高知工科大学

基礎数学ワークブック

(2004年度版)

初級編

No. 2

解答

＜ 1 ページ. 弧度法の練習 ＞

問 1 の解答

度数法	45°	60°	90°	120°	180°	360°
弧度法 θ	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	2π
弧の長さ ℓ	$\frac{1}{4}\pi r$	$\frac{\pi}{3}r$	$\frac{\pi}{2}r$	$\frac{2\pi}{3}r$	πr	$2\pi r$
面積 S	$\frac{\pi r^2}{8}$	$\frac{\pi r^2}{6}$	$\frac{1}{4}\pi r^2$	$\frac{\pi r^2}{3}$	$\frac{\pi r^2}{2}$	πr^2

問 2 の解答

$$\ell = \theta r$$

$$S = \frac{1}{2}\theta r^2$$

< 2ページ. 三角関数の極限 (1) >

問 1 の解答

(1) $\ell_1 = \sin\theta$, $\ell_2 = \tan\theta$

問 2 の解答

$\ell_2 = \theta$

問 3 の解答

$\sin\theta < \theta < \tan\theta$

問 4 の解答

$0 < \theta < \dfrac{\pi}{2}$ のとき $\boxed{\sin\theta} < \theta < \boxed{\dfrac{\sin\theta}{\cos\theta}}$

＜ 3ページ. 三角関数の極限 (2) ＞

問 以下の証明中の $\boxed{}$ 内に適当な数または数式を記入せよ。

[定理の証明]

[1] $\displaystyle\lim_{\theta\to+0}\frac{\sin\theta}{\theta}=1$ を示す。

前ページの結果より $0<\theta<\dfrac{\pi}{2}$ のとき

$$(*)\quad \sin\theta<\theta<\frac{\sin\theta}{\cos\theta}$$

がわかる。$\cos\theta>0$，$\theta>0$ より

$$\theta<\frac{\sin\theta}{\cos\theta}\quad\Rightarrow\quad\boxed{\cos\theta}<\frac{\sin\theta}{\theta}\qquad\cdots ①$$

また

$$\sin\theta<\theta\quad\Rightarrow\quad\frac{\sin\theta}{\theta}<\boxed{1}\qquad\cdots ②$$

①，②より

$$\boxed{\cos\theta}<\frac{\sin\theta}{\theta}<\boxed{1}\qquad\cdots ③$$

ここで $\theta\to+0$ のとき $\cos\theta\to\cos 0=1$ より③から

$$\lim_{\theta\to+0}\frac{\sin\theta}{\theta}=1$$

がわかる。

[2] $\displaystyle\lim_{\theta\to-0}\frac{\sin\theta}{\theta}=1$ を示す。

$\theta\to-0$ のとき $\theta=-\theta_1$ $(\theta_1>0)$ とおくと，$\theta_1\to+0$ より

$$\lim_{\theta\to-0}\frac{\sin\theta}{\theta}=\lim_{\theta_1\to+0}\frac{\sin(-\theta_1)}{-\theta_1}=\lim_{\theta_1\to+0}\frac{\boxed{-\sin\theta_1}}{-\theta_1}=\lim_{\theta_1\to+0}\frac{\sin\theta_1}{\theta_1}=1$$

[3] [1] と [2] より右極限値と左極限値が一致するので定理の極限が証明された。

(証明終)

< 4ページ. 三角関数の極限 (3) >

問の解答

(1) $\displaystyle \lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\left(\frac{\sin x}{x}\times\frac{1}{\cos x}\right) = 1$

(2) $\displaystyle \lim_{x\to 0}\frac{\sin(2x)}{3x} = \lim_{x\to 0}\frac{\sin(2x)}{2x}\times\frac{2}{3} = \frac{2}{3}$

(3) $\displaystyle \lim_{x\to 0}\frac{\sin(3x)}{\sin(5x)} = \lim_{x\to 0}\frac{\frac{\sin(3x)}{3x}}{\frac{\sin(5x)}{5x}}\times\frac{3}{5} = \frac{3}{5}$

(4) $\displaystyle \lim_{x\to 0}\frac{1-\cos x}{x\sin x} = \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{(x\sin x)(1+\cos x)} = \lim_{x\to 0}\frac{1-\cos^2 x}{x\sin x(1+\cos x)}$

$\displaystyle = \lim_{x\to 0}\frac{\sin^2 x}{x\sin x(1+\cos x)} = \lim_{x\to 0}\frac{\sin x}{x}\times\frac{1}{1+\cos x} = \frac{1}{2}$

(5) $\displaystyle \lim_{x\to 0}\frac{\cos x-1}{x} = \lim_{x\to 0}\frac{\cos^2 x-1}{x(\cos x+1)} = \lim_{x\to 0}\frac{-\sin^2 x}{x(\cos x+1)} = \lim_{x\to 0}\left(\frac{-\sin x}{\cos x+1}\times\frac{\sin x}{x}\right) = 0$

＜ 5ページ. 三角関数の極限 (4) ＞

問の解答

(1) $\displaystyle\lim_{h\to 0}\frac{\sin(\frac{\pi}{3}+h)-\sin\frac{\pi}{3}}{h}=\frac{1}{2}$

(2) $\displaystyle\lim_{h\to 0}\frac{\cos(\pi+h)-\cos\pi}{h}=0$

(3) $\displaystyle\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

$$=\lim_{h\to 0}\left\{\frac{\sin x(\cos h-1)}{h}+\frac{\cos x\sin h}{h}\right\}$$

$$=\lim_{h\to 0}\left\{\sin x\times\left(\frac{\cos h-1}{h}\right)+\cos x\times\frac{\sin h}{h}\right\}=\cos x$$

(4) $\displaystyle\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=\lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}$

$$=\lim_{h\to 0}\left\{\frac{\cos x(\cos h-1)}{h}-\frac{\sin x\sin h}{h}\right\}$$

$$=\lim_{h\to 0}\left\{\cos x\times\left(\frac{\cos h-1}{h}\right)-\sin x\times\left(\frac{\sin h}{h}\right)\right\}=-\sin x$$

< 6ページ. 関数の連続性 >

問の解答

(1) $x = \dfrac{\pi}{2}$ が定義域にないので

$x = \dfrac{\pi}{2}$ で連続でない.

従って不連続

(2) $\displaystyle\lim_{x \to +0} |x| = \lim_{x \to +0} x = 0, \quad \lim_{x \to -0} |x| = \lim_{x \to -0} (-x) = 0$

より $\displaystyle\lim_{x \to 0} |x| = 0 = |0|$

よって $\displaystyle\lim_{x \to 0} f(x) = f(0)$ 従って $x = 0$ で連続

(3) $$\left.\begin{aligned} \lim_{x \to 1+0} f(x) &= \lim_{x \to 1+0} (x - [x]) = 1 - 1 = 0 \\ \lim_{x \to 1-0} f(x) &= \lim_{x \to 1-0} (x - [x]) = 1 - 0 = 1 \end{aligned}\right\}$$ より極限 $\displaystyle\lim_{x \to 1} f(x)$ は存在しない.

