

高知工科大学

基礎数学ワークブック

(2003年度版)

初級編

No. 3

解答

< 1 ページ. 微分の練習 >

問 1 の解答

(1) 0

(2) rx^{r-1}

(3) $\cos x$

(4) $-\sin x$

(5) $\frac{1}{\cos^2 x}$

(6) $\frac{1}{x}$

(7) $\frac{1}{x} \log_a e$

(8) e^x

(9) $a^x \log_e a$

(10) $\frac{1}{\sqrt{1-x^2}}$

(11) $-\frac{1}{\sqrt{1-x^2}}$

(12) $\frac{1}{1+x^2}$

問 2 の解答

(1) $f'(x) + g'(x)$

(2) $f'(x) - g'(x)$

(3) $kf'(x)$

(4) $f'(x)g(x) + f(x)g'(x)$

(5) $-\frac{f'(x)}{\{f(x)\}^2}$

(6) $\frac{g'(x)f(x) - g(x)f'(x)}{\{f(x)\}^2}$

(7) $r(f(x))^{r-1} \times f'(x)$

(8) $-\frac{rf'(x)}{(f(x))^{r+1}}$

(9) $\frac{f'(x)}{2\sqrt{f(x)}}$

(10) $\cos(f(x)) \times f'(x)$
 $= f'(x) \cos(f(x))$

(11) $-\sin(f(x)) \times f'(x)$
 $= -f'(x) \sin(f(x))$

(12) $\frac{f'(x)}{\cos^2(f(x))}$

(13) $e^{f(x)} \times f'(x)$
 $= f'(x)e^{f(x)}$

(14) $\frac{f'(x)}{f(x)}$

問 3 の解答

(1) $-\frac{1}{2x\sqrt{x}}$

(2) $\frac{3}{2}\sqrt{x}$

(3) $\frac{4}{3}\sqrt[3]{x}$

(4) $4 \cos(4x - 3)$

(5) $\frac{1}{4 \cos^2\left(\frac{x}{4}\right)}$

(6) $-xe^{-\frac{x^2}{2}}$

(7) $\log x$

(8) $\tan x$

(9) $\left(2 \sin(3x) + 3 \cos(3x)\right)e^{2x}$

< 2 ページ. 関数の増減 >

問の解答

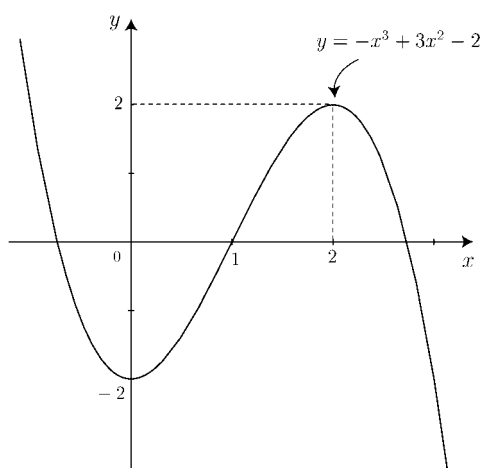
