高知工科大学

# 基礎数学ワークブック

(2003年度版)

# 初級編

# *No.* 4

# 解答

# ＜１ページ. 数列の類推＞

## 問１の解答

$a_1 = 1$ $a_2 = 5$ $a_3 = 14$ $a_4 = 30$ $a_5 = 55$

$b_1 = 2$ $b_2 = 3$ $b_3 = 4$ $b_4 = 5$ $b_5 = 6$

$b_n = n + 1$

## 問２の解答

$a_1 = 1$ $a_2 = 3$ $a_3 = 6$ $a_4 = 10$ $a_5 = 15$

$b_1 = 1$ $b_2 = 9$ $b_3 = 36$ $b_4 = 100$ $b_5 = 225$

$b_n = (a_n)^2$

# < 2ページ. 和の記号 $\sum$ 1 >

## 問 1 の解答

(1) $\displaystyle\sum_{k=1}^{7} k = 1 + 2 + 3 + 4 + 5 + 6 + 7$

(2) $\displaystyle\sum_{k=1}^{5} 2^k = 2 + 4 + 8 + 16 + 32$

(3) $\displaystyle\sum_{k=1}^{6} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$

(4) $\displaystyle\sum_{k=1}^{4} (3k - 1) = 2 + 5 + 8 + 11$

## 問 2 の解答

(1) $\displaystyle 1 + 2 + 3 + 4 + \cdots + n = \sum_{k=1}^{n} k$

(2) $\displaystyle 1 \times 2 + 3 \times 4 + 5 \times 6 + \cdots + (2n - 1)(2n) = \sum_{k=1}^{n} (2k - 1)(2k)$

(3) $\displaystyle 5 + 10 + 15 + \cdots + 100 = \sum_{k=1}^{20} 5k$

## 問 3 の解答

(1) $\displaystyle\sum_{k=3}^{7} (k^2 - 8) = (9 - 8) + (16 - 8) + (25 - 8) + (36 - 8) + (49 - 8)$

(2) $\displaystyle\sum_{k=4}^{8} (2k - 1)(2k + 1) = 7 \times 9 + 9 \times 11 + 11 \times 13 + 13 \times 15 + 15 \times 17$

(3) $\displaystyle\sum_{k=0}^{5} 4^k = 1 + 4 + 4^2 + 4^3 + 4^4 + 4^5$

# ＜ 3ページ. 和の記号 $\sum$ 2 ＞

## 問 1 の解答

(1) $\displaystyle\sum_{k=1}^{n}(2k+3) = 2\sum_{k=1}^{n}k + 3\sum_{k=1}^{n}1 = 2\times\frac{n(n+1)}{2} + 3\times n = n^2+4n$

(2) $\displaystyle\sum_{k=1}^{n}(8k-5) = 8\times\frac{n(n+1)}{2} - 5\times n = 4n^2+4n-5n = 4n^2-n$

## 問 2 の解答

(1) $\displaystyle\sum_{k=1}^{n}(2k-1) = 2\times\frac{n(n+1)}{2} - n = n^2$

(2) $\displaystyle\sum_{k=1}^{n}(5k-3) = 5\times\frac{n(n+1)}{2} - 3n = \frac{5}{2}n^2 - \frac{n}{2}$

(3) $\displaystyle\sum_{k=1}^{n}(7n-4) = 7\times\frac{n(n+1)}{2} - 4n = \frac{7}{2}n^2 - \frac{1}{2}n$

# ＜ 4ページ. 和の記号 $\sum$ 3 ＞

## 問 1 の解答

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

## 問 2 の解答

(1) $$\sum_{k=1}^{7} k^2 = \frac{7 \times (7+1) \times (2 \times 7 + 1)}{6} = 140$$

(2) $$\sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

## 問 3 の解答

(1) $$\begin{aligned}\sum_{k=1}^{n}(5 - 2k + 12k^2) &= 5n - 2 \times \frac{n(n+1)}{2} + 12 \times \frac{n(n+1)(2n+1)}{6} \\ &= 5n - n(n+1) + 2n(n+1)(2n+1) \\ &= n(4n^2 + 5n + 6)\end{aligned}$$

(2) $$\begin{aligned}\sum_{k=1}^{n}(3 + 2k)^2 &= \sum_{k=1}^{n}(9 + 12k + 4k^2) \\ &= 9n + 12\left(\frac{n(n+1)}{2}\right) + 4\left(\frac{n(n+1)(2n+1)}{6}\right) \\ &= 9n + 6n(n+1) + \frac{2}{3}n(n+1)(2n+1)\end{aligned}$$

# ＜5ページ. 和の記号 $\sum$ 4 ＞

## 問 1 の解答

$$\sum_{k=1}^{n} k^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

## 問 2 の解答

(1) $$\sum_{k=1}^{7} k^3 = \left( \frac{7 \times 8}{2} \right)^2 = 784$$

(2) $$\sum_{k=1}^{n-1} k^3 = \left\{ \frac{(n-1)n}{2} \right\}^2$$

## 問 3 の解答

(1) $$\begin{aligned} \sum_{k=1}^{n} (1-k)^3 &= \sum_{k=1}^{n} (1 - 3k + 3k^2 - k^3) \\ &= n - \frac{3}{2} n(n+1) + \frac{n(n+1)(2n+1)}{2} - \frac{n^2(n+1)^2}{4} \end{aligned}$$

(2) $$\begin{aligned} \sum_{k=1}^{n} (2+k)^3 &= \sum_{k=1}^{n} (8 + 12k + 6k^2 + k^3) \\ &= 8n + 6n(n+1) + n(n+1)(2n+1) + \frac{n^2(n+1)^2}{4} \end{aligned}$$

# ＜ 6ページ. 極限 ＞

## 問の解答

(1) $\displaystyle\lim_{n\to\infty}\frac{3}{2n+4}=0$

(2) $\displaystyle\lim_{n\to\infty}\frac{3n}{2n-1}=\frac{3}{2}$

(3) $\displaystyle\lim_{n\to\infty}\frac{2n^2+3n-5}{3n^2-2n+4}=\frac{2}{3}$

(4) $\displaystyle\lim_{n\to\infty}\frac{(2n-3)(n+2)}{(n-1)(n+1)}=\lim_{n\to\infty}\frac{2n^2+n-6}{n^2-1}=2$

(5) $\displaystyle\lim_{n\to\infty}\frac{1}{n^3}(1^2+2^2+3^2+\cdots+n^2)=\lim_{n\to\infty}\frac{1}{n^3}\times\frac{n(n+1)(2n+1)}{6}=\frac{1}{3}$

(6) $\displaystyle\lim_{n\to\infty}\frac{1}{n^4}(1^3+2^3+3^3+\cdots+n^3)=\lim_{n\to\infty}\frac{1}{n^4}\times\frac{n^2(n+1)^2}{4}=\frac{1}{4}$

# ＜ 7ページ. 和の極限値 ＞

## 問の解答

(1) $$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}1=\lim_{n\to\infty}\frac{1}{n}\times n=1$$

(2) $$\lim_{n\to\infty}\frac{1}{n^2}\sum_{k=1}^{n}k=\lim_{n\to\infty}\frac{1}{n^2}\times\frac{n(n+1)}{2}=\frac{1}{2}$$

(3) $$\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}k^2=\lim_{n\to\infty}\frac{1}{n^3}\times\frac{n(n+1)(2n+1)}{6}=\frac{1}{3}$$

(4) $$\lim_{n\to\infty}\frac{1}{n^4}\sum_{k=1}^{n}k^3=\lim_{n\to\infty}\frac{1}{n^4}\times\frac{n^2(n+1)^2}{4}=\frac{1}{4}$$

(5) $$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)=\lim_{n\to\infty}\frac{1}{n}\left\{n+\frac{1}{n}\times\frac{n(n+1)}{2}\right\}=\frac{3}{2}$$

(6) $$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(4+\frac{4k}{n}+\frac{k^2}{n^2}\right)=\lim_{n\to\infty}\frac{1}{n}\left\{4n+2(n+1)+\frac{n(2n+1)}{6n}\right\}=\frac{19}{3}$$

(7) $$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^3=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(1+\frac{3k}{n}+\frac{3k^2}{n^2}+\frac{k^3}{n^3}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\left\{n+\frac{3(n+1)}{2}+\frac{(n+1)(2n+1)}{2n}+\frac{(n+1)^2}{4n}\right\}$$

$$=\frac{15}{4}$$

# ＜ 9ページ. 区分求積法 2 ＞

## 問 1 の解答

$$\sum_{k=1}^{n-1} k^2 = \boxed{\frac{(n-1)n(2n-1)}{6}}$$

$$S_n = \left\{\frac{1}{6}(n-1)n(2n-1)\right\}\left(\frac{1}{n}\right)^3 = \frac{1}{6}\left(1-\boxed{\frac{1}{n}}\right)\left(2-\boxed{\frac{1}{n}}\right)$$

## 問 2 の解答

$$S_1 = \frac{1}{6} \times (1-1)(2-1) = 0$$

$$S_2 = \frac{1}{6} \times \left(1-\frac{1}{2}\right) \times \left(2-\frac{1}{2}\right) = \frac{1}{6} \times \frac{1}{2} \times \frac{3}{2} = \frac{1}{8}$$

$$S_3 = \frac{1}{6} \times \left(1-\frac{1}{3}\right) \times \left(2-\frac{1}{3}\right) = \frac{1}{6} \times \frac{2}{3} \times \frac{5}{3} = \frac{5}{27}$$