従って $x = 1$ で不連続

< 7ページ. 微分可能性 >

問の解答

$$\lim_{h\to+0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to+0}\frac{|h|}{h}=1$$

$$\lim_{h\to-0}\frac{f(-1+h)-f(-1)}{h}=\lim_{h\to-0}\frac{|h|}{h}=-1$$

より極限 $\displaystyle\lim_{h\to0}\frac{f(-1+h)-f(-1)}{h}$ が存在しないので, $x=-1$ で微分可能ではない.

< 8ページ. 導関数 (1) >

問の解答

(1) $$\left(\sqrt{x+1}\right)' = \lim_{h\to 0}\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$$

$$= \lim_{h\to 0}\frac{(\sqrt{x+h+1})^2-(\sqrt{x+1})^2}{h(\sqrt{x+h+1}+\sqrt{x+1})} = \lim_{h\to 0}\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

(2) $$\left(\frac{1}{x}\right)' = \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0}\frac{-h}{(x+h)xh} = -\frac{1}{x^2}$$

< 9 ページ. 導関数 (2) >

問の解答

(1) $y' = 5x^4$

(2) $y' = 6x^5$

(3) $y' = -12x^3$

(4) $y' = 5x^4 + 8x^3$

(5) $y' = 8x^3 - 15x^4$

(6) $y' = 3x^2 - 2x + 1$

(7) $y' = 3x^2 - 6x - 4$

(8) $y' = 4x^3 + 3x^2 - 1$

< 10 ページ. 積の微分 (1) >

問の解答

(1) $y' = 3x^2 - 2x + 1$

(2) $y' = 3x^2 - 6x - 4$

(3) $y' = 4x^3 + 3x^2 - 1$

(4) $y' = 4x^3 + 12x^2 + 12x + 4$ または $y' = 4(x+1)^3$

＜ 11 ページ. 積の微分 (2) ＞

問 1 の解答

(1) $\left(x\sqrt{x}\right)' = (x)' \times \sqrt{x} + x \times \left(\sqrt{x}\right)' = \sqrt{x} + \dfrac{x}{2\sqrt{x}} = \dfrac{3}{2}\sqrt{x}$

(2) $\left(k\sqrt{x}\right)' = (k)' \times \sqrt{x} + k \times (\sqrt{x})' = \dfrac{k}{2\sqrt{x}}$

問 2 の解答

$(k)' = 0$ より

$(k \times f(x))' = (k)' \times f(x) + k \times (f(x))' = k \times f'(x)$

問 3 の解答

$\left(f(x)g(x)h(x)\right)' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

< 12 ページ. 商の微分 >

問 1 の解答

$$\left\{\frac{f(x)}{g(x)}\right\}' = (f(x))' \times \frac{1}{g(x)} + f(x) \times \left(\frac{1}{g(x)}\right)' = f'(x) \times \frac{1}{g(x)} + f(x) \times \left\{-\frac{g'(x)}{(g(x))^2}\right\}$$

$$= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{\{g(x)\}^2} = \frac{f'(x)g(x)}{\{g(x)\}^2} - \frac{f(x)g'(x)}{\{g(x)\}^2} = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}$$

問 2 の解答

(1) $\left(\dfrac{1}{x^2}\right)' = -\dfrac{(x^2)'}{x^4} = -\dfrac{2}{x^3}$

(2) $\left(\dfrac{1}{2x^2}\right)' = -\dfrac{(2x^2)'}{(2x^2)^2} = -\dfrac{4x}{4x^4} = -\dfrac{1}{x^3}$

(3) $\left(\dfrac{x+1}{x^2}\right)' = \dfrac{(x+1)'x^2 - (x+1)(x^2)'}{(x^2)^2} = \dfrac{x^2 - 2(x+1)x}{x^4} = -\dfrac{x+2}{x^3}$

(4) $\left(\dfrac{x^3}{x+1}\right)' = \dfrac{(x^3)'(x+1) - x^3(x+1)'}{(x+1)^2} = \dfrac{3x^2(x+1) - x^3 \times 1}{(x+1)^2} = \dfrac{2x^3 + 3x^2}{(x+1)^2}$

< 13 ページ. 三角関数の微分 >

問 1 の解答

(1) $(3\sin x + 4\cos x)' = 3\cos x - 4\sin x$

(2) $(-3\cos x + 5\tan x)' = 3\sin x + \dfrac{5}{\cos^2 x}$

(3) $(\sin x \cos x)' = \cos^2 x - \sin^2 x$

(4) $(\sin^2 x)' = 2\sin x \cos x$

(5) $(\cos^2 x)' = -2\sin x \cos x$

(6) $(x\tan x)' = \tan x + \dfrac{x}{\cos^2 x}$

(7) $\left(\dfrac{\sin x}{x}\right)' = \dfrac{x\cos x - \sin x}{x^2}$

(8) $\left(\dfrac{\cos x}{x}\right)' = \dfrac{-x\sin x - \cos x}{x^2}$

問 2 の解答

(1) $(\mathrm{cosec}\, x)' = \left(\dfrac{1}{\sin x}\right)' = -\dfrac{(\sin x)'}{(\sin x)^2} = -\dfrac{\cos x}{\sin^2 x}$

(2) $(\sec x)' = \left(\dfrac{1}{\cos x}\right)' = -\dfrac{(\cos x)'}{(\cos x)^2} = \dfrac{\sin x}{\cos^2 x}$

(3) $(\cot x)' = \left(\dfrac{1}{\tan x}\right)' = -\dfrac{(\tan x)'}{(\tan x)^2} = -\dfrac{\frac{1}{\cos^2 x}}{\tan^2 x} = -\dfrac{1}{\sin^2 x}$

＜ 14 ページ. 導関数と極限 (1) ＞

問の解答

(1) $\displaystyle \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=(\cos x)'=-\sin x$

(2) $\displaystyle \lim_{h\to 0}\frac{\tan(x+h)-\tan x}{h}=(\tan x)'=\frac{1}{\cos^2 x}$

(3) $\displaystyle \lim_{h\to 0}\frac{\frac{1}{\sin(x+h)}-\frac{1}{\sin x}}{h}=\left(\frac{1}{\sin x}\right)'=-\frac{\cos x}{\sin^2 x}$

(4) $\displaystyle \lim_{h\to 0}\frac{(x+h)\cos(x+h)-x\cos x}{h}=(x\cos x)'=\cos x-x\sin x$

(5) $\displaystyle \lim_{h\to 0}\frac{\frac{\sin(x+h)}{x+h}-\frac{\sin x}{x}}{h}=\left(\frac{\sin x}{x}\right)'=\frac{x\cos x-\sin x}{x^2}$

(6) $\displaystyle \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\left(\frac{1}{x}\right)'=-\frac{1}{x^2}$

(7) $\displaystyle \lim_{h\to 0}\frac{\frac{1}{\cos(x+h)}-\frac{1}{\cos x}}{h}=\left(\frac{1}{\cos x}\right)'=\frac{\sin x}{\cos^2 x}$

(8) $\displaystyle \lim_{h\to 0}\frac{(x+h)^2\sin(x+h)-x^2\sin x}{h}=(x^2\sin x)'=2x\sin x+x^2\cos x$

(9) $\displaystyle \lim_{h\to 0}\frac{\sin(x+h)\cos(x+h)-\sin x\cos x}{h}=(\sin x\cos x)'=\cos^2 x-\sin^2 x$

