくと

$$\sqrt{1.1} \fallingdotseq 1 + \frac{1}{2}\times 0.1 - \frac{1}{8}\times(0.1)^2 = 1 + 0.05 - 0.00125 = 1.04875$$

問３の解答

$$\log x \fallingdotseq \log 1 + \frac{1}{1}(x-1) - \frac{1}{2\times 1^2}(x-1)^2 = (x-1) - \frac{1}{2}(x-1)^2$$

$x = 1.1$ とおくと

$$\log 1.1 \fallingdotseq 0.1 - \frac{1}{2}\times(0.1)^2 = 0.1 - 0.005 = 0.095$$

＜ 32 ページ. 近似の練習 (2) No.2 ＞

問 4 の解答

$$\cos x \fallingdotseq \cos 0 - \sin 0 (x-0) - \frac{1}{2}\cos 0 (x-0)^2 = 1 - \frac{1}{2}x^2$$

$$\cos(0.1) \fallingdotseq 1 - \frac{1}{2} \times (0.1)^2 = 1 - 0.005 = 0.995$$

問 5 の解答

$$e^x \fallingdotseq e^0 + e^0 x + \frac{1}{2!}e^0 x^2 + \frac{1}{3!}e^0 x^3 = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

$$\sqrt{e} = e^{\frac{1}{2}} \fallingdotseq 1 + \frac{1}{2} + \frac{1}{2} \times \left(\frac{1}{2}\right)^2 + \frac{1}{6} \times \left(\frac{1}{2}\right)^3 = \frac{79}{48} \fallingdotseq 1.6458$$

問 6 の解答

$$e^x \fallingdotseq 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!} + \frac{1}{4!}x^4$$

$$\frac{1}{e} = e^{-1} \fallingdotseq 1 - 1 + \frac{1}{2} \times (-1)^2 + \frac{1}{6} \times (-1)^3 + \frac{1}{24} \times (-1)^4 = \frac{3}{8} = 0.375$$

問 7 の解答

(1) e

(2) $e^{-1} = \dfrac{1}{e}$

(3) sin(1)

(4) cos(1)

< 33 ページ. 不定積分 (1) >

問の解答

(1) $\left(\dfrac{1}{\alpha+1}x^{\alpha+1}\right)' = x^{\alpha}$ $\Rightarrow$ $\displaystyle\int x^{\alpha}dx = \frac{1}{\alpha+1}x^{\alpha+1} + C$

(2) $(\log|x|)' = \dfrac{1}{x}$ $\Rightarrow$ $\displaystyle\int \frac{1}{x}dx = \log|x| + C$

(3) $(\sin x)' = \cos x$ $\Rightarrow$ $\displaystyle\int \cos x dx = \sin x + C$

(4) $(-\cos x)' = \sin x$ $\Rightarrow$ $\displaystyle\int \sin x dx = -\cos x + C$

(5) $(e^x)' = e^x$ $\Rightarrow$ $\displaystyle\int e^x dx = e^x + C$

< 34 ページ. 不定積分 (2) >

問の解答

(1) $\dfrac{1}{7}x^7 + C$

(2) $\dfrac{4}{5}x^{\frac{5}{4}} + C = \dfrac{4}{5}x\sqrt[4]{x} + C$

(3) $2x^{\frac{1}{2}} + C = 2\sqrt{x} + C$

(4) $\displaystyle\int x^{-3}dx = \frac{1}{-2}x^{-2} + C = -\frac{1}{2x^2} + C$

(5) $\displaystyle\int x^{\frac{2}{3}}dx = \frac{3}{5}x^{\frac{5}{3}} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$

(6) $\displaystyle\int x^{-\frac{1}{4}}dx = \frac{4}{3}x^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{x^3} + C$

＜ 35 ページ. 不定積分 (3) ＞

問の解答

(1) $\displaystyle\int\left(\frac{1}{x}-\frac{4}{x^2}+\frac{1}{x^3}\right)dx=\log|x|+\frac{4}{x}-\frac{1}{2x^2}+C$

(2) $\displaystyle\int\frac{x^4-4x^2+3}{x^4}dx=\int\left(1-\frac{4}{x^2}+\frac{3}{x^4}\right)dx=x+\frac{4}{x}-\frac{1}{x^3}+C$