## 問 3 の解答

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = \frac{1}{6} \times 1 \times 2 = \frac{1}{3}$$

# < 10 ページ. 区分求積法 3 >

## 問の解答

(1) $S_n^* = {x_1}^2 \varDelta x + {x_2}^2 \varDelta x + \cdots + {x_n}^2 \varDelta x = ({x_1}^2 + {x_2}^2 + \cdots + {x_n}^2)\varDelta x$

(2)
$$\begin{aligned} S_n^* &= \left((\varDelta x)^2 + (2\varDelta x)^2 + \cdots + (n\varDelta x)^2\right) \varDelta x \\ &= \left(1^2 + 2^2 + \cdots + n^2\right)(\varDelta x)^3 \\ &= \left(\sum_{k=1}^{n} k^2\right) \times (\varDelta x)^3 \end{aligned}$$

(3) $S_n^* = \dfrac{n(n+1)(2n+1)}{6} \times \left(\dfrac{1}{n}\right)^3 = \dfrac{(n+1)(2n+1)}{6n^2}$

(4) $S_1^* = \dfrac{1 \times 3}{6} = 1$ $\quad S_2^* = \dfrac{3 \times 5}{6 \times 4} = \dfrac{5}{8}$ $\quad S_3^* = \dfrac{4 \times 7}{6 \times 9} = \dfrac{14}{27}$

(5) $\displaystyle\lim_{n\to\infty} S_n^* = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2} = \frac{1}{3}$

(6) $S = \dfrac{1}{3}$

# ＜ 11 ページ. 区分求積法 4 ＞

## 問 1 の解答

(1) $$S_n = (\Delta x)^3\Delta x + (2\Delta x)^3\Delta x + \cdots + ((n-1)\Delta x)^3\,\Delta x$$
$$= \left\{1 + 2^3 + 3^3 + \cdots + (n-1)^3\right\}(\Delta x)^4$$
$$= \left\{\sum_{k=1}^{n-1} k^3\right\}(\Delta x)^4$$

(2) $$S_n = \frac{(n-1)^2 n^2}{2^2} \times \frac{1}{n^4} = \frac{(n-1)^2}{4n^2}$$

(3) $$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n^2 - 2n + 1}{4n^2} = \frac{1}{4}$$

## 問 2 の解答

$$S_n^* = \left\{\sum_{k=1}^{n} k^3\right\}(\Delta x)^4$$

$$\lim_{n\to\infty} S_n^* = \lim_{n\to\infty} \left\{\frac{n(n+1)}{2}\right\}^2 \times \left(\frac{1}{n}\right)^4 = \lim_{n\to\infty} \frac{n^2 + 2n + 1}{4n^2} = \frac{1}{4}$$

## 問 3 の解答

$$S = \frac{1}{4}$$

# ＜ 12 ページ. 区分求積法 5 ＞

## 問 1 の解答

(1) $S_n(x) = \left({x_1}^2 + {x_2}^2 + \cdots + {x_{n-1}}^2\right) \Delta x$

(2) $S_n(x) = \left\{\Delta x^2 + (2\Delta x)^2 + \cdots + ((n-1)\Delta x)^2\right\} \Delta x = \left\{\sum_{k=1}^{n-1} k^2\right\} (\Delta x)^3$

(3) $S_n(x) = \dfrac{(n-1)n(2n-1)}{6} \times \dfrac{x^3}{n^3} = \dfrac{(n-1)(2n-1)}{6n^2} x^3$

(4) $\lim_{n\to\infty} S_n(x) = \lim_{n\to\infty} \left(\dfrac{2n^2 - 3n + 1}{6n^2}\right) x^3 = \dfrac{1}{3} x^3$

## 問 2 の解答

(1) $S_n^*(x) = \sum_{k=1}^{n} {x_k}^2 \Delta x = \sum_{k=1}^{n} k^2 (\Delta x)^3 = \dfrac{(n+1)(2n+1)}{6n^2} x^3$

(2) $\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty} \dfrac{2n^2 + 3n + 1}{6n^2} x^3 = \dfrac{1}{3} x^3$

## 問 3 の解答

$S(x) = \dfrac{1}{3} x^3$

# ＜ 13 ページ. 区分求積法 6 ＞

## 問 1 の解答

(1) $S_n(x) = \left({x_1}^3 + {x_2}^3 + \cdots + {x_{n-1}}^3\right)\Delta x$

(2) $S_n(x) = \displaystyle\sum_{k=1}^{n-1} {x_k}^3 \Delta x = \left\{\sum_{k=1}^{n-1} k^3\right\}(\Delta x)^4$

(3) $S_n(x) = \left\{\dfrac{(n-1)n}{2}\right\}^2 \times \left(\dfrac{x}{n}\right)^4 = \dfrac{(n-1)^2}{2^2n^2}x^4$

(4) $\displaystyle\lim_{n\to\infty} S_n(x) = \lim_{n\to\infty} \frac{n^2-2n+1}{4n^2}x^4 = \frac{1}{4}x^4$

## 問 2 の解答

(1) $S_n^*(x) = \dfrac{(n+1)^2}{4n^2}x^4$

(2) $\displaystyle\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2}x^4 = \frac{1}{4}x^4$

## 問 3 の解答

$S(x) = \dfrac{1}{4}x^4$

# < 14 ページ. 面積関数 1 >

## 問 1 の解答

(1) $S(x) = x$

(2) $S(x) = \dfrac{1}{2}x^2$

## 問 2 の解答

(1) $S(x) = \dfrac{1}{3}x^3$

(2) $S(x) = \dfrac{1}{4}x^4$

## 問 3 の解答

(1) $S(x) = \dfrac{1}{5}x^5$

(2) $S(x) = \dfrac{1}{n+1}x^{n+1}$

## 問 4 の解答

$S'(x) = f(x) \qquad \left(\int f(x)dx = S(x)\right)$

# < 15 ページ. 面積関数 2 >

## 問の解答

[証明]

$$f(x+\varDelta x)\varDelta x \leqq S(x+\varDelta x) - S(x) \leqq f(x)\varDelta x$$

より

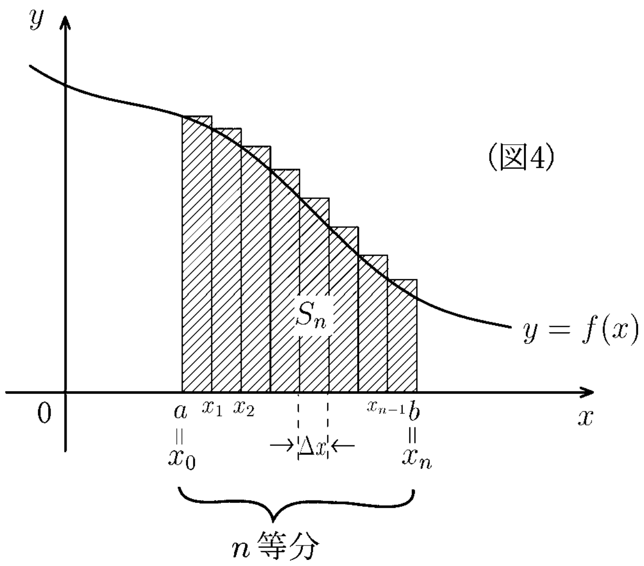
$$\lim_{\varDelta x \to 0} f(x+\varDelta x) \leqq \lim_{\varDelta x \to 0} \frac{S(x+\varDelta x)-S(x)}{\varDelta x} \leqq f(x)$$

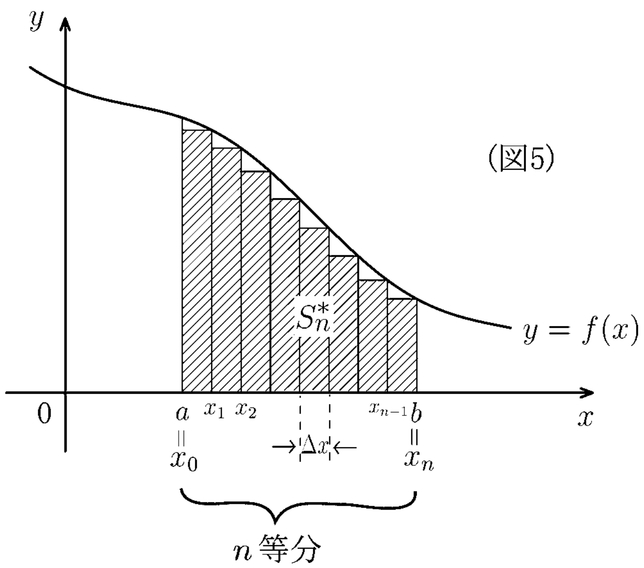
よって $f(x) \leqq S'(x) \leqq f(x)$ より $S'(x) = f(x)$ が成立する。

(証明終)

# ＜ 16 ページ. 定積分の定義 ＞

## 問の解答



# < 17ページ. 和の極限の定積分表示 1 >

## 問の解答

(1) $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)\frac{1}{n}=\int_1^2 x\,dx$