< 15 ページ. 微分係数と極限 (1) >

問 1 の解答

(1) $\displaystyle\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-\sin\frac{\pi}{2}}{h}=\cos\frac{\pi}{2}=0$

(2) $\displaystyle\lim_{h\to 0}\frac{\sin(0+h)-\sin 0}{h}=\cos 0=1$

(3) $\displaystyle\lim_{h\to 0}\frac{\cos(0+h)-\cos 0}{h}=-\sin 0=0$

(4) $\displaystyle\lim_{h\to 0}\frac{\cos(\frac{\pi}{2}+h)-\cos\frac{\pi}{2}}{h}=-\sin\left(\frac{\pi}{2}\right)=-1$

(5) $\displaystyle\lim_{h\to 0}\frac{\tan(\frac{\pi}{4}+h)-\tan\frac{\pi}{4}}{h}=\frac{1}{\cos^2\left(\frac{\pi}{4}\right)}=2$

(6) $\displaystyle\lim_{h\to 0}\frac{(\frac{\pi}{4}+h)\sin(\frac{\pi}{4}+h)-\frac{\pi}{4}\sin\frac{\pi}{4}}{h}=\sin\frac{\pi}{4}+\frac{\pi}{4}\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{8}\pi$

(7) $\displaystyle\lim_{h\to 0}\frac{\cos^2(\frac{\pi}{3}+h)-\cos^2(\frac{\pi}{3})}{h}=-2\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$

< 16 ページ. 微分の練習 (1) >

問 1 の解答

(1) $\Big((x^2-2x)(3x+1)\Big)' = 9x^2-10x-2$

(2) $\Big((2x^3+1)(3x^2-1)\Big)' = 30x^4-6x^2+6x$

(3) $\Big((x+1)(x+2)(x+3)\Big)' = 3x^2+12x+11$

(4) $\left(\dfrac{2x}{x+1}\right)' = \dfrac{2}{(x+1)^2}$

(5) $\left(\dfrac{x-1}{x^2+1}\right)' = \dfrac{-x^2+2x+1}{(x^2+1)^2}$

(6) $\left(\dfrac{3x}{(x+1)^2}\right)' = \dfrac{-3x+3}{(x+1)^3}$

(7) $(4\sin x - 5\cos x)' = 4\cos x + 5\sin x$

(8) $(x^2\sin x)' = 2x\sin x + x^2\cos x$

(9) $(x^3\cos x)' = 3x^2\cos x - x^3\sin x$

(10) $\left(\dfrac{\tan x}{x}\right)' = \dfrac{\frac{x}{\cos^2 x}-\tan x}{x^2} = \dfrac{x-\sin x\cos x}{x^2\cos^2 x}$

(11) $(2\sec x + 3\cot x)' = \left(\dfrac{2}{\cos x}+\dfrac{3}{\tan x}\right)' = \dfrac{2\sin x}{\cos^2 x} - \dfrac{3}{\sin^2 x}$

(12) $\left(\dfrac{\cos x}{3+\sin x}\right)' = \dfrac{-1-3\sin x}{(3+\sin x)^2}$

問 2 の解答

(1) $(x^{-3})' = \left(\dfrac{1}{x^3}\right)' = -\dfrac{3x^2}{x^6} = -\dfrac{3}{x^4} = -3x^{-4}$

(2) $(x^{-4})' = \left(\dfrac{1}{x^4}\right)' = -\dfrac{4x^3}{x^8} = -\dfrac{4}{x^5} = -4x^{-5}$

(3) $(x^{-n})' = \left(\dfrac{1}{x^n}\right)' = -\dfrac{nx^{n-1}}{x^{2n}} = -\dfrac{n}{x^{n+1}} = -nx^{-n-1}$

< 17ページ. 微分記号 >

問の解答

(1) $\dfrac{dy}{dx} = 2x + 1$

(2) $\dfrac{dy}{dt} = -9.8$

(3) $\dfrac{d\ell}{dt} = 6t - 2$

(4) $\dfrac{dS}{dr} = 2\pi r$

(5) $\dfrac{dV}{dr} = 4\pi r^2$

< 18 ページ. 増分記号 Δ(デルタ) >

問の解答

(1) $\displaystyle \lim_{\Delta x \to 0} \frac{(x+\Delta x)^5 - x^5}{\Delta x} = (x^5)' = 5x^4$

(2) $\displaystyle \lim_{\Delta t \to 0} \frac{\sin(t+\Delta t) - \sin(t)}{\Delta t} = (\sin t)' = \cos t$

(3) $\displaystyle \lim_{\Delta u \to 0} \frac{\cos(u+\Delta u) - \cos(u)}{\Delta u} = (\cos u)' = -\sin u$

< 19 ページ. 合成関数の微分 (1) >

問 1 の解答

$$\frac{dy}{dx} = \left(\lim_{\Delta u \to 0} \frac{\cos(u+\Delta u) - \cos u}{\Delta u}\right) \times \left(\lim_{\Delta x \to 0} \frac{(x+\Delta x)^4 - x^4}{\Delta x}\right)$$

$$= (\cos u)' \times (x^4)'$$

$$= -\sin u \times 4x^3 = -4x^3 \sin u = -4x^3 \sin(x^4)$$

問 2 の解答

$$u = x^3 + 2x^2$$

$$\Delta u = (x+\Delta x)^3 + 2(x+\Delta x)^2 - x^3 - 2x^2$$

$$\Delta y = \sin(u+\Delta u) - \sin u$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sin\{(x+\Delta x)^3 + 2(x+\Delta x)^2\} - \sin(x^3 + x^2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(u+\Delta u) - \sin u}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(u+\Delta u) - \sin u}{\Delta u} \times \frac{\Delta u}{\Delta x}$$

$$= \lim_{\Delta u \to 0} \frac{\sin(u+\Delta u) - \sin u}{\Delta u} \times \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + 2(x+\Delta x)^2 - x^3 - 2x^2}{\Delta x}$$

$$= (\sin u)' \times (x^3 + 2x^2)' = \cos u \times (3x^2 + 4x) = (3x^2 + 4x)\cos(x^3 + 2x^2)$$