(3) $\displaystyle\int\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)dx=\int\left(x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\right)dx=\frac{2}{3}x^{\frac{3}{2}}+2\times\frac{1}{\frac{1}{2}}x^{\frac{1}{2}}+C=\frac{2}{3}x\sqrt{x}+4\sqrt{x}+C$

(4) $\displaystyle\int\frac{x-2\sqrt{x}+1}{x}dx=\int\left(1-\frac{2}{\sqrt{x}}+\frac{1}{x}\right)dx=x-4\sqrt{x}+\log|x|+C$

＜ 36 ページ. 不定積分 (4) ＞

問１の解答

(1) $(\tan x)' = \dfrac{1}{\cos^2 x} \quad \Rightarrow \quad \displaystyle\int \frac{dx}{\cos^2 x} = \tan x + C$

(2) $\left(\dfrac{1}{\tan x}\right)' = -\dfrac{\frac{1}{\cos^2 x}}{\tan^2 x} = -\dfrac{1}{\sin^2 x} \quad \Rightarrow \quad \displaystyle\int \frac{dx}{\sin^2 x} = -\frac{1}{\tan x} + C \quad \left(= -\cot x + C \right)$

(3) $(a^x)' = a^x \log_e a \quad \Rightarrow \quad \displaystyle\int a^x dx = \frac{a^x}{\log_e a} + C$

(4) $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}} \quad \Rightarrow \quad \displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

(5) $(\tan^{-1} x)' = \dfrac{1}{1+x^2} \quad \Rightarrow \quad \displaystyle\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

問２の解答

(1) $-4\cos x - 3\sin x + C$

(2) $\displaystyle\int \left(3\cos x - \frac{1}{\cos^2 x}\right) dx = 3\sin x - \tan x + C$

(3) $\displaystyle\int (2\cos x - \sin x) dx = 2\sin x + \cos x + C$

(4) $\displaystyle\int \frac{1}{-\cos^2 x} dx = -\tan x + C$

(5) $\displaystyle\int \left(\frac{1}{\sin^2 x} - 1\right) dx = -\frac{1}{\tan x} - x + C \quad \left(= -\cot x - x + C \right)$

(6) $\dfrac{3^x}{\log_e 3} - 2e^x + C$

(7) $3\sin^{-1} x + C$

(8) $5\tan^{-1} x + C$

< 37ページ.積分記号 >

問の解答

(1) $10t - 4.9t^2 + C$

(2) $\frac{4}{3}\pi r^3 + C$

(3) $e^u + C$

(4) $\log|y| + C$

(5) $\sin u + C$

< 38 ページ. 置換積分法 (1) >

問の解答

(1) $\displaystyle\int u^3 \times \frac{1}{5}du = \frac{1}{20}u^4 + C = \frac{1}{20}(5x+6)^4 + C$

$$\left(\begin{array}{l} u = 5x + 6 \\ \dfrac{du}{dx} = 5 \quad \Rightarrow \quad dx = \dfrac{1}{5}du \end{array} \right)$$