(1) $y = -x^3 + 3x^2 - 2$

$$y' = -3x^2 + 6x$$

$$= -3x(x - 2)$$

 $x = 2$ のとき極大値 $y = 2$ $x = 0$ のとき極小値 $y = -2$

x	...	0	...	2	...
y'	-	0	+	0	-
y	↘	-2	↗	2	↘



(2) $y = -x^4 + 2x^2$

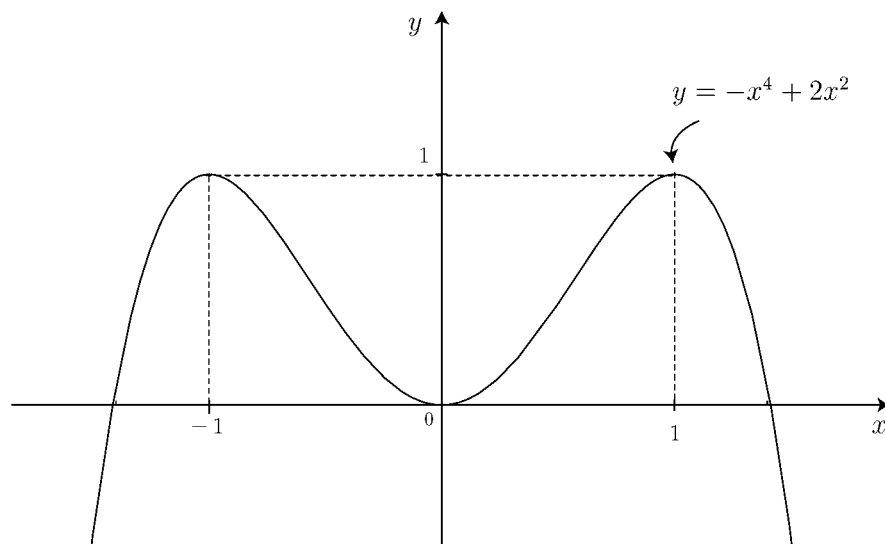
$$y' = -4x^3 + 4x$$

$$= -4x(x^2 - 1)$$

$$= -4x(x - 1)(x + 1)$$

 $x = \pm 1$ のとき極大値 $y = 1$ $x = 0$ のとき極小値 $y = 0$

x	...	-1	...	0	...	1	...
y'	+	0	-	0	+	0	-
y	↗	1	↘	0	↗	1	↘



< 3 ページ. 高階導関数 >

問 1 の解答

(1) $f'(x) = 3x^2 - 3$ (2) $f'(x) = \cos x$ (3) $f'(x) = \frac{1}{x}$

$$f''(x) = 6x$$
$$f''(x) = -\sin x$$
$$f''(x) = -\frac{1}{x^2}$$

問 2 の解答

(1) $f'(x) = 5x^4 - 3x^2 + 1$ (2) $f'(x) = -\sin x$

$$f''(x) = 20x^3 - 6x$$
$$f''(x) = -\cos x$$

$$f'''(x) = 60x^2 - 6$$
$$f'''(x) = \sin x$$

(3) $f'(x) = \log x + 1$ (4) $f'(x) = 2e^{2x}$

$$f''(x) = \frac{1}{x}$$
$$f''(x) = 4e^{2x}$$

$$f'''(x) = -\frac{1}{x^2}$$
$$f'''(x) = 8e^{2x}$$

< 4 ページ. グラフの凹凸 1 >

問の解答

(1) $y = x^3 + 3x^2 - 9x$

$y' = 3x^2 + 6x - 9$

$y'' = 6(x + 1)$

x	...	-1	...
y''	-	0	+
y	凸	11	凹

(2) $y = -x^4 + 2x^3 + 12x^2 - 10$

$y' = -4x^3 + 6x^2 + 24x$

$y'' = -12x^2 + 12x + 24$

$= -12(x - 2)(x + 1)$