< 20 ページ. 合成関数の微分 (2) >

問 1 の解答

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$

問 2 の解答

(1) $\dfrac{dy}{dx} = 6(x-1)(x^2-2x+5)^2$

(2) $\dfrac{dy}{dx} = -2\sin(2x-3)$

(3) $\dfrac{dy}{dx} = (5x^4-4x)\cos(x^5-2x^2)$

< 21 ページ. 合成関数の微分 (3) >

問 1 の解答

(1) $\left((3x+5)^7\right)' = 21(3x+5)^6$

(2) $\left((4x^2+5x)^8\right)' = 8(8x+5)(4x^2+5x)^7$

問 2 の解答

$\left((f(x))^n\right)' = n(f(x))^{n-1} \times f'(x)$

問 3 の解答

(1) $\left((3x+4)^5\right)' = 15(3x+4)^4$

(2) $\left((4x^2+9x)^6\right)' = 6(8x+9)(4x^2+9x)^5$

(3) $\left((x^4-2x^3)^{10}\right)' = 10(4x^3-6x^2)(x^4-2x^3)^9$

(4) $\left((3+4\sin x)^5\right)' = 20(\cos x)\ (3+4\sin x)^4$

(5) $\left((x-3\cos x)^7\right)' = 7(1+3\sin x)(x-3\cos x)^6$

< 22 ページ. 合成関数の微分 (4) >

問 1 の解答

(1) $\Big(\sin(5x-4)\Big)' = 5\cos(5x-4)$

(2) $\Big(\sin(x^6+7x^2-3)\Big)' = (6x^5+14x)\cos(x^6+7x^2-3)$

(3) $\Big(\cos(4x+3)\Big)' = -4\sin(4x+3)$

(4) $\Big(\cos(x^5-2x+1)\Big)' = -(5x^4-2)\sin(x^5-2x+1)$

問 2 の解答

$\Big(\cos(f(x))\Big)' = -\sin(f(x)) \times f'(x)$

問 3 の解答

(1) $\Big(\sin(x^6+7x^5-3x^2+4x)\Big)'$

$= (6x^5+35x^4-6x+4)\cos(x^6+7x^5-3x^2+4x)$

(2) $\Big(\sin(x^7-8x^5+4x^3-6x+1)\Big)'$

$= (7x^6-40x^4+12x^2-6)\cos(x^7-8x^5+4x^3-6x+1)$

< 23 ページ. 対関数の導関数 (1) >

問の解答

$\lim_{x \to 0}(1+x)^{\frac{1}{x}} = e$

< 24 ページ. 対関数の導関数 (2) >

問 1 の解答

(1) $f'(3) = \lim_{\Delta x \to 0} \dfrac{f(3+\Delta x) - f(3)}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{\log_{10}(3+\Delta x) - \log_{10} 3}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \log_{10}\left(\frac{3+\Delta x}{3}\right) = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \log_{10}\left(1 + \frac{\Delta x}{3}\right)$$

ここで $\dfrac{\Delta x}{3} = h$ とおくと $\Delta x \to 0$ のとき $h \to 0$ より

$$f'(3) = \lim_{h \to 0} \frac{1}{3h} \log_{10}(1+h) = \lim_{h \to 0} \frac{1}{3} \log_{10}(1+h)^{\frac{1}{h}} = \frac{1}{3} \log_{10} e$$

(2) $f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{\log_{10}(x+\Delta x) - \log_{10} x}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \log_{10}\left(\frac{x+\Delta x}{x}\right) = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \log_{10}\left(1 + \frac{\Delta x}{x}\right)$$

ここで $\dfrac{\Delta x}{x} = h$ とおくと $\Delta x \to 0$ のとき $h \to 0$ より

$$f'(x) = \lim_{h \to 0} \frac{1}{xh} \log_{10}(1+h) = \lim_{h \to 0} \frac{1}{x} \log_{10}(1+h)^{\frac{1}{h}} = \frac{1}{x} \log_{10} e$$

問 2 の解答

$$f'(x) = \lim_{\Delta x \to 0} \frac{\log_a(x+\Delta x) - \log_a(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \log_a\left(1 + \frac{\Delta x}{x}\right)$$

ここで $\dfrac{\Delta x}{x} = h$ とおくと $\Delta x \to 0$ のとき $h \to 0$ より

$$f'(x) = \lim_{h \to 0} \frac{1}{xh} \log_a(1+h) = \lim_{h \to 0} \frac{1}{x} \log_a(1+h)^{\frac{1}{h}} = \frac{1}{x} \log_a e$$

＜25 ページ. 自然対数＞

問 1 の解答

(1) $(\log_{10} x)' = \dfrac{1}{x}\log_{10} e$

(2) $(\log_a x)' = \dfrac{1}{x}\log_a e$

問 2 の解答

(答) $(\log_e x)' = \dfrac{1}{x}\log_e e = \dfrac{1}{x}$

問 3 の解答

(1) $\log e = 1$　　(2) $\log(\sqrt[3]{e}) = \dfrac{1}{3}$　　(3) $\log\left(\dfrac{1}{e}\right) = -1$　　(4) $\log 1 = 0$

(5) $\ln\left(\dfrac{1}{e}\right) = -1$　　(6) $\ln(\sqrt[4]{e}) = \dfrac{1}{4}$　　(7) $\ln(e) = 1$　　(8) $\ln(e\sqrt{e}) = \dfrac{3}{2}$

問 4 の解答

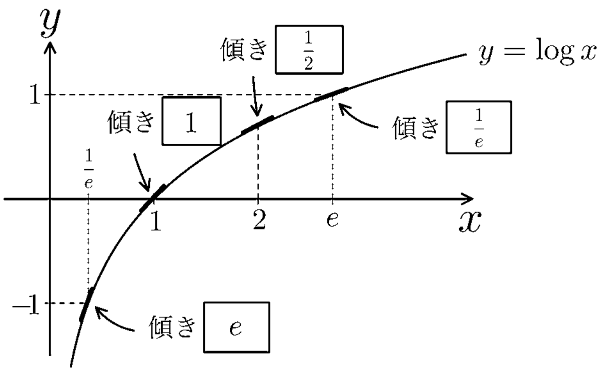
(1) $(\log x)' = \dfrac{1}{x}$

(2) $(\ln x)' = \dfrac{1}{x}$

問 5 の解答

$f'\left(\dfrac{1}{e}\right) = e,\qquad f'(1) = 1$

$f'(2) = \dfrac{1}{2},\qquad f'(e) = \dfrac{1}{e}$



< 26 ページ.$\log f(x)$ の導関数 >

問 1 の解答

(1) $\dfrac{dy}{dx} = \dfrac{3x^2+2}{x^3+2x-5}$

(2) $\dfrac{dy}{dx} = \dfrac{\cos x}{1+\sin x}$

(3) $\dfrac{dy}{dx} = \dfrac{\sin x}{5-\cos x}$

問 2 の解答

$$\Big(\log(f(x))\Big)' = \frac{f'(x)}{f(x)}$$

問 3 の解答

(1) $\Big(\log(x^2+2x)\Big)' = \dfrac{2x+2}{x^2+2x}$

(2) $\Big(\log(x^6+3x^4)\Big)' = \dfrac{6x^2+12}{x^3+3x}$

(3) $\Big(\log(\sin x)\Big)' = \dfrac{\cos x}{\sin x} \quad (=\cot x)$

< 27 ページ. 逆関数の微分 (1) >

問 1 の解答

$$y = \cos^{-1} x \iff x = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\cos y)'} = \frac{1}{-\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$$

問 2 の解答

$$y = \tan^{-1} x \iff x = \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\tan y)'} = \frac{1}{\frac{1}{\cos^2 y}} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

< 28 ページ. 逆関数の微分 (2) >

問 1 の解答

(1) $y = x^{\frac{1}{4}} \Longleftrightarrow x = y^4$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(y^4)'} = \frac{1}{4y^3} = \frac{1}{4(x^{\frac{1}{4}})^3} = \frac{1}{4}x^{-\frac{3}{4}}$$