(2) $\displaystyle -\frac{1}{21(7x+5)^3} + C$

(3) $\displaystyle \frac{2}{15}(5x-3)\sqrt{5x-3} + C$

(4) $\displaystyle -\frac{1}{3}\cos(3x+2) + C$

(5) $\displaystyle -\frac{1}{3}e^{-3x+2} + C$

(6) $\displaystyle \frac{1}{4}\tan(4x+3) + C$

< 39 ページ. 置換積分法 (2) >

問の解答

(1) $\frac{1}{2}e^{x^2+1}+C$

(2) $\frac{1}{4}e^{x^4}+C$

(3) $\frac{1}{3}\sin(x^3+2)+C$

(4) $-\frac{1}{2}\cos(x^2+3)+C$

(5) $\frac{1}{2}\log(x^2+3)+C$

(6) $\frac{1}{12}(x^2+1)^6+C$

< 40 ページ. 分数関数の積分 (1) >

問の解答

(1) $\log|x+1|+C$

(2) $-\log|2-x|+C\ \left(=-\log|x-2|+C\right)$

(3) $\dfrac{1}{2}\log|2x+3|+C$

(4) $-\dfrac{1}{4}\log|3-4x|+C\ \left(=-\dfrac{1}{4}\log|4x-3|+C\right)$

(5) $-\dfrac{1}{x-3}+C\ \left(=\dfrac{1}{3-x}+C\right)$

(6) $\dfrac{1}{4-x}+C$

(7) $-\dfrac{1}{3(3x-4)}+C$

(8) $\dfrac{1}{5(3-5x)}+C$

(9) $\tan^{-1}(x-2)+C$

(10) $\dfrac{1}{5}\tan^{-1}\left(\dfrac{x}{5}\right)+C$

(11) $\dfrac{1}{6}\tan^{-1}\left(\dfrac{2x+1}{3}\right)+C$

(12) $\dfrac{1}{6}\tan^{-1}\left(\dfrac{3x-5}{2}\right)+C$

＜ 41 ページ. 分数関数の積分 (2) ＞

問の解答

(1) $\displaystyle\int\left(\frac{1}{x}-\frac{1}{x+1}\right)dx = \log\left|\frac{x}{x+1}\right| + C$

(2) $\displaystyle\int\frac{1}{2}\left\{\frac{1}{x-1}-\frac{1}{x+1}\right\}dx = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$

(3) $\displaystyle\int\left(\frac{1}{x-3}-\frac{1}{x-2}\right)dx = \log\left|\frac{x-3}{x-2}\right| + C$

(4) $\displaystyle\int\frac{1}{7}\left\{\frac{1}{x-3}-\frac{1}{x+4}\right\}dx = \frac{1}{7}\log\left|\frac{x-3}{x+4}\right| + C$

(5) $$\int\frac{1}{(2x+1)(3x+4)}dx = \int\left(\frac{\frac{2}{5}}{2x+1}-\frac{\frac{3}{5}}{3x+4}\right)dx$$

$$= \frac{1}{5}\int\left(\frac{2}{2x+1}-\frac{3}{3x+4}\right)dx$$

$$= \frac{1}{5}\left\{\log|2x+1| - \log|3x+4|\right\} + C$$

$$= \frac{1}{5}\log\left|\frac{2x+1}{3x+4}\right| + C$$

< 42 ページ. 部分積分法 (1) >

問の解答

(1) $\int x \times (-\cos x)' dx = -x\cos x + \int \cos x dx = -x\cos x + \sin x + C$

(2) $\int x \times (e^x)' dx = xe^x - \int e^x dx = xe^x - e^x + C$

(3) $\int x \times \left(\frac{\sin(2x)}{2}\right)' dx = \frac{x}{2}\sin(2x) - \int \frac{1}{2}\sin(2x) dx = \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$

(4) $\int x \times \left(-\frac{\cos(2x)}{2}\right)' dx = x \times \left(-\frac{\cos(2x)}{2}\right) - \int \left(-\frac{\cos(2x)}{2}\right) dx$

$= -\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + C$

(5) $\int x \times \left(\frac{e^{3x}}{3}\right)' dx = x \times \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx = \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C$

＜ 43 ページ. 部分積分法 (2) ＞

問１の解答

(1) $\int x \log x dx = \int \left(\frac{x^2}{2}\right)' \times \log x dx = \frac{x^2}{2} \times \log x - \int \frac{x^2}{2} \times (\log x)' dx$

$= \frac{x^2}{2} \log x - \int \frac{x}{2} dx = \frac{x^2}{2} \log x - \frac{x^2}{4} + C$

(2) $\int x^2 \log x dx = \int \left(\frac{x^3}{3}\right)' \log x dx = \frac{x^3}{3} \log x - \int \frac{x^3}{3} \times (\log x)' dx$

$= \frac{x^3}{3} \log x - \int \frac{x^2}{3} dx = \frac{x^3}{3} \log x - \frac{x^3}{9} + C$

問２

(1) $\int x^2 \sin x dx = \int x^2 (-\cos x)' dx = -x^2 \cos x + \int 2x \cos x dx$

$= -x^2 \cos x + \int 2x (\sin x)' dx = -x^2 \cos x + 2x \sin x - \int 2 \sin x dx$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$

(2) $\int x^2 e^x dx = \int x^2 (e^x)' dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - \int 2x (e^x)' dx$