(2) $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{k}{n}\right)^2\frac{1}{n}=\int_2^3 x^2\,dx$

(3) $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^3\frac{1}{n}=\int_1^2 x^3\,dx$

(4) $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{2}{n}k\right)^4\frac{2}{n}=\int_3^5 x^4\,dx$

# < 18 ページ. 和の極限の定積分表示 2 >

## 問の解答

(1) $$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{2+\frac{k}{n}}\right)\frac{1}{n}=\int_2^3\frac{1}{x}\,dx$$

(2) $$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{(4+\frac{2k}{n})^5}\right)\frac{2}{n}=\int_4^6\frac{1}{x^5}\,dx$$

(3) $$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{\sqrt{1+\frac{3k}{n}}}\right)\frac{3}{n}=\int_1^4\frac{1}{\sqrt{x}}\,dx$$

(4) $$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\sqrt[3]{2+\frac{4k}{n}}\right)\frac{4}{n}=\int_2^6\sqrt[3]{x}\,dx$$

# < 20 ページ. 定積分 1 >

## 問の解答

(1) $\displaystyle\int_a^b x^n\,dx = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$

(2) $\displaystyle\int_a^b \frac{1}{x}\,dx = \log\left|\frac{b}{a}\right|$

(3) $\displaystyle\int_a^b dx = b - a$

(4) $\displaystyle\int_a^b e^x\,dx = e^b - e^a$

(5) $\displaystyle\int_a^b \cos x\,dx = \sin b - \sin a$

(6) $\displaystyle\int_a^b \sin x\,dx = -\cos b + \cos a$

(7) $\displaystyle\int_a^b \frac{dx}{\cos^2 x} = \tan b - \tan a$

(8) $\displaystyle\int_a^b \frac{dx}{1+x^2} = \tan^{-1} b - \tan^{-1} a$

(9) $\displaystyle\int_4^{10} dx = 6$

(10) $\displaystyle\int_1^5 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^5 = \frac{4}{5}$

(11) $\displaystyle\int_1^4 \sqrt{x}\,dx = \left[\frac{2}{3}x\sqrt{x}\right]_1^4 = \frac{14}{3}$

(12) $\displaystyle\int_1^8 \frac{1}{\sqrt[3]{x}}\,dx = \left[\frac{3}{2}\sqrt[3]{x^2}\right]_1^8 = \frac{9}{2}$

(13) $\displaystyle\int_1^e \frac{1}{x}\,dx = \log e - \log 1 = 1$

(14) $\displaystyle\int_0^2 e^x\,dx = e^2 - 1$

(15) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x\,dx = \sin\frac{\pi}{2} - \sin 0 = 1$

(16) $\displaystyle\int_0^{\pi} \sin x\,dx = -\cos\pi + \cos 0 = 2$

(17) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = \tan\frac{\pi}{4} - \tan 0 = 1$

(18) $\displaystyle\int_0^1 \frac{dx}{1+x^2} = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4}$

# < 21 ページ. 定積分 2 >

## 問の解答

(1)
$$\int_1^2 \frac{2x^2-3x+1}{x^2}\,dx = \int_1^2 \left(2-\frac{3}{x}+\frac{1}{x^2}\right)\,dx$$
$$= \left[2x-3\log|x|-\frac{1}{x}\right]_1^2$$
$$= 4-3\log 2-\frac{1}{2}-(2-1)$$
$$= \frac{5}{2}-3\log 2$$

(2)
$$\int_{-1}^0 \left(x^3+x^4\right)\,dx + \int_0^1 \left(x^3+x^4\right)\,dx = \int_{-1}^1 \left(x^3+x^4\right)\,dx = \left[\frac{1}{4}x^4+\frac{1}{5}x^5\right]_{-1}^1 = \frac{2}{5}$$

(3)
$$\int_1^2 \frac{dx}{x(x+1)} = \int_1^2 \left(\frac{1}{x}-\frac{1}{x+1}\right)dx = \left[\log\left|\frac{x}{x+1}\right|\right]_1^2 = \log\frac{2}{3}-\log\frac{1}{2} = \log\frac{4}{3}$$

(4)
$$\int_0^\pi \cos^2 x\,dx = \int_0^\pi \frac{1+\cos(2x)}{2}dx = \left[\frac{x}{2}+\frac{\sin(2x)}{4}\right]_0^\pi = \frac{\pi}{2}$$

(5)
$$\int_0^{\frac{\pi}{2}} \sin(2x)\cos(4x)\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin(6x)+\sin(-2x)}{2}dx$$
$$= \left[\frac{-\cos(6x)}{12}+\frac{\cos(-2x)}{4}\right]_0^{\frac{\pi}{2}}$$
$$= -\frac{\cos(3\pi)}{12}+\frac{\cos(-\pi)}{4}-\left(\frac{-1}{12}+\frac{1}{4}\right)$$
$$= -\frac{1}{3}$$

# < 22ページ. 定積分 3 >

## 問の解答

(1) $\displaystyle\int_{-1}^{1}\left(x^3+x^4+x^5\right)dx = 2\int_{0}^{1} x^4\,dx = \frac{2}{5}$

(2) $\displaystyle\int_{-1}^{1}\left(x+x^3+x^6\right)dx = 2\int_{0}^{1} x^6\,dx = \frac{2}{7}$

(3) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin x+\cos x)\,dx = 2\int_{0}^{\pi}\cos x\,dx = 2$

(4) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 2\int_{0}^{\frac{\pi}{4}}\frac{1}{\cos^2 x}dx = 2$

# < 23 ページ. 定積分の積分変数 >

## 問の解答

(1) $\displaystyle\int_1^3 (4-9.8t)dt = \left[4t - 4.9t^2\right]_1^3 = (12 - 4.9 \times 9) - (4 - 4.9) = -31.2$

(2) $\displaystyle\int_0^R 2\pi r\ dr = [\pi r^2]_0^R = \pi R^2$

(3) $\displaystyle\int_0^\pi \sin\theta\ d\theta = [-\cos\theta]_0^\pi = -\cos\pi + \cos 0 = 2$

(4) $\displaystyle\int_a^b u^n\ du = \left[\frac{1}{n+1}u^{n+1}\right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$

(5) $\displaystyle\int_1^9 \sqrt{u}\ du = \left[\frac{2}{3}u\sqrt{u}\right]_1^9 = \frac{2}{3}9\sqrt{9} - \frac{2}{3} = \frac{52}{3}$

# ＜ 24 ページ. 定積分の置換積分法 1 ＞

## 問の解答

(1) $\displaystyle\int_{-1}^{1} 3x^2(x^3+1)^4 dx = \int_0^2 u^4\, du = \left[\frac{1}{5}u^5\right]_0^2 = \frac{32}{5}$

(2) $\displaystyle\int_0^2 2x\sqrt{x^2+1}\, dx = \int_1^5 \sqrt{u}\, du = \left[\frac{2}{3}u\sqrt{u}\right]_1^5 = \frac{10\sqrt{5}}{3} - \frac{2}{3}$

(3) $\displaystyle\int_0^1 \frac{4x^3}{(x^4+1)^2} dx = \int_1^2 \frac{1}{u^2} du = \left[-\frac{1}{u}\right]_1^2 = \frac{1}{2}$

# < 25 ページ. 定積分の置換積分法 2 >

## 問の解答

(1) $\displaystyle \int_0^1 x(x^2+2)^3dx = \int_2^3 \frac{1}{2}u^3du = \left[\frac{1}{8}u^4\right]_2^3 = \frac{1}{8}(81-16) = \frac{65}{8}$

(2) $\displaystyle \int_0^3 xe^{x^2}dx = \int_0^9 \frac{1}{2}e^u du = \left[\frac{1}{2}e^u\right]_0^9 = \frac{e^9}{2} - \frac{1}{2}$

(3) $\displaystyle \int_{-1}^2 \frac{x^2}{x^3+2}dx = \int_1^{10} \frac{1}{u} \times \frac{1}{3}du = \left[\frac{1}{3}\log u\right]_1^{10} = \frac{1}{3}\log 10$

(4) $\displaystyle \int_0^2 \frac{x}{(x^2+1)^3}du = \int_1^5 \frac{1}{2u^3}du = \left[-\frac{1}{4u^2}\right]_1^5 = -\frac{1}{100} + \frac{1}{4} = \frac{6}{25}$

# < 26 ページ. 定積分の置換積分法 3 >

## 問の解答

(1) $\displaystyle\int_0^1 x(1-x)^7\,dx = \int_1^0 (1-u)u^7(-1)\,du = \int_1^0 (u-1)u^7\,du$

$$= \int_1^0 (u^8 - u^7)\,du = \left[\frac{u^9}{9} - \frac{u^8}{8}\right]_1^0$$

$$= 0 - \left(\frac{1}{9} - \frac{1}{8}\right) = \frac{1}{72}$$

$$\left(\begin{array}{c} u = 1-x \\ \Downarrow \\ \dfrac{du}{dx} = -1 \\ x = 1-u \end{array}\right)$$