(2) $y = x^{\frac{1}{n}} \Longleftrightarrow x = y^n$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(y^n)'} = \frac{1}{ny^{n-1}} = \frac{1}{n(x^{\frac{1}{n}})^{n-1}} = \frac{1}{n}x^{\frac{1}{n}-1}$$

問 2 の解答

(1) $y = 2^x \Longleftrightarrow x = \log_2 y$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\log_2 y)'} = \frac{1}{\frac{1}{y}\log_2 e} = y\log_e 2 == 2^x \log_e 2$$

(2) $y = a^x \Longleftrightarrow x = \log_a y$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\log_a y)'} = \frac{1}{\frac{1}{y}\log_a e} = y\log_e a = a^x \log_e a$$

< 29 ページ. 指数関数の微分 >

問 1 の解答

(1) $\left(e^{3x}\right)' = 3e^{3x}$

(2) $\left(e^{x^2+3}\right)' = 2xe^{x^2+3}$

(3) $\left(e^{-x^2+2x}\right)' = (-2x+2)e^{-x^2+2x}$

問 2 の解答

$\left(e^{f(x)}\right)' = e^{f(x)} \times f'(x)$

問 3 の解答

(1) $\left(e^{-3x}\right)' = -3e^{-3x}$

(2) $\left(e^{-\frac{x^2}{2}}\right)' - xe^{-\frac{x^2}{2}}$

< 30 ページ. 対数微分法 (1) >

問 1 の解答

(解) $y = 3^x$ の両辺の自然対数をとると

$\log y = x \log 3$ となる. この両辺を x で微分すると

$$\frac{y'}{y} = \log 3 \Rightarrow y' = y \times \log 3 = 3^x \log 3$$

(答) $(3^x)' = 3^x \log 3$

問 2 の解答

(解) $y = a^x$ の両辺の自然対数をとると

$\log y = x \log a$ となる. この両辺を x で微分すると

$$\frac{y'}{y} = \log a \Rightarrow y' = a^x \log a$$

(答) $(a^x)' = a^x \log a$

問 3 の解答

(答) $(e^x)' = e^x \log e = e^x$

< 31 ページ. 対数微分法 (2) >

問 1 の解答

(解) $y = x^{\frac{4}{3}}$ の両辺の自然対数をとると

$\log y = \dfrac{4}{3}\log x$ である. この両辺を x で微分すると

$$\frac{y'}{y} = \frac{4}{3x} \Rightarrow y' = y \times \frac{4}{3x} = x^{\frac{4}{3}} \times \frac{4}{3x} = \frac{4}{3}x^{\frac{1}{3}}$$

(答) $\left(x^{\frac{4}{3}}\right)' = \dfrac{4}{3}x^{\frac{1}{3}}$

問 2 の解答

(解) $y = x^r$ の両辺の自然対数をとると

$\log y = r\log x$ である. この両辺を x で微分すると

$$\frac{y'}{y} = \frac{r}{x} \Rightarrow y' = y \times \frac{r}{x} = x^{r-1} \times r$$

(答) $(x^r)' = rx^{r-1}$

＜ 32 ページ.x^r の導関数 ＞

問 1 の解答

(1) $\left(\sqrt[4]{x^5}\right)' = \dfrac{5}{4}\sqrt[4]{x}$

(2) $\left(\sqrt[5]{x^7}\right)' = \dfrac{7}{5}\sqrt[5]{x^2}$

(3) $\left(\sqrt{x^3}\right)' = \dfrac{3}{2}\sqrt{x}$

問 2 の解答

(1) $\left(\dfrac{1}{x^3}\right)' = -\dfrac{3}{x^4}$

(2) $\left(\dfrac{1}{x^4}\right)' = -\dfrac{4}{x^5}$

(3) $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$

問 3 の解答

(1) $\left(\sqrt[4]{x}\right)' = \dfrac{1}{4\sqrt[4]{x^3}}$

(2) $\left(\sqrt[5]{x^4}\right)' = \dfrac{4}{5\sqrt[5]{x}}$

(3) $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$

問 4 の解答

(1) $\left(\dfrac{1}{\sqrt[3]{x^2}}\right)' = -\dfrac{2}{3x\sqrt[3]{x^2}}$

(2) $\left(\dfrac{1}{\sqrt[4]{x}}\right)' = -\dfrac{1}{4x\sqrt[4]{x}}$

(3) $\left(\dfrac{1}{\sqrt{x}}\right)' = -\dfrac{1}{2x\sqrt{x}}$

< 33 ページ. $\log|x|$ の導関数 >

問の解答

(1) $\dfrac{dy}{dx} = \dfrac{\frac{1}{\cos^2 x}}{\tan x} = \dfrac{1}{\cos x \sin x}$

(2) $\dfrac{dy}{dx} = \dfrac{2x+3}{x^2+3x}$

(3) $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

< 34 ページ. 微分の練習 (2) >

問 1 の解答

(1) $(k)' = 0$ (2) $(x^n)' = nx^{n-1}$

(3) $(\sin x)' = \cos x$ (4) $(\cos x)' = -\sin x$

(5) $(\log x)' = \dfrac{1}{x}$ (6) $(e^x)' = e^x$

(7) $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$ (8) $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$

問 2 の解答

(1) $\Big(f(x)+g(x)\Big)' = f'(x)+g'(x)$ (2) $\Big(f(x)-g(x)\Big)' = f'(x)-g'(x)$

(3) $\Big(kf(x)\Big)' = kf'(x)$ (4) $\Big(f(x)\times g(x)\Big)' = f'(x)g(x)+f(x)g'(x)$

(5) $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x)-f(x)g'(x)}{\{g(x)\}^2}$

問 3 の解答

(1) $\Big((f(x))^n\Big)' = n(f(x))^{n-1}\times f'(x)$ (2) $\Big(\sin(f(x))\Big)' = \cos(f(x))\times f'(x)$

(3) $\Big(\cos(f(x))\Big)' = -\sin(f(x))\times f'(x)$ (4) $\Big(\log|f(x)|\Big)' = \dfrac{f'(x)}{f(x)}$

(5) $\Big(e^{f(x)}\Big)' = e^{f(x)}\times f'(x)$

問 4 の解答

(1) $(x^4-5x^3+6x^2-7x+8)'$
$= 4x^3-15x^2+12x-7$

(2) $\Big(\sqrt{x}\Big)' = \dfrac{1}{2\sqrt{x}}$

(3) $\Big(x\sqrt{x}\Big)' = \dfrac{3}{2}\sqrt{x}$ (4) $\left(\dfrac{\sin x}{x}\right)' = \dfrac{x\cos x-\sin x}{x^2}$

(5) $(\sin x\cos x)' = \cos^2 x-\sin^2 x\ \ (=\cos(2x))$ (6) $(\tan x)' = \dfrac{1}{\cos^2 x}$

(7) $(x\log x - x)' = \log x$ (8) $(-\log|\cos x|)' = \tan x$

(9) $(e^{2x}\sin(3x))' = 2e^{2x}\sin(3x)+3e^{2x}\cos(3x)$

(10) $\Big(\log(x+\sqrt{x^2+1})\Big)' = \dfrac{1+\frac{2x}{2\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \dfrac{1}{\sqrt{x^2+1}}$