$= x^2 e^x - 2x e^x + \int 2 e^x dx$

$= x^2 e^x - 2x e^x + 2e^x + C$

＜ 44 ページ. 三角関数の不定積分 ＞

問の解答

(1) $\displaystyle\int\left(\frac{1}{2}-\frac{1}{2}\cos(2x)\right)dx = \frac{1}{2}x-\frac{1}{4}\sin(2x)+C$

(2) $\displaystyle\int\frac{1}{2}\left\{\cos(5x)+\cos x\right\}dx = \frac{1}{10}\sin(5x)+\frac{1}{2}\sin x+C$

(3) $\displaystyle\int\frac{1}{2}\left\{\cos(3x)-\cos(5x)\right\}dx = \frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)+C$

(4) $\displaystyle\int\frac{1}{2}\left\{\sin(7x)+\sin x\right\}dx = -\frac{1}{14}\cos(7x)-\frac{1}{2}\cos x+C$

(5) $\displaystyle\int\frac{1}{2}\left\{1+\cos(6x)\right\}dx = \frac{x}{2}+\frac{1}{12}\sin(6x)+C$

(6) $\displaystyle\int\frac{1}{2}\left\{1-\cos(8x)\right\}dx = \frac{x}{2}-\frac{1}{16}\sin(8x)+C$

＜ 45 ページ. 上半円の積分 ＞

問の解答

(1) $x = \sin\theta$ とおくと

$$\begin{aligned}\int \sqrt{1-x^2}dx &= \int \sqrt{1-\sin^2\theta}\cos\theta d\theta \\ &= \int \cos^2\theta d\theta \\ &= \int \frac{1+\cos(2\theta)}{2}d\theta \\ &= \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C \\ &= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\sqrt{1-\sin^2\theta} + C \\ &= \frac{1}{2}\sin^{-1}x + \frac{1}{2}x\sqrt{1-x^2} + C\end{aligned}$$

(2) $x = a\sin\theta$ とおくと

$$\begin{aligned}\int \sqrt{a^2-x^2}dx &= \int \sqrt{a^2-a^2\sin^2\theta}a\cos\theta d\theta \\ &= \int a^2\cos^2\theta d\theta \\ &= a^2\int \cos^2\theta d\theta \\ &= a^2\left\{\frac{1}{2}\theta + \frac{1}{2}\sin\theta\sqrt{1-\sin^2\theta}\right\} + C \\ &= a^2\left\{\frac{1}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}\times\frac{x}{a}\sqrt{1-\frac{x^2}{a^2}}\right\} + C \\ &= \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}x\sqrt{a^2-x^2} + C\end{aligned}$$

＜ 46 ページ. 不定積分の検証 ＞

問の解答

(1) $\left(\frac{1}{4}(x^4-1)^4\right)' = \frac{1}{4} \times 4(x^4-1)^3 \times 4x^3$

$= 4x^3(x^4-1)^3$　　より正しくない

(2) $\left(\frac{1}{2}\log|x^2-1|\right)' = \frac{1}{2} \times \frac{2x}{x^2-1}$

$= \frac{x}{x^2-1}$　　より正しい

(3) $(x^2e^x - 2xe^x + 2e^x)' = 2xe^x + x^2e^x - 2e^x - 2xe^x + 2e^x$

$= x^2e^x$　　より正しい

＜ 47 ページ. 不定積分の練習 (1) ＞

問の解答

(1) $\dfrac{1}{6}x^6 + \dfrac{1}{8}x^8 + C$

(2) $-x^{-1} + C = -\dfrac{1}{x} + C$

(3) $\dfrac{3}{4}x^{\frac{4}{3}} + C = \dfrac{3}{4}x\sqrt[3]{x} + C$

(4) $\dfrac{1}{-2}x^{-2} + C = -\dfrac{1}{2x^2} + C$

(5) $\dfrac{2}{3}x^{\frac{3}{2}} + C = \dfrac{2}{3}x\sqrt{x} + C$

(6) $\dfrac{4}{5}x^{\frac{5}{4}} + C = \dfrac{4}{5}x\sqrt[4]{x} + C$

(7) $2\sqrt{x} + C$

(8) $\dfrac{x^2}{2} - 2x + \log|x| + C$

(9) $\log|x| + \dfrac{2}{x} - \dfrac{1}{2x^2} + C$

(10) $\displaystyle\int\left(2\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx = \frac{4}{3}x\sqrt{x} - 2\sqrt{x} + C$