(2) $\displaystyle\int_2^5 x\sqrt{x-1}\,dx = \int_1^4 (u+1)\sqrt{u}\,du = \int_1^4 (u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du$

$$= \left[\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\right]_1^4$$

$$= \left(\frac{2}{5} \times 4^{\frac{5}{2}} + \frac{2}{3} \times 4^{\frac{3}{2}}\right) - \left(\frac{2}{5} + \frac{2}{3}\right)$$

$$= \frac{64}{5} + \frac{16}{3} - \frac{2}{5} - \frac{2}{3} = \frac{62}{5} + \frac{14}{3}$$

$$= \frac{186+70}{15} = \frac{256}{15}$$

$$\left(\begin{array}{c} u = x-1 \\ \Downarrow \\ \dfrac{du}{dx} = 1 \\ x = u+1 \end{array}\right)$$

# < 27 ページ. 定積分の置換積分法 4 >

## 問の解答

(1) $\displaystyle\int_{-1}^{\sqrt{3}} \sqrt{4-x^2}\ dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{4-4\sin^2\theta}\ 2\cos\theta\ d\theta = \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} 4\cos^2\theta\ d\theta$

$\left(\begin{array}{l} x = 2\sin\theta \\ \dfrac{dx}{d\theta} = 2\cos\theta \end{array}\right)$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (2+2\cos(2\theta))\ d\theta = \Big[2\theta + \sin(2\theta)\Big]_{-\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \frac{2\pi}{3} + \sin\left(\frac{2\pi}{3}\right) - \left(-\frac{\pi}{3} + \sin\left(-\frac{\pi}{3}\right)\right)$$

$$= \frac{3\pi}{3} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \pi + \sqrt{3}$$

(2) $\displaystyle\int_{0}^{\sqrt{2}} \frac{dx}{\sqrt{4-x^2}}\ dx = \int_{0}^{\frac{\pi}{4}} \frac{1}{\sqrt{4-4\sin^2\theta}}\ 2\cos\theta\ d\theta = \int_{0}^{\frac{\pi}{4}}\ d\theta$

( $x = 2\sin\theta$ )

$$= \Big[\ \theta\ \Big]_{0}^{\frac{\pi}{4}} = \frac{\pi}{4}$$

# ＜ 28 ページ. 定積分の部分積分法 1 ＞

## 問の解答

$$(1)\int_{-1}^{1}(x+1)(x-1)^3\ dx=\left[\frac{1}{4}(x+1)(x-1)^4\right]_{-1}^{1}-\int_{-1}^{1}\frac{1}{4}(x-1)^4\ dx$$

$$=0-\left[\frac{1}{20}(x-1)^5\right]_{-1}^{1}$$

$$=-\left(0-\frac{-32}{20}\right)=-\frac{8}{5}$$

$$(2)\int_{0}^{\frac{\pi}{2}}x\sin x\ dx=\Big[-x\cos x\Big]_{0}^{\frac{\pi}{2}}-\int_{0}^{\frac{\pi}{2}}(-\cos x)\ dx=0+\int_{0}^{\frac{\pi}{2}}\cos x\ dx$$

$$=\Big[\sin x\Big]_{0}^{\frac{\pi}{2}}=1$$

$$(3)\int_{0}^{1}xe^x\ dx=\big[xe^x\big]_0^1-\int_0^1 e^x\ dx$$

$$=(1e^1-0)-\big[e^x\big]_0^1$$

$$=e-(e-1)=1$$

# < 29 ページ. 定積分の部分積分法 2 >

## 問の解答

(1) $$\int_{\frac{1}{e}}^{e} x\log x\ dx = \left[\frac{x^2}{2}\log x\right]_{\frac{1}{e}}^{e} - \int_{\frac{1}{e}}^{e}\frac{x}{2}\ dx$$
$$= \frac{e^2}{2}\log e - \frac{1}{2e^2}\log\left(\frac{1}{e}\right) - \left[\frac{x^2}{4}\right]_{\frac{1}{e}}^{e}$$
$$= \frac{e^2}{2} + \frac{1}{2e^2} - \left(\frac{e^2}{4} - \frac{1}{4e^2}\right)$$
$$= \frac{e^2}{4} + \frac{3}{4e^2}$$

(2) $$\int_{1}^{\sqrt{e}} x^3\log x\ dx = \left[\frac{x^4}{4}\log x\right]_{1}^{\sqrt{e}} - \int_{1}^{\sqrt{e}}\frac{x^3}{4}\ dx$$
$$= \frac{e^2}{4}\log\sqrt{e} - 0 - \left[\frac{x^4}{16}\right]_{1}^{\sqrt{e}}$$
$$= \frac{e^2}{8} - \left(\frac{e^2}{16} - \frac{1}{16}\right)$$
$$= \frac{e^2}{16} + \frac{1}{16}$$

(3) $$\int_{0}^{1} x^2e^{x+1}\ dx = \left[x^2e^{x+1}\right]_{0}^{1} - \int_{0}^{1} 2xe^{x+1}\ dx$$
$$= (1e^2 - 0) - \left\{\left[2xe^{x+1}\right]_{0}^{1} - \int_{0}^{1} 2e^{x+1}\ dx\right\}$$
$$= e^2 - (2e^2 - 0) + \left[2e^{x+1}\right]_{0}^{1}$$
$$= -e^2 + (2e^2 - 2e^1) = e^2 - 2e$$

(4) $$\int_{0}^{1} x^3e^{x}\ dx = \left[x^3e^{x}\right]_{0}^{1} - \int_{0}^{1} 3x^2e^{x}\ dx$$
$$= (1e^1 - 0) - \left\{\left[3x^2e^{x}\right]_{0}^{1} - \int_{0}^{1} 6xe^{x}\ dx\right\}$$
$$= e - (3e^1 - 0) + \left[6xe^{x}\right]_{0}^{1} - \int_{0}^{1} 6e^{x}\ dx$$
$$= -2e + (6e^1 - 0) - \left[6e^{x}\right]_{0}^{1}$$
$$= 4e - (6e^1 - 6) = 6 - 2e$$

# < 30 ページ. 定積分の部分積分法 3 >

## 問の解答

$$(1)\int_0^{\frac{\pi}{2}} x^2\cos x\ dx = \Big[x^2\sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x\sin x\ dx$$

$$= \left(\frac{\pi}{2}\right)^2\sin\left(\frac{\pi}{2}\right) - 0 - \left\{\Big[2x\times(-\cos x)\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2\times(-\cos x)\ dx\right\}$$

$$= \frac{\pi^2}{4} - 0 - \int_0^{\frac{\pi}{2}} 2\cos x\ dx$$

$$= \frac{\pi^2}{4} - \Big[2\sin x\Big]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4} - 2$$

$$(2)\int_0^{\frac{\pi}{2}} x^2\sin(2x)\ dx = \left[\frac{-x^2\cos 2x}{2}\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}\big(-x\cos(2x)\big)\ dx$$

$$= -\frac{\left(\frac{\pi}{2}\right)^2}{2}\cos\pi + \int_0^{\frac{\pi}{2}} x\cos(2x)\ dx$$

$$= \frac{\pi^2}{8} + \left[x\frac{\sin(2x)}{2}\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}\frac{\sin(2x)}{2}\ dx$$

$$= \frac{\pi^2}{8} + \frac{\pi}{2}\times\frac{\sin(\pi)}{2} - 0 + \left[\frac{\cos(2x)}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi^2}{8} - \frac{1}{2}$$

# < 31 ページ. 定積分の練習 1 >

## 問の解答

(1) $\displaystyle\int_{-1}^{3} dx = 3+1=4$

(2) $\displaystyle\int_{1}^{\sqrt{e}} \frac{dx}{x} = \Big[\log x\Big]_1^{\sqrt{e}} = \log\sqrt{e} - \log 1 = \frac{1}{2}$

(3) $\displaystyle\int_{0}^{1} \sqrt[3]{x}\ dx = \Big[\frac{3}{4}x^{\frac{4}{3}}\Big]_0^1 = \frac{3}{4}$

(4) $\displaystyle\int_{0}^{\pi} (3\sin x - 4\cos x)\ dx = \Big[-3\cos x - 4\sin x\Big]_0^{\pi}$

$\displaystyle = -3\cos\pi - 4\sin\pi + 3\cos 0 + 4\sin 0 = 6$

(5) $\displaystyle\int_{1}^{2} \frac{3x^2-4x+1}{x^2}\ dx = \int_1^2 \Big(3-\frac{4}{x}+\frac{1}{x^2}\Big)\ dx = \Big[3x - 4\log|x| - \frac{1}{x}\Big]_1^2$

$\displaystyle = \Big(6-4\log 2-\frac{1}{2}\Big) - \Big(3-4\log 1-\frac{1}{1}\Big) = \frac{7}{2} - 4\log 2$

(6) $\displaystyle\int_{1}^{9} \frac{dx}{\sqrt{x}} = \Big[2\sqrt{x}\Big]_1^9 = 2\sqrt{9}-2=4$

(7) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = \Big[\tan x\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{4}} = 1+\sqrt{3}$

(8) $\displaystyle\int_{1}^{2} \frac{1}{3x+1}\ dx = \Big[\frac{1}{3}\log|3x+1|\Big]_1^2 = \frac{1}{3}\log 7 - \frac{1}{3}\log 4 = \frac{1}{3}\log\big(\frac{7}{4}\big)$