< 35 ページ. 導関数と極限 (2) >

問の解答

(1) $\displaystyle \lim_{h\to 0}\frac{\log(x+h)-\log x}{h}=(\log x)'=\frac{1}{x}$

(2) $\displaystyle \lim_{h\to 0}\frac{e^{x+h}-e^{x}}{h}=(e^{x})'=e^{x}$

(3) $\displaystyle \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=(\sqrt{x})'=\frac{1}{2\sqrt{x}}$

(4) $\displaystyle \lim_{h\to 0}\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}=(\sqrt[3]{x})'=\frac{1}{3\sqrt[3]{x^2}}$

(5) $\displaystyle \lim_{h\to 0}\frac{\sqrt{(x+h)^3}-\sqrt{x^3}}{h}=(\sqrt{x^3})'=\frac{3}{2}\sqrt{x}$

(6) $\displaystyle \lim_{h\to 0}\frac{\sqrt[4]{(x+h)^5}-\sqrt[4]{x^5}}{h}=(\sqrt[4]{5})'=\frac{4}{5}\sqrt[4]{x}$

(7) $\displaystyle \lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\left(\frac{1}{\sqrt{x}}\right)'=-\frac{1}{2x\sqrt{x}}$

(8) $\displaystyle \lim_{h\to 0}\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}=\left(\frac{1}{x^3}\right)'=-\frac{3}{x^4}$

(9) $\displaystyle \lim_{h\to 0}\frac{\log|\cos(x+h)|-\log|\cos x|}{h}=(\log|\sin x|)'=-\tan x$

(10) $\displaystyle \lim_{h\to 0}\frac{e^{-(x+h)^2}-e^{-x^2}}{h}=(e^{-x^2})'=-2xe^{-x^2}$

＜ 36ページ. 微分係数と極限 (2) ＞

問の解答

(1) $\displaystyle\lim_{h\to0}\frac{\log(2+h)-\log 2}{h}=\frac{1}{2}$

(2) $\displaystyle\lim_{h\to0}\frac{\log(1+h)}{h}=\lim_{h\to0}\frac{\log(1+h)-\log 1}{h}=1$

(3) $\displaystyle\lim_{h\to0}\frac{e^{3+h}-e^3}{h}=e^3$

(4) $\displaystyle\lim_{h\to0}\frac{e^h-1}{h}=\lim_{h\to0}\frac{e^h-e^0}{h}=e^0=1$

(5) $\displaystyle\lim_{h\to0}\frac{\sqrt{1+h}-\sqrt{1}}{h}=\frac{1}{2\sqrt{1}}=\frac{1}{2}$

(6) $\displaystyle\lim_{h\to0}\frac{\sqrt[3]{8+h}-\sqrt[3]{8}}{h}=\frac{1}{3\sqrt[3]{8^2}}=\frac{1}{12}$

(7) $\displaystyle\lim_{h\to0}\frac{\frac{1}{\sqrt{4+h}}-\frac{1}{\sqrt{4}}}{h}=-\frac{1}{2\times4\sqrt{4}}=-\frac{1}{16}$

(8) $\displaystyle\lim_{h\to0}\frac{\frac{1}{(1+h)^3}-\frac{1}{1^3}}{h}=-\frac{3}{1^4}=-3$

(9) $\displaystyle\lim_{h\to0}\frac{\log\left|\cos\left(\frac{\pi}{4}+h\right)\right|-\log\left|\cos\frac{\pi}{4}\right|}{h}=-\tan\frac{\pi}{4}=-1$

(10) $\displaystyle\lim_{h\to0}\frac{e^{-(1+h)^2}-e^{-1}}{h}=-2\times1\times e^{-1^2}=-\frac{2}{e}$

< 37 ページ. 微分係数と傾き >

問 1 の解答

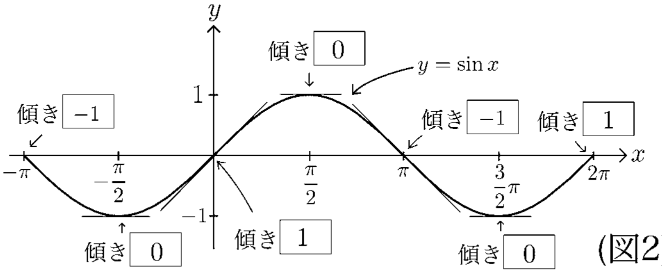
$f'(x) = \cos x$

$f'(-\pi) = -1$ $\quad$ $f'(-\frac{\pi}{2}) = 0$

$f'(0) = 1$ $\quad$ $f'(\frac{\pi}{2}) = 0$

$f'(\pi) = -1$ $\quad$ $f'(\frac{3}{2}\pi) = 0$

$f'(2\pi) = 1$



(図2)

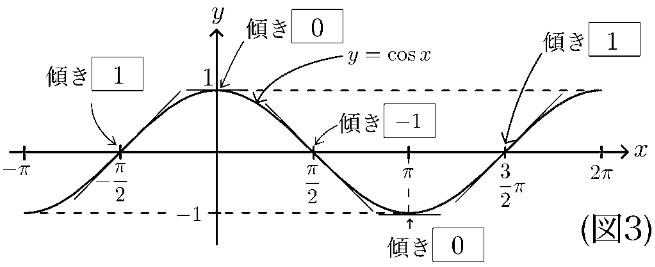
問 2 の解答

$f'(x) = -\sin x$

$f'(-\frac{\pi}{2}) = 1$ $\quad$ $f'(0) = 0$

$f'(\frac{\pi}{2}) = -1$ $\quad$ $f'(\pi) = 0$

$f'(\frac{3}{2}\pi) = 1$



(図3)

問 3 の解答

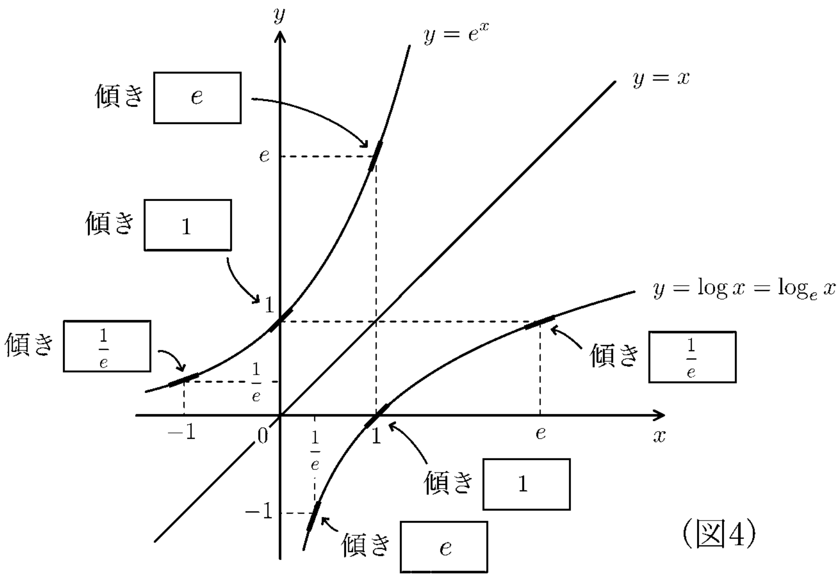
(1) $f^{-1}(x) = \log_e x$

(2) $f'(x) = e^x$ $\quad$ $g'(x) = \dfrac{1}{x}$

$f'(-1) = \dfrac{1}{e}$ $\quad$ $g'\left(\dfrac{1}{e}\right) = e$

$f'(0) = 1$ $\quad$ $g'(1) = 1$

$f'(1) = e$ $\quad$ $g'(e) = \dfrac{1}{e}$



(図4)