(11) $\displaystyle\int \frac{x + 6\sqrt{x} + 9}{x}dx = x + 12\sqrt{x} + 9\log|x| + C$

(12) $2\sin x + 3\cos x + C$

(13) $\displaystyle\int(2\sin x + 3\cos x)dx = -2\cos x + 3\sin x + C$

(14) $\displaystyle\int\left(\frac{1}{\cos^2 x} + 2\right)dx = \tan x + 2x + C$

＜ 48 ページ. 不定積分の練習 (2) ＞

問の解答

(1) $\displaystyle\int \frac{1}{\sin^2 x}dx = -\cot x + C$

(2) $\displaystyle\int \frac{1}{\cos^2 x}dx = \tan x + C$

(3) $\displaystyle\int \frac{1}{\sin^2 x}dx = -\cot x + C$

(4) $\displaystyle\int \frac{\cos^2 x}{\sin^2 x}dx = \int \left(\frac{1}{\sin^2 x} - 1\right)dx = -\cot x - x + C$

(5) $3\sin^{-1} x + C$

(6) $4\tan^{-1} x + C$

(7) $\dfrac{4^x}{\log_e 4} - e^x + C$

(8) $\dfrac{t^3}{3} - 3t^2 + 5t + C$

(9) $\dfrac{1}{5}u^5 - u^3 + C$

(10) $-\cos t + C$

(11) $\sin u + C$

(12) $e^u + C$

(13) $\log |u| + C$

(14) $\tan\theta + C$

＜ 49 ページ. 不定積分の練習 (3) ＞

問の解答

(1) $-\dfrac{1}{3}e^{-3x+1}+C$

(2) $\dfrac{1}{4}\sin(4x-2)+C$

(3) $-\dfrac{1}{3}\cos(3x+5)+C$

(4) $\dfrac{1}{5}\tan(5x+6)+C$

(5) $\dfrac{1}{4}\log|4x+3|+C$

(6) $\dfrac{1}{20}(5x-2)^4+C$

(7) $-\dfrac{1}{14(7x-5)^2}+C$

(8) $\dfrac{2}{15}(5x+3)\sqrt{5x+3}+C$

(9) $\dfrac{2}{7}\sqrt{7x-6}+C$

(10) $\dfrac{3}{20}(5x+1)\sqrt[3]{5x+1}+C$

(11) $-\dfrac{1}{2}e^{-x^2}+C$

(12) $-\dfrac{1}{3}e^{-x^3}+C$

(13) $\dfrac{1}{3}\sin(x^3+4)+C$

(14) $-\dfrac{1}{4}\cos(x^4)+C$

(15) $\dfrac{3}{2}\log(1+x^2)+C$

(16) $\dfrac{4}{3}\log|x^3+2|+C$

＜ 50 ページ. 不定積分の練習 (4) ＞

問の解答

(1) $\dfrac{1}{10}(x^2+3)^5+C$

(2) $\dfrac{1}{3}(x^2+1)\sqrt{x^2+1}+C$

(3) $3x+2\log|x+1|+C$

(4) $\dfrac{x^2}{2}+3x+4\log|x-1|+C$

(5) $-\dfrac{1}{x+1}+C$

(6) $\dfrac{1}{2}\log\left|\dfrac{x}{x+2}\right|+C$

(7) $\tan^{-1}(x+1)+C$

(8) $\dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right|+C$

(9) $-xe^{-x}-e^{-x}+C$

(10) $\dfrac{x^2}{2}\log x-\dfrac{x^2}{4}+C$

(11) $\dfrac{x}{3}\sin(3x)+\dfrac{1}{9}\cos(3x)+C$

(12) $-\dfrac{1}{6}\cos(3x)+\dfrac{1}{2}\cos x+C$

(13) $\dfrac{1}{8}\sin(4x)+\dfrac{1}{4}\sin(2x)+C$

(14) $\dfrac{1}{4}\sin(2x)-\dfrac{1}{8}\sin(4x)+C$

(15) $\dfrac{x}{2}-\dfrac{1}{8}\sin(4x)+C$

(16) $\dfrac{x}{2}+\dfrac{1}{8}\sin(4x)+C$