x	...	-1	...	2	...
y''	-	0	+	0	-
y	凸	-1	凹	38	凸



< 5 ページ. グラフの凹凸 2 >

問の解答

(1) $y = x^2 - 5x + 6$

$y' = 2x - 5$



$y'' = 2$

x	...	$\frac{5}{2}$...
y'	-	0	+
y''	+	+	+
y		$-\frac{1}{4}$	

(2) $y = -x^2 + 4x - 5$

$y' = -2x + 4 = -2(x - 2)$

$y'' = -2$

x	...	2	...
y'	+	0	-
y''	-	-	-
y		-1	

< 6 ページ. グラフの凹凸 3 >

問の解答





(1) $y = x^3 + 3x^2 - 9x - 10$

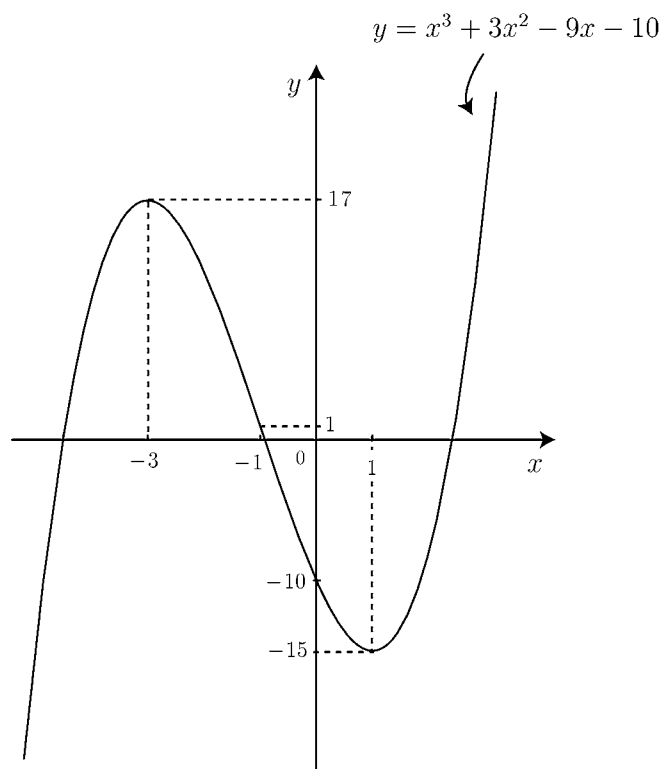
$$y' = 3x^2 + 6x - 9$$

$$= 3(x+3)(x-1)$$

$$y'' = 6(x+1)$$

変曲点は $(-1, 1)$

x	...	-3	...	-1	...	1	...
y'	+	0	-	-	-	0	+
y''	-	-	-	0	+	+	+
y		17		1		-15	

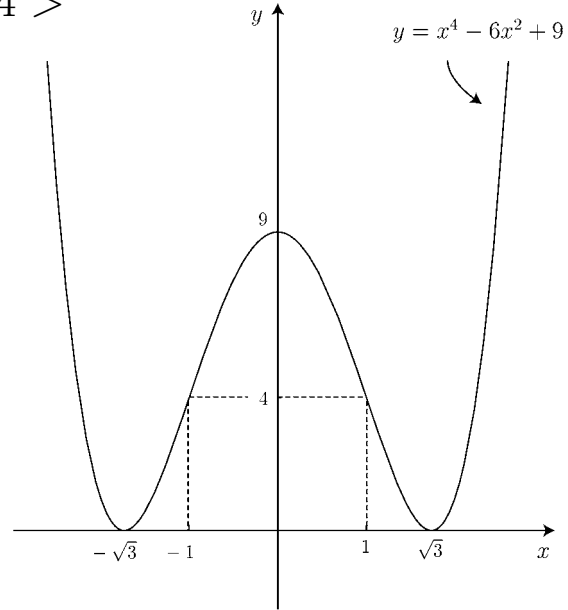


< 7ページ. グラフの凹凸 4 >

問の解答

$$\begin{aligned}
 (1) \quad y &= x^4 - 6x^2 + 9 \\
 y' &= 4x^3 - 12x \\
 &= 4x(x - \sqrt{3})(x + \sqrt{3}) \\
 y'' &= 12x^2 - 12 \\
 &= 12(x - 1)(x + 1)
 \end{aligned}$$

変曲点は $(-1, 4)$ と $(1, 4)$

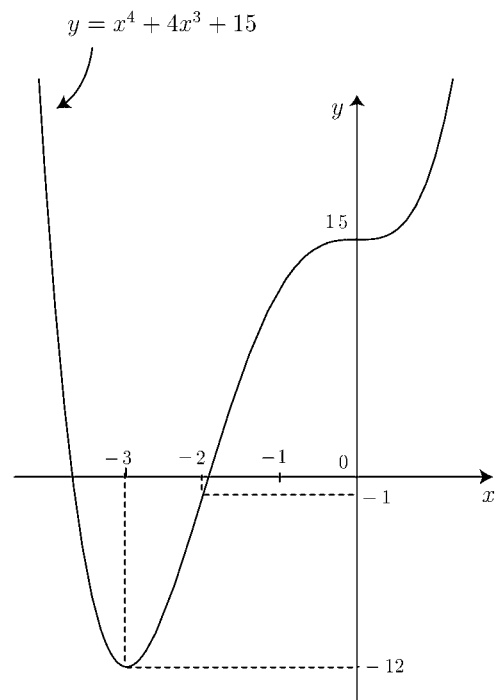


x	...	$-\sqrt{3}$...	-1	...	0	...	1	...	$\sqrt{3}$...
y'	-	0	+	+	+	0	-	-	-	0	+
y''	+	+	+	0	-	-	-	0	+	+	+
y	↪	0	↩	4	↪	9	↩	4	↪	0	↩

$$\begin{aligned}
 (2) \quad y &= x^4 + 4x^3 + 15 \\
 y' &= 4x^3 + 12x^2 = 4x^2(x + 3) \\
 y'' &= 12x^2 + 24x = 12x(x + 2)
 \end{aligned}$$