＜ 38 ページ. 接線の方程式 (1) ＞

問の解答

(1) $y = x + 1$

(2) $y = x - 1$

(3) $y = x$

(4) $y = \frac{1}{4}x + 1$

(5) $y = -x + 2$

< 39 ページ. 接線の方程式 (2) >

問の解答

(1) $y = 3x + \sqrt{3} - \dfrac{\pi}{3}$

(2) $y = -5\sqrt{3}x + \dfrac{5\sqrt{3}\pi}{6} + \dfrac{5}{2}$

(3) $y = 4x$

(4) $y = -\dfrac{1}{2}x - 1$

(5) $y = \dfrac{1}{6}x + \dfrac{3}{2}$

(6) $y = \dfrac{2}{3}x + \dfrac{5}{3}$

(7) $y = -\dfrac{x}{16} + \dfrac{3}{4}$

(8) $y = -2x + 3$

(9) $y = 2x + 1$

(10) $y = 2ex - e$

(11) $y = \dfrac{1}{e}x$

(12) $y = x - 1 + \log 2$

< 40 ページ. 接線の方程式 (3) >

問の解答

$x^2 + y^2 = 16$ の両辺を x で微分する

$2x + 2yy' = 0$

$\Downarrow$

$y' = -\dfrac{x}{y} = -\dfrac{2}{2\sqrt{3}} = -\dfrac{1}{\sqrt{3}}$ が接線の傾きである

よって点 $(2,\ 2\sqrt{3})$ を通り傾き $-\frac{1}{\sqrt{3}}$ の直線の式は

$y = -\dfrac{1}{\sqrt{3}}(x-2) + 2\sqrt{3}$

(答) $y = -\dfrac{\sqrt{3}}{3}x + \dfrac{8\sqrt{3}}{3}$

< 41ページ. 接線の方程式 (4) >

問1の解答

楕円の方程式 $\dfrac{x^2}{8}+\dfrac{y^2}{2}=1$ の両辺を x で微分すると

$\dfrac{2x}{8}+\dfrac{2yy'}{2}=0$ より

$y'=-\dfrac{x}{4y}=-\dfrac{-2}{4\times 1}=\dfrac{1}{2}$ が接線の傾きである. よって接線の式は

$y=\dfrac{1}{2}(x+2)+1$ より

(答) $y=\dfrac{1}{2}x+2$

問2の解答

円の方程式 $x^2+y^2=5^2$ の両辺を x で微分すると

$2x+2yy'=0$

$y'=-\dfrac{x}{y}=-\dfrac{-3}{-4}=-\dfrac{3}{4}$ が接線の傾きである. よって接線の式は

$y=-\dfrac{3}{4}(x+3)-4$

(答) $y=-\dfrac{3}{4}x-\dfrac{25}{4}$

問3の解答

円の方程式 $x^2+y^2=r^2$ の両辺を x で微分すると

$2x+2yy'=0$

$y'=-\dfrac{x}{y}=-\dfrac{r\cos\theta}{r\sin\theta}=-\dfrac{1}{\tan\theta}$ が接線の傾きである. よって接線の式は

$$y=-\frac{1}{\tan\theta}(x-r\cos\theta)+r\sin\theta$$

$$=-\frac{x}{\tan\theta}+\frac{r(\cos^2\theta+\sin^2\theta)}{\sin\theta}$$

(答) $y=-\dfrac{x}{\tan\theta}+\dfrac{r}{\sin\theta}$

＜ 42 ページ.2 階導関数 ＞

問 1 の解答

(1) $f''(x) = 24x - 10$

(2) $f''(x) = -\sin x$

(3) $f''(x) = -\dfrac{1}{x^2}$

(4) $\dfrac{d^2y}{dx^2} = 20x^3 - 12x^2$

(5) $\dfrac{d^2y}{dx^2} = -\cos x$

(6) $\dfrac{d^2y}{dx^2} = 4e^{2x}$

問 2 の解答

(1) $\dfrac{d^2y}{dt^2} = -9.8$

(2) $\dfrac{d^2y}{dt^2} = -4\sin(2t)$

(3) $\dfrac{d^2y}{dt^2} = -9\cos(3t)$

< 43 ページ. 直線上の運動 >

問の解答

(1) $v(t) = 4 - 10t$ $\qquad a(t) = -10$

(2) $v(t) = -6\sin(2t)$ $\qquad a(t) = -12\cos(2t)$

(3) $v(t) = 2e^{2t}\sin(4t) + 4e^{2t}\cos(4t)$ $\qquad a(t) = -12e^{2t}\sin(4t) + 16e^{2t}\cos(4t)$

< 44 ページ. 平面上の運動 (1) >

問の解答

$$\vec{v} = \left(\frac{dx}{dt},\ \frac{dy}{dt}\right) = (2,\ -2t)$$

$$|\vec{v}| = \sqrt{2^2 + (-2t)^2} = \sqrt{4 + 4t^2} = 2\sqrt{1 + t^2}$$

＜ 45 ページ. 平面上の運動 (2) ＞

問の解答

(1) $$\begin{cases} v_x(t) = \dfrac{dx}{dt} = k_1 \\ \\ v_y(t) = \dfrac{dy}{dt} = k_2 - gt \end{cases}$$

(2) $\dfrac{v_y(t)}{v_x(t)} = \dfrac{k_2 - gt}{k_1}$

(3) $y = \dfrac{k_2}{k_1}x - \dfrac{g}{2k_1^2}x^2$

(4) $f(x) = \dfrac{k_2}{k_1}x - \dfrac{g}{2k_1^2}x^2$ より

$$f'(x) = \frac{k_2}{k_1} - \frac{g}{2k_1^2} \times 2x = \frac{k_2}{k_1} - \frac{g}{k_1^2}x$$

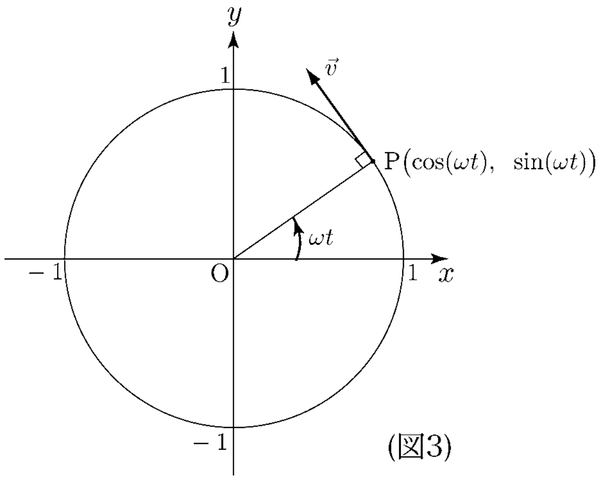
(5) $f'(x(t)) = f'(k_1 t) = \dfrac{k_2}{k_1} - \dfrac{g}{k_1^2}(k_1 t) = \dfrac{k_2 - gt}{k_1} = \dfrac{v_y(t)}{v_x(t)}$