(9) $\displaystyle\int_{2}^{3} \frac{dx}{x^2-1} = \int_2^3 \frac{1}{2}\Big(\frac{1}{x-1}-\frac{1}{x+1}\Big)\ dx = \Big[\frac{1}{2}\log\Big|\frac{x-1}{x+1}\Big|\Big]_2^3 = \frac{1}{2}\log\big(\frac{3}{2}\big)$

(10) $\displaystyle\int_{0}^{\pi} \sin 2x\cos x\ dx = \int_0^{\pi}\frac{1}{2}\Big\{\sin(3x)+\sin x\Big\}\ dx = \Big[-\frac{1}{6}\cos(3x) - \frac{1}{2}\cos x\Big]_0^{\pi}$

$\displaystyle = \Big(-\frac{1}{6}\cos(3\pi) - \frac{1}{2}\cos\pi\Big) + \Big(\frac{1}{6}\cos 0 + \frac{1}{2}\cos 0\Big) = \frac{4}{3}$

(11) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx = 2\int_0^{\frac{\pi}{2}} \frac{1}{2}\big(1-\cos(2x)\big)\ dx = \Big[x - \frac{1}{2}\sin(2x)\Big]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$

(12) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(2x)\ dx = 2\int_0^{\frac{\pi}{2}} \frac{1}{2}\big(1+\cos(2x)\big)\ dx = \Big[x + \frac{1}{2}\sin(2x)\Big]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$

(13) $\displaystyle\int_{-2}^{2} e^{3x-1}\ dx = \Big[\frac{1}{3}e^{3x-1}\Big]_{-2}^2 = \frac{1}{3}e^5 - \frac{1}{3}e^{-7} = \frac{1}{3}\Big(e^5 - \frac{1}{e^7}\Big)$

(14) $\displaystyle\int_{-1}^{1} xe^{-x^2}\ dx = \Big[-\frac{1}{2}e^{-x^2}\Big]_{-1}^1 = -\frac{1}{2}e^{-1} + \frac{1}{2}e^{-1} = 0$

# < 32 ページ. 定積分の練習 2 >

## 問の解答

(1) $\int_0^1 \frac{1}{(3x+1)^5}\ dx = \int_1^4 \frac{1}{3}\times\frac{1}{u^5}\ du = \Big[-\frac{1}{12}u^{-4}\Big]_1^4 = -\frac{1}{12}\Big\{\frac{1}{4^4}-\frac{1}{1^4}\Big\} = \frac{85}{1024}$

(2) $\int_1^{10} \sqrt{5x-1}\ dx = \int_4^{49} \frac{1}{5}\sqrt{u}\ du = \Big[\frac{2}{15}u^{\frac{3}{2}}\Big]_4^{49} = \frac{2}{15}\Big\{49^{\frac{3}{2}} - 4^{\frac{3}{2}}\Big\} = \frac{2}{15}\Big\{343-8\Big\} = \frac{134}{3}$

(3) $\int_0^1 \frac{x^2}{(x^3+1)^4}\ dx = \int_1^2 \frac{1}{3}\times\frac{1}{u^4}\ du = \Big[-\frac{1}{9}u^{-3}\Big]_1^2 = -\frac{1}{9}\Big\{\frac{1}{2^3}-\frac{1}{1^3}\Big\} = \frac{1}{9}\times\frac{7}{8} = \frac{7}{72}$

(4) $\int_0^1 \frac{3x}{x^2+1}\ dx = \Big[\frac{3}{2}\log(x^2+1)\Big]_0^1 = \frac{3}{2}\log 2$

(5) $\int_{-\sqrt{3}}^2 \sqrt{4-x^2}\ dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{4-4\sin^2\theta}\ 2\cos\theta\ d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} 4\cos^2\theta\ d\theta$

$\Big(x = 2\sin\theta\Big)$ $= \int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \Big\{2+2\cos(2\theta)\Big\}\ d\theta = \Big[2\theta+\sin(2\theta)\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \pi + \sin\pi - \Big(-\frac{2\pi}{3} + \sin\big(-\frac{2\pi}{3}\big)\Big) = \frac{5\pi}{3} + \frac{\sqrt{3}}{2}$

(6) $\int_0^1 x(x+1)^4\ dx = \Big[x\frac{(x+1)^5}{5}\Big]_0^1 - \int_0^1 \frac{(x+1)^5}{5}\ dx = \frac{2^5}{5} - 0 - \Big[\frac{(x+1)^6}{30}\Big]_0^1$

$= \frac{32}{5} - \Big(\frac{2^6}{30} - \frac{1^6}{30}\Big) = \frac{43}{10}$

(7) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} x\sin x\ dx = \Big[-x\cos x\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{3}} - \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} (-\cos x)\ dx = -\frac{\pi}{3}\cos\frac{\pi}{3} + \frac{\pi}{4}\cos\Big(-\frac{\pi}{4}\Big) + \Big[\sin x\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= -\frac{\pi}{6} + \frac{\sqrt{2}}{8}\pi + \sin\frac{\pi}{3} - \sin\Big(-\frac{\pi}{4}\Big) = \Big(\frac{\sqrt{2}}{8} - \frac{1}{6}\Big)\pi + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}$

(8) $\int_{\frac{1}{e}}^1 \log x\ dx = \Big[x\log x - x\Big]_{\frac{1}{e}}^1 = (1\log 1 - 1) - \Big(\frac{1}{e}\log\frac{1}{e} - \frac{1}{e}\Big) = \frac{2}{e} - 1$

(9) $\int_{-1}^0 x^2e^x\ dx = \Big[x^2e^x\Big]_{-1}^0 - \int_{-1}^0 2xe^x\ dx = (0-e^{-1}) - \Big\{\Big[2xe^x\Big]_{-1}^0 - \int_{-1}^0 2e^x\ dx\Big\}$

$= -\frac{1}{e} - (0+2e^{-1}) + \Big[2e^x\Big]_{-1}^0 = -\frac{3}{e} + 2e^0 - 2e^{-1} = 2 - \frac{5}{e}$

(10) $\int_0^\pi x^2\cos x\ dx = \Big[x^2\sin x\Big]_0^\pi - \int_0^\pi 2x\sin x\ dx = 0 - 0 + \int_0^\pi 2x(\cos x)'\ dx$

$= \Big[2x\cos x\Big]_0^\pi - \int_0^\pi 2\cos x\ dx = 2\pi\cos\pi - 0 - \Big[2\sin x\Big]_0^\pi = -2\pi$

# < 33 ページ. 和の極限と定積分 1 >

## 問の解答

(1) $\displaystyle\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\left(2+\frac{k}{n}\right)^2}=\int_2^3\frac{1}{x^2}\,dx=\left[-\frac{1}{x}\right]_2^3=-\frac{1}{3}+\frac{1}{2}=\frac{1}{6}$

(2) $\displaystyle\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^3=\int_1^3 x^3\,dx=\left[\frac{1}{4}x^4\right]_1^3=\frac{81}{4}-\frac{1}{4}=\frac{80}{4}=20$

(3) $\displaystyle\lim_{n\to\infty}\frac{5}{n}\sum_{k=1}^{n}\sqrt{4+\frac{5k}{n}}=\int_4^9\sqrt{x}\,dx=\left[\frac{2}{3}x\sqrt{x}\right]_4^9=\frac{2}{3}(9\sqrt{9}-4\sqrt{4})=\frac{38}{3}$

(4) $\displaystyle\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}\frac{1}{\sqrt{1+\frac{3k}{n}}}=\int_1^4\frac{1}{\sqrt{x}}\,dx=\left[2\sqrt{x}\right]_1^4=2\sqrt{4}-2=2$

# < 34 ページ. 和の極限と定積分 2 >

## 問の解答

(1) $\displaystyle\lim_{n\to\infty}\frac{3}{n}\left\{\sqrt{1+\frac{3}{n}}+\sqrt{1+\frac{6}{n}}+\sqrt{1+\frac{9}{n}}+\cdots+\sqrt{1+\frac{3n}{n}}\right\}$

$$=\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}\sqrt{1+\frac{3}{n}k}=\int_1^4\sqrt{x}\ dx=\left[\frac{2}{3}x\sqrt{x}\right]_1^4=\frac{2}{3}(4\sqrt{4}-1)=\frac{14}{3}$$

(2) $\displaystyle\lim_{n\to\infty}\frac{\pi}{n}\left\{\cos\left(-\frac{\pi}{2}+\frac{\pi}{n}\right)+\sin\left(-\frac{\pi}{2}+\frac{2\pi}{n}\right)+\cos\left(-\frac{\pi}{2}+\frac{3\pi}{n}\right)+\cdots+\cos\left(-\frac{\pi}{2}+\frac{n\pi}{n}\right)\right\}$

$$=\lim_{n\to\infty}\frac{\pi}{n}\sum_{k=1}^{n}\cos\left(-\frac{\pi}{2}+\frac{k\pi}{n}\right)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(x)\ dx=\Big[\sin x\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=2$$

(3) $\displaystyle\lim_{n\to\infty}\frac{5}{n}\left\{\frac{1}{\sqrt{4+\frac{5}{n}}}+\frac{1}{\sqrt{4+\frac{10}{n}}}+\frac{1}{\sqrt{4+\frac{15}{n}}}+\cdots+\frac{1}{\sqrt{4+\frac{5n}{n}}}\right\}$

$$=\lim_{n\to\infty}\frac{5}{n}\sum_{k=1}^{n}\frac{1}{\sqrt{4+\frac{5}{n}k}}=\int_4^9\frac{1}{\sqrt{x}}\ dx=\Big[2\sqrt{x}\Big]_4^9=2\sqrt{9}-2\sqrt{4}=6-4=2$$

# < 35 ページ. 和の極限と定積分 3 >

## 問の解答

(1) $\displaystyle \lim_{n\to\infty}\frac{2}{n}\left\{\left(-1+\frac{2}{n}\right)^4+\left(-1+\frac{4}{n}\right)^4+\left(-1+\frac{6}{n}\right)^4+\cdots+\left(-1+\frac{2n}{n}\right)^4\right\}$

$$=\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}\left(-1+\frac{2}{n}k\right)^4=\int_{-1}^{1}x^4\ dx=2\int_0^1 x^4\ dx=2\left[\frac{1}{5}x^5\right]_0^1=\frac{2}{5}$$

(2) $\displaystyle \lim_{n\to\infty}\frac{3}{n}\left\{\frac{1}{(1+\frac{3}{n})^3}+\frac{1}{(1+\frac{6}{n})^3}+\frac{1}{(1+\frac{9}{n})^3}+\cdots+\frac{1}{(1+\frac{3n}{n})^3}\right\}$

$$=\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}\frac{1}{(1+\frac{3}{n}k)^3}=\int_1^4\frac{1}{x^3}\ dx=\left[-\frac{1}{2x^2}\right]_1^4=-\frac{1}{32}+\frac{1}{2}=\frac{15}{32}$$

(3) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\frac{1}{1+\frac{3}{n}}+\cdots+\frac{1}{1+\frac{n}{n}}\right\}$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+\frac{k}{n}}=\int_1^2\frac{1}{x}\ dx=\Big[\log x\Big]_1^2=\log 2$$

(4) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\left\{\left(4+\frac{1}{n}\right)+\left(4+\frac{2}{n}\right)+\left(4+\frac{3}{n}\right)+\cdots+\left(4+\frac{n}{n}\right)\right\}$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(4+\frac{k}{n}\right)=\int_4^5 x\ dx=\left[\frac{x^2}{2}\right]_4^5=\frac{1}{2}(25-16)=\frac{9}{2}$$

(5) $\displaystyle \lim_{n\to\infty}\frac{4}{n}\left\{\sqrt{\frac{4}{n}}+\sqrt{\frac{8}{n}}+\sqrt{\frac{12}{n}}+\cdots+\sqrt{\frac{4n}{n}}\right\}$

$$=\lim_{n\to\infty}\frac{4}{n}\sum_{k=1}^{n}\sqrt{\frac{4}{n}k}=\int_0^4\sqrt{x}\ dx=\left[\frac{2}{3}x\sqrt{x}\right]_0^4=\frac{2}{3}4\sqrt{4}=\frac{16}{3}$$

(6) $\displaystyle \lim_{n\to\infty}\frac{\pi}{n}\left\{\sin\left(\frac{\pi}{n}\right)+\sin\left(\frac{2\pi}{n}\right)+\sin\left(\frac{3\pi}{n}\right)+\cdots+\sin\left(\frac{n\pi}{n}\right)\right\}$

$$=\lim_{n\to\infty}\frac{\pi}{n}\sum_{k=1}^{n}\sin\left(\frac{\pi}{n}k\right)=\int_0^{\pi}\sin x\ dx=\Big[-\cos x\Big]_0^{\pi}=2$$

(7) $\displaystyle \lim_{n\to\infty}\frac{\pi}{2n}\left\{\cos\left(\frac{\pi}{2n}\right)+\cos\left(\frac{2\pi}{2n}\right)+\cos\left(\frac{3\pi}{2n}\right)+\cdots+\cos\left(\frac{n\pi}{2n}\right)\right\}$

$$=\lim_{n\to\infty}\frac{\pi}{2n}\sum_{k=1}^{n}\cos\left(\frac{\pi}{2n}k\right)=\int_0^{\frac{\pi}{2}}\cos x\ dx=\Big[\sin x\Big]_1^{\frac{\pi}{2}}=1$$

(8) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{1+\left(\frac{1}{n}\right)^2}+\frac{1}{1+\left(\frac{2}{n}\right)^2}+\frac{1}{1+\left(\frac{3}{n}\right)^2}+\cdots+\frac{1}{1+\left(\frac{n}{n}\right)^2}\right\}$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+\left(\frac{k}{n}\right)^2}=\int_0^1\frac{1}{1+x^2}\ dx=\Big[\tan^{-1}x\Big]_0^1=\tan^{-1}(1)-\tan^{-1}(0)=\frac{\pi}{4}$$

(9) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\left\{\frac{\frac{1}{n}}{1+\left(\frac{1}{n}\right)^2}+\frac{\frac{2}{n}}{1+\left(\frac{2}{n}\right)^2}+\frac{\frac{3}{n}}{1+\left(\frac{3}{n}\right)^2}+\cdots+\frac{\frac{n}{n}}{1+\left(\frac{n}{n}\right)^2}\right\}$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{\frac{k}{n}}{1+\left(\frac{k}{n}\right)^2}=\int_0^1\frac{x}{1+x^2}\ dx=\left[\frac{1}{2}\log(1+x^2)\right]_0^1=\frac{1}{2}\log 2$$

# < 36 ページ. 面積 1 >

## 問の解答

(1) $\displaystyle\int_0^1 e^x dx = e - 1$

(2) $\displaystyle\int_1^9 \sqrt{x}\, dx = \left[\frac{2}{3}x\sqrt{x}\right]_1^9 = \frac{2}{3}\left(9\sqrt{9} - 1\right) = \frac{52}{3}$

(3) $\displaystyle\int_1^2 \frac{1}{x^2} dx = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

(4) $\displaystyle\int_1^2 \frac{1}{x} dx = \left[\log x\right]_1^2 = \log 2$

# ＜ 37 ページ. 面積 2 ＞

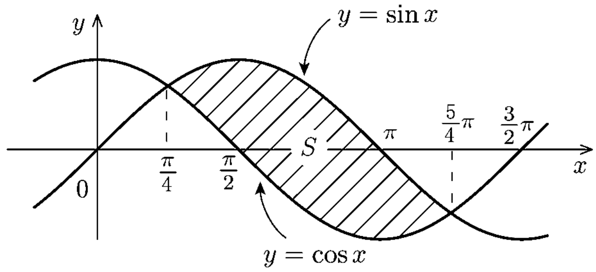
## 問 1 の解答

$$S = \int_a^b \{0 - g(x)\}dx = -\int_a^b g(x)dx$$

## 問 2 の解答

$$S = \int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} (\sin x - \cos x)dx = \Big[-\cos x - \sin x\Big]_{\frac{\pi}{4}}^{\frac{5}{4}\pi}$$

$$= -\cos\left(\tfrac{5}{4}\pi\right) - \sin\left(\tfrac{5}{4}\pi\right) + \cos\left(\tfrac{\pi}{4}\right) + \sin\left(\tfrac{\pi}{4}\right) = 2\sqrt{2}$$

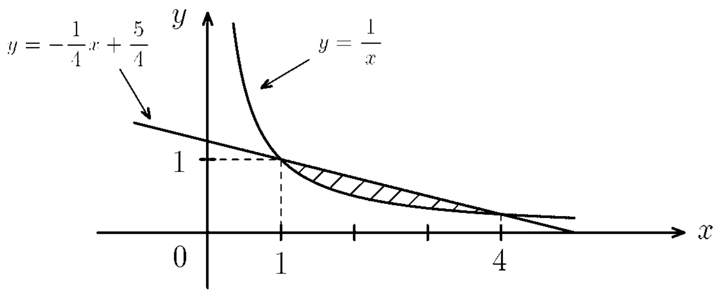



## 問 3 の解答

(1) $\displaystyle\int_0^1 (\sqrt{x} - x^2)dx = \left[\frac{2}{3}x\sqrt{x} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

(2) $\displaystyle\int_1^4 \left(-\frac{1}{4}x + \frac{5}{4} - \frac{1}{x}\right)dx = \left[-\frac{x^2}{8} + \frac{5}{4}x - \log x\right]_1^4$

$$= -\frac{16}{8} + 5 - \log 4 - \left(-\frac{1}{8} + \frac{5}{4}\right) = 3 + \frac{1}{8} - \frac{5}{4} - \log 4 = \frac{15}{8} - \log 4$$