< 46 ページ. 平面上の運動 (3) >

問の解答

$$\vec{v} = \Big(-\omega\sin(\omega t),\ \ \omega\cos(\omega t) \Big)$$

$$|\vec{v}| = \sqrt{\Big(-\omega\sin(\omega t) \Big)^2 + \Big(\omega\cos(\omega t) \Big)^2} = \sqrt{\omega^2 \Big(\sin^2(\omega t) + \cos^2(\omega t) \Big)} = \omega$$



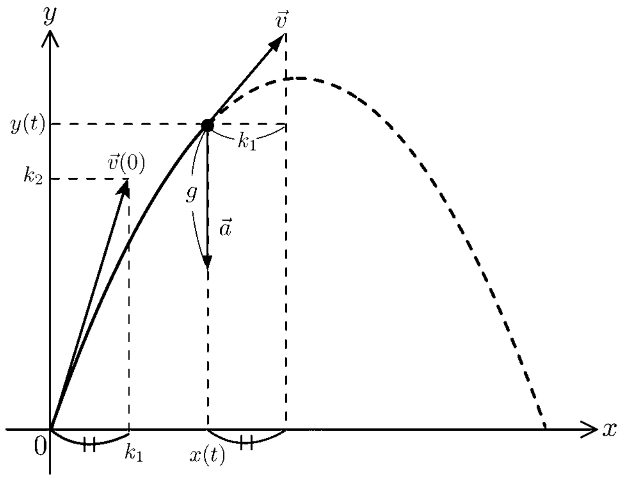
(図3)

< 47 ページ. 平面上の運動 (4) >

問の解答

(1) $\vec{v}(t) = (k_1,\ k_2 - gt)$

(2) $\vec{a}(t) = (0,\ -g)$



< 48 ページ. 平面上の運動 (5) >

問 1 の解答

$$|\vec{v}| = \sqrt{(-r\sin t)^2 + (r\cos t)^2} = r$$

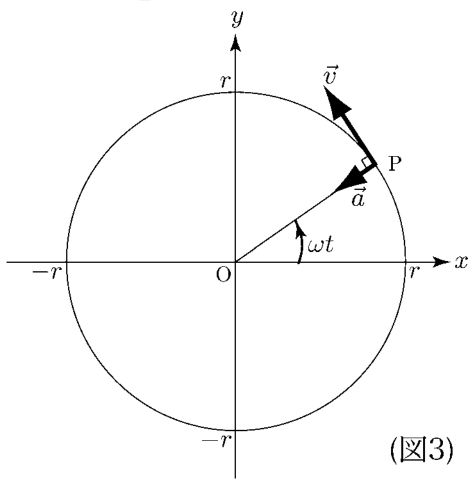
$$|\vec{a}| = \sqrt{(-r\cos t)^2 + (-r\sin t)^2} = r$$

問 2 の解答

$$\vec{v} = \Big(-r\omega\sin(\omega t),\ r\omega\cos(\omega t)\Big)\ , \qquad |\vec{v}| = r|\omega|$$

$$\vec{a} = \Big(-r\omega^2\cos(\omega t),\ -r\omega^2\sin(\omega t)\Big)\ , \quad |\vec{a}| = r\omega^2$$

$w = \dfrac{1}{2}$ のとき $\vec{v}$ と $\vec{a}$ は図 3 のベクトル.



(図3)

$<$ 49 ページ. 微分の練習 (3) $>$

問 1 の解答

(1) $\lim_{x\to 0}\frac{\sin(3x)}{2x}=\frac{3}{2}$　(2) $\lim_{x\to 0}\frac{\tan(2x)}{3x}=\frac{2}{3}$

(3) $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$　(4) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$

問 2 の解答

(1) $y'=\frac{1}{3\sqrt[3]{x^2}}$　(2) $y'=-\frac{3}{x^4}$　(3) $y'=-\frac{1}{2x\sqrt{x}}$

(4) $y'=2\cos(2x)$　(5) $y'=-8\sin(4x)$　(6) $y'=\frac{5}{\cos^2(5x)}$

(7) $y'=\frac{1}{x}$　(8) $y'=\frac{3}{x}$　(9) $y'=-\tan x$

(10) $y'=4e^{4x+1}$　(11) $y'=-xe^{-\frac{x^2}{2}}$　(12) $y'=\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

(13) $y'=\frac{3}{2}\sqrt{x}$　(14) $y'=\sin x+x\cos x$

(15) $y'=\cos^2 x-\sin^2 x\ \ (=\cos 2x)$　(16) $y'=e^x\sin x+e^x\cos x$

(17) $y'=-e^{-x}\cos x-e^{-x}\sin x$　(18) $y'=3e^{3x}\sin(2x)+2e^{3x}\cos(2x)$

(19) $y'=\frac{-x\sin x-\cos x}{x^2}$　(20) $y'=\frac{1-x}{2\sqrt{x}(1+x)^2}$

(21) $y'=-\frac{1}{2x\sqrt{x}}+\frac{1}{2\sqrt{x}}\ \left(=\frac{x-1}{2x\sqrt{x}}\right)$

＜50ページ.微分の応用＞

問1の解答

(1) $y = \dfrac{1}{2}x + 1$

(2) $y = -2x + 3$

(3) $y = \dfrac{\sqrt{3}}{2}x - \dfrac{\sqrt{3}\pi}{12} + \dfrac{1}{2}$

(4) $y = 2x - \dfrac{\pi}{2} + 1$

(5) $y = -\dfrac{x}{\sqrt{e}} + \dfrac{2}{\sqrt{e}}$

(6) $y = x$

(7) $y = -\dfrac{\sqrt{3}}{6}x + \dfrac{2\sqrt{3}}{3}$

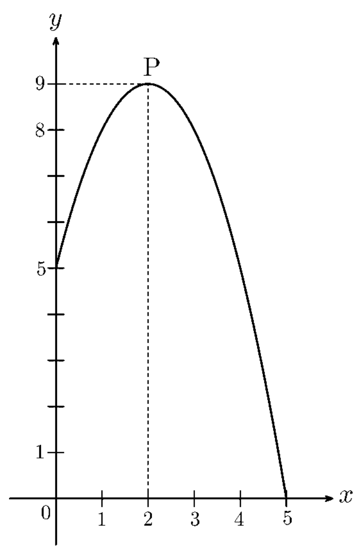
問2の解答

(1) $y = -4t^2 + 8t + 5$

$= -(2t)^2 + 4(2t) + 5$

$= -x^2 + 4x + 5$

$= -(x-2)^2 + 9$ 軌道は右図の放物線



(2) $\vec{v} = \Big(2,\ -8t + 8\Big)$

$|\vec{v}| = \sqrt{2^2 + (-8t+8)^2} = \sqrt{64t^2 - 128t + 68}$

$\Big(= 2\sqrt{16t^2 - 32t + 17}\Big)$

(3) $\vec{a} = (0,\ -8)$

$|\vec{a}| = 8$

(4) 右図のベクトル