# ＜ 38ページ. 面積 3 ＞

## 問の解答

$$
\left( \begin{array}{l} x = a\sin\theta \\ \begin{array}{c|l} x & 0 \to \frac{\sqrt{3}}{2}a \\ \hline \theta & 0 \to \frac{\pi}{3} \end{array} \end{array} \right)
$$

$$
\begin{aligned}
S &= \int_0^{\frac{\sqrt{3}}{2}a} \sqrt{a^2 - x^2}\,dx = \int_0^{\frac{\pi}{3}} \sqrt{a^2 - a^2\sin^2\theta}\;a\cos\theta\,d\theta \\
&= \int_0^{\frac{\pi}{3}} a^2\cos^2\theta\,d\theta = a^2\int_0^{\frac{\pi}{3}} \frac{1+\cos(2\theta)}{2}d\theta \\
&= a^2\left[\frac{\theta}{2} + \frac{1}{4}\sin(2\theta)\right]_0^{\frac{\pi}{3}} = a^2\left\{\frac{\pi}{6} + \frac{1}{4}\sin\left(\frac{2\pi}{3}\right) - 0\right\} \\
&= \frac{a^2\pi}{6} + \frac{\sqrt{3}}{8}a^2
\end{aligned}
$$

# < 39 ページ. 面積 4 >

## 問 1 の解答

(1) $$\int_0^r \sqrt{r^2 - x^2}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2\sin^2\theta}\; r\cos\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} r^2\cos^2\theta\,d\theta = \int_0^{\frac{\pi}{2}} \frac{r^2}{2}\big\{1+\cos(2\theta)\big\}d\theta$$

$$= \left[\frac{r^2}{2}\theta + \frac{r^2}{4}\sin(2\theta)\right]_0^{\frac{\pi}{2}} = \frac{\pi r^2}{4}$$

(2) $S = 4 \times \dfrac{S}{4} = 4 \times \dfrac{\pi r^2}{4} = \pi r^2$

## 問 2 の解答

(1) $y = \sqrt{b^2 - \left(\frac{b}{a}x\right)^2}$

(2) $\dfrac{S}{4} = \displaystyle\int_0^a \sqrt{b^2 - \left(\tfrac{b}{a}x\right)^2}\,dx$

(3) $x = a\sin\theta$ とおくと

$$\frac{S}{4} = \int_0^{\frac{\pi}{2}} \sqrt{b^2 - b^2\sin^2\theta}\,a\cos\theta\,d\theta = \int_0^{\frac{\pi}{2}} ab\cos^2\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{ab}{2}\big\{1+\cos(2\theta)\big\}d\theta = \left[\frac{ab}{2}\theta + \frac{ab}{4}\sin(2\theta)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{ab}{4}\pi$$

よって $S = 4 \times \dfrac{S}{4} = 4 \times \dfrac{ab\pi}{4} = ab\pi$

# < 40 ページ. 体積 1 >

## 問の解答

(1) $\mathrm{A'C'} = \dfrac{ax}{h}$ , $\mathrm{B'C'} = \dfrac{bx}{h}$

(2) $S(x) = \dfrac{1}{2}\mathrm{A'C'} \times \mathrm{B'C'} = \dfrac{abx^2}{2h^2}$

(3) $V = \displaystyle\int_0^h S(x)dx = \int_0^h \frac{abx^2}{2h^2}dx = \left[\frac{abx^3}{6h^2}\right]_0^h = \frac{abh}{6}$

# < 41 ページ. 体積 2 >

## 問 1 の解答

$$S(x) = \left(\frac{x}{2}\right)^2$$

$$V = \int_0^4 S(x)\ dx = \int_0^4 \frac{x^2}{4}\ dx = \left[\frac{x^3}{12}\right]_0^4 = \frac{4^3}{12} = \frac{16}{3}$$

## 問 2 の解答

(1) $S(x) = \left(\frac{ax}{h}\right) \times \left(\frac{bx}{h}\right) = \frac{abx^2}{h^2}$

$$V = \int_0^h S(x)\ dx = \int_0^h \frac{abx^2}{h^2}\ dx = \left[\frac{abx^3}{3h^2}\right]_0^h = \frac{abh}{3}$$

(2) $S = ab$ より

$$\frac{S(x)}{S} = \frac{\frac{abx^2}{h^2}}{ab} = \frac{x^2}{h^2}$$

## 問 3 の解答

(1) $S(x) = \pi \times \left(\frac{rx}{h}\right)^2 = \frac{\pi r^2 x^2}{h^2}$

(2) $V = \int_0^h S(x)\ dx = \int_0^h \frac{\pi r^2 x^2}{h^2}\ dx = \left[\frac{\pi r^2 x^3}{3h^2}\right]_0^h = \frac{\pi r^2 h}{3}$

(3) $S = \pi r^2$ より

$$\frac{S(x)}{S} = \frac{\frac{\pi r^2 x^2}{h^2}}{\pi r^2} = \frac{x^2}{h^2}$$

# < 42 ページ. 体積 3 >

## 問 1 の解答

(1) $S(x) = \pi\{f(x)\}^2$

(2) $V = \displaystyle\int_a^b \pi\{f(x)\}^2\,dx$

## 問 2 の解答

$$V = \int_0^h \pi\left(\frac{r}{h}x\right)^2 dx = \int_0^h \frac{\pi r^2 x^2}{h^2}\,dx = \left[\frac{\pi r^2 x^3}{3h^2}\right]_0^h = \frac{\pi r^2 h}{3}$$

## 問 3 の解答

$$V = \int_{-r}^{r} \pi\left\{\sqrt{r^2 - x^2}\right\}^2 dx = \int_{-r}^{r} \pi(r^2 - x^2)\,dx = \left[\pi r^2 x - \frac{\pi}{3}x^3\right]_{-r}^{r}$$

$$= \left(\pi r^3 - \frac{\pi}{3}r^3\right) - \left(-\pi r^3 + \frac{\pi}{3}r^3\right) = \frac{4}{3}\pi r^3$$

# ＜ 43 ページ. 体積 4 ＞

## 問の解答

(1) $A_1'C_1' = \dfrac{5x}{7}$

(2) $A_2'C_2' = \dfrac{5x}{7}$

(3) $S_1(x) = \dfrac{1}{2} \times \left(\dfrac{5x}{7}\right)^2 = \dfrac{25x^2}{98}$

(4) $S_2(x) = \dfrac{25x^2}{98}$

(5) $V_1 = \displaystyle\int_0^7 \frac{25x^2}{98}\,dx = \left[\frac{25x^3}{3 \times 98}\right]_0^7 = \frac{175}{6}$

$V_2 = \displaystyle\int_0^7 \frac{25x^2}{98}\,dx = \frac{175}{6}$

# ＜ 44 ページ. 体積 5 ＞

## 問 1 の解答

(1) $S(x) = S$

(2) $V = \int_0^h S(x)\ dx = Sh$

## 問 2 の解答

(1) $S(x) = \dfrac{x^2}{h^2}S$

(2) $V = \int_0^h S(x)\ dx = \int_0^h \dfrac{Sx^2}{h^2}\ dx = \left[\dfrac{Sx^3}{3h^2}\right]_0^h = \dfrac{1}{3}Sh$

# < 45 ページ. 平面上の運動 >

## 問の解答

(1) $v(t) = (\ 6t^2\ ,\ 6t\ )$

$$|v(t)| = \sqrt{(6t^2)^2 + (6t)^2} = 6t\sqrt{t^2+1}$$

(2) $v(t) = (\ -2\sin t\ ,\ 2\cos t\ )$

$$|v(t)| = \sqrt{(-2\sin t)^2 + (2\cos t)^2} = 2$$

(3) $v(t) = (\ -3\sin t\cos^2 t\ ,\ 3\cos t\sin^2 t\ )$

$$|v(t)| = \sqrt{(-3\sin t\cos^2 t)^2 + (3\cos t\sin^2 t)^2} = 3\,|\sin t\cos t|$$

(4) $v(t) = (\ -e^{-t}\cos t - e^{-t}\sin t\ ,\ -e^{-t}\sin t + e^{-t}\cos t\ )$

$$|v(t)| = \sqrt{(-e^{-t}\cos t - e^{-t}\sin t)^2 + (-e^{-t}\sin t + e^{-t}\cos t)^2}$$

$$= -e^{-t}\sqrt{2\cos^2 t + 2\sin^2 t} = \sqrt{2}e^{-t}$$

# < 47ページ.道のり 2 >

## 問 1 の解答

(1) $\big( \ x(t) \ , \ y(t) \ \big) = \big( \ \sin t \ , \ 0 \ \big)$

(2) $$\int_0^{\pi} |v(t)| \ dt = \int_0^{\pi} |\cos t| \ dt$$
$$= \int_0^{\frac{\pi}{2}} \cos t \ dt + \int_{\frac{\pi}{2}}^{\pi} (-\cos t) \ dt$$
$$= \Big[\sin t\Big]_0^{\frac{\pi}{2}} + \Big[-\sin t\Big]_{\frac{\pi}{2}}^{\pi} = 2$$

## 問 2 の解答

(1) $\big( \ x(t) \ , \ y(t) \ \big) = \big( \ \cos t \ , \ \sin t \ \big)$

(2) $$\int_0^{2\pi} |v(t)| \ dt = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2} \ dt$$
$$= \int_0^{2\pi} 1 \ dt = 2\pi$$

# ＜ 48 ページ. 道のり 3 ＞

## 問の解答

(1) $$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=(-e^{-t}\cos t-e^{-t}\sin t)^2+(-e^{-t}\sin t+e^{-t}\cos t)^2$$
$$=(e^{-t})^2\{2\cos^2 t+2\sin^2 t\}=2(e^{-t})^2$$

$$\ell=\int_0^{2\pi}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^{2\pi}\sqrt{2(e^{-t})^2}\,dt$$
$$=\int_0^{2\pi}\sqrt{2}e^{-t}\,dt=\Big[-\sqrt{2}e^{-t}\Big]_0^{2\pi}$$
$$=-\sqrt{2}e^{-2\pi}+\sqrt{2}e^0=\sqrt{2}\left(1-\frac{1}{e^{2\pi}}\right)$$

(2) $$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=(-3\sin t\cos^2 t)^2+(3\cos t\sin^2 t)^2$$
$$=9\sin^2 t\cos^2 t\{\cos^2 t+\sin^2 t\}$$
$$=9\sin^2 t\cos^2 t$$

$$\ell=\int_0^{\frac{\pi}{2}}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^{\frac{\pi}{2}}3|\sin t\cos t|\,dt$$
$$=\int_0^{\frac{\pi}{2}}\frac{3}{2}\sin 2t\,dt=\Big[-\frac{3}{4}\cos 2t\Big]_0^{\frac{\pi}{2}}$$
$$=-\frac{3}{4}\cos\pi+\frac{3}{4}\cos 0=\frac{3}{2}$$

# ＜ 49 ページ. 定積分の応用 1 ＞

## 問 1 の解答

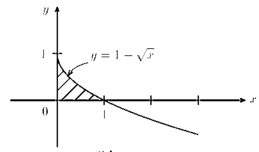
(1) $\displaystyle\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+\cdots+\sqrt{1+\frac{n}{n}}\right)=\int_1^2\sqrt{x}\ dx=\left[\frac{2}{3}x\sqrt{x}\right]_1^2=\frac{2}{3}(2\sqrt{2}-1)$

(2) $\displaystyle\lim_{n\to\infty}\frac{4}{n}\left(\frac{1}{\sqrt{\frac{4}{n}}}+\frac{1}{\sqrt{\frac{8}{n}}}+\cdots+\frac{1}{\sqrt{\frac{4n}{n}}}\right)=\int_0^4\frac{1}{\sqrt{x}}\ dx=\Big[2\sqrt{x}\Big]_0^4=4$

(3) $\displaystyle\lim_{n\to\infty}\frac{\pi}{2n}\left(\sin\left(\frac{\pi}{2n}\right)+\sin\left(\frac{2\pi}{2n}\right)+\cdots+\sin\left(\frac{n\pi}{2n}\right)\right)=\int_0^{\frac{\pi}{2}}\sin(x)\ dx=\Big[-\cos x\Big]_0^{\frac{\pi}{2}}=1$

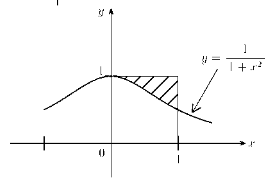
## 問 2 の解答

(1) $\displaystyle\int_0^1(1-\sqrt{x})\ dx=\left[x-\frac{2}{3}x\sqrt{x}\right]_0^1=1-\frac{2}{3}=\frac{1}{3}$

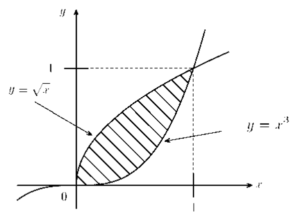



(2) $\displaystyle\int_0^1\left(1-\frac{1}{1+x^2}\right)\ dx=\Big[x-\tan^{-1}(x)\Big]_0^1$

$\displaystyle=\big(1-\tan^{-1}(x)\big)-\big(0-\tan^{-1}0\big)=1-\frac{\pi}{4}$

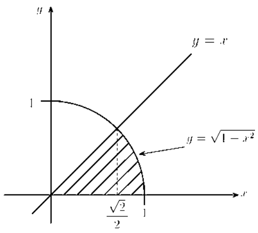


(3) $\displaystyle\int_0^1(\sqrt{x}-x^3)\ dx=\left[\frac{2}{3}x\sqrt{x}-\frac{1}{4}x^4\right]_0^1=\frac{2}{3}-\frac{1}{4}=\frac{5}{12}$



## 問 3 の解答

(1) $\displaystyle\int_0^{\frac{\sqrt{2}}{2}}\pi x^2\ dx+\int_{\frac{\sqrt{2}}{2}}^1\pi\left(\sqrt{1-x^2}\right)^2\ dx=\left[\frac{\pi x^3}{3}\right]_0^{\frac{\sqrt{2}}{2}}+\pi\left[x-\frac{x^3}{3}\right]_{\frac{\sqrt{2}}{2}}^1$



$\displaystyle=\frac{\pi}{3\times2\sqrt{2}}+\pi\left\{\left(1-\frac{1}{3}\right)-\left(\frac{1}{\sqrt{2}}-\frac{1}{3\times2\sqrt{2}}\right)\right\}$

$\displaystyle=\pi\left\{\frac{1}{3\sqrt{2}}+\frac{2}{3}-\frac{1}{\sqrt{2}}\right\}=\left(\frac{2-\sqrt{2}}{3}\right)\pi$

(2) $\displaystyle\int_{-a}^a\pi\left(\sqrt{b^2-(\frac{b}{c}-x)^2}\right)^2\ dx=2\int_0^a\pi\left(b^2-\frac{b^2}{c^2}x^2\right)\ dx=2\pi\left[b^2x-\frac{b^2}{3a^2}x^3\right]_0^a=2\pi\left(ab^2-\frac{ab^2}{3}\right)$

$\displaystyle=\frac{4\pi}{3}ab^2$

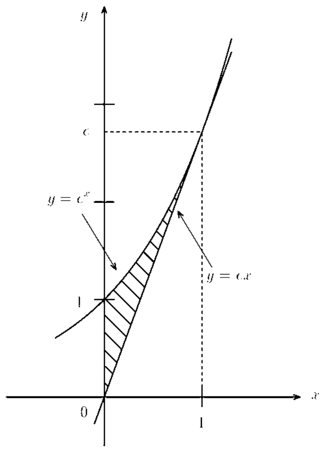
## 問 4 の解答

$\displaystyle S=\int_0^1(e^x-ex)\ dx=\left[e^x-\frac{e}{2}x^2\right]_0^1$

$\displaystyle=\left(e^1-\frac{e}{2}\right)-\left(e^0-0\right)=\frac{e}{2}-1$

$\displaystyle V=\int_0^1\pi(e^x)^2)\ dx-\int_0^1\pi(ex)^2\ dx$

$\displaystyle=\pi\left\{\left[\frac{1}{2}e^{2x}\right]_0^1-\left[\frac{e^2}{3}x^3\right]_0^1\right\}=\left(\frac{e^2}{6}-\frac{1}{2}\right)\pi$



# ＜ 50 ページ. 定積分の応用 2 ＞

## 問 1 の解答

(1) $\Big( x(t)\ ,\ y(t) \Big) = (\ 1 - \cos t\ ,\ 0\ )$

(2) $$\int_0^{2\pi} |v(t)|\ dt = \int_0^{2\pi} |\sin t|\ dt = \int_0^{\pi} \sin t\ dt + \int_{\pi}^{2\pi} (-\sin t)\ dt$$

$$= \Big[-\cos t\Big]_0^{\pi} + \Big[\cos t\Big]_{\pi}^{2\pi}$$

$$= -\cos\pi + \cos 0 + \cos 2\pi - \cos\pi = 4$$

## 問 2 の解答

(1) $\Big( x(t)\ ,\ y(t) \Big) = \Big( t\ ,\ \dfrac{2}{3} t\sqrt{t} \Big)$

(2) $$\int_0^{1} |v(t)|\ dt = \int_0^{1} \sqrt{1+t}\ dt = \Big[\frac{2}{3}(1+t)\sqrt{1+t}\Big]_0^{1}$$

$$= \frac{2}{3}(2\sqrt{2} - 1) = \frac{4\sqrt{2}}{3} - \frac{2}{3}$$

## 問 3 の解答

(1) $$\ell = \int_0^1 \sqrt{\Big(\frac{dx}{dt}\Big)^2 + \Big(\frac{dy}{dt}\Big)^2}\ dt = \int_0^1 \sqrt{(6t^2)^2 + (6t)^2}\ dt$$

$$= \int_0^1 6t\sqrt{t^2+1}\ dt = \Big[2(t^2+1)^{\frac{3}{2}}\Big]_0^1 = 2 \times 2^{\frac{3}{2}} - 2 = 4\sqrt{2} - 2$$

(2) $$\Big(\frac{dx}{dt}\Big)^2 + \Big(\frac{dy}{dt}\Big)^2 = (1-\cos t)^2 + (\sin t)^2 = 1 - 2\cos t + \cos^2 t + \sin^2 t$$

$$= 2 - 2\cos t = 4\sin^2(\frac{t}{2})$$

$$\ell = \int_0^{2\pi} \sqrt{\Big(\frac{dx}{dt}\Big)^2 + \Big(\frac{dy}{dt}\Big)^2}\ dt = \int_0^{2\pi} \sqrt{4\sin^2(\frac{t}{2})}\ dt$$

$$= \int_0^{2\pi} 2\sin(\frac{t}{2})\ dt = \Big[-4\cos(\frac{t}{2})\Big]_0^{2\pi} = -4\cos\pi + 4\cos 0 = 8$$