変曲点は $(-2, -1)$ と $(0, 15)$

x	...	-3	...	-2	...	0	...
y'	-	0	+	+	+	0	+
y''	+	+	+	0	-	0	+
y	↪	-12	↩	-1	↪	15	↩



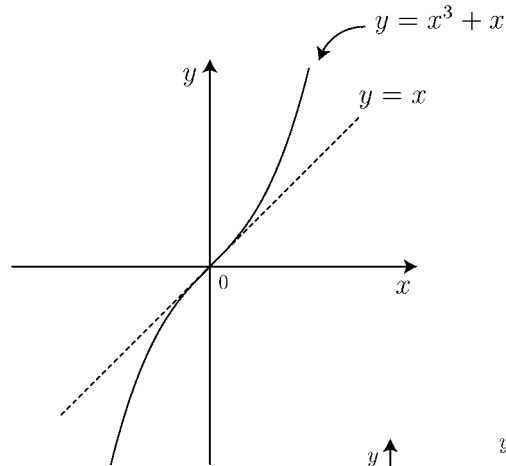
< 8 ページ. グラフの凹凸 5 >

問の解答

(1) $y' = 3x^2 + 1$, $y'' = 6x$

x	...	0	...
y'	+	1	+
y''	-	0	+
y	↪	0	↩

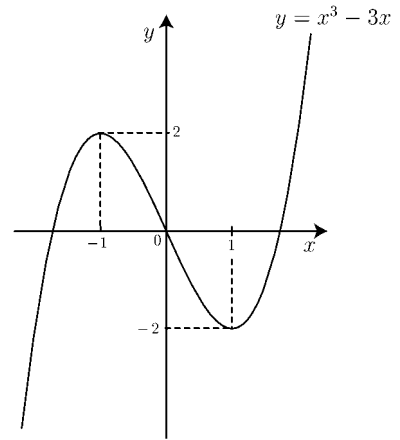
変曲点は $(0, 0)$



(2) $y' = 3(x^2 - 1)$, $y'' = 6x$

x	...	-1	...	0	...	1	...
y'	+	0	-	-	-	0	+
y''	-	-	-	0	+	+	+
y	↪	2	↩	0	↪	-2	↩

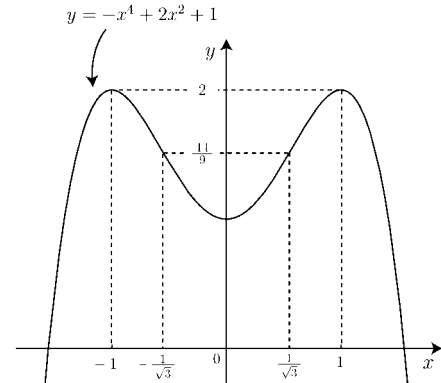
変曲点は $(0, 0)$



(3) $y' = -4x(x^2 - 1)$, $y'' = -4(3x^2 - 1)$

x	...	-1	...	$-\frac{1}{\sqrt{3}}$...	0	...	$\frac{1}{\sqrt{3}}$...	1	...
y'	+	0	-	-	-	0	+	+	+	0	-
y''	-	-	-	0	+	+	+	0	-	-	-
y	↪	2	↩	$\frac{14}{9}$	↩	1	↪	$\frac{14}{9}$	↪	2	↩

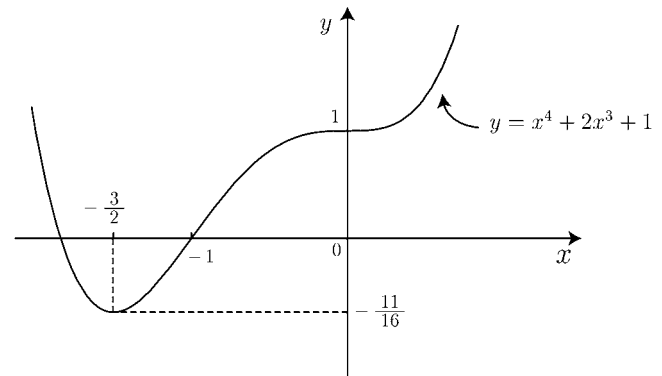
変曲点は $(\frac{1}{\sqrt{3}}, \frac{14}{9})$ と $(-\frac{1}{\sqrt{3}}, \frac{14}{9})$



(4) $y' = 2x^2(2x + 3)$, $y'' = 12x(x + 1)$

x	...	$-\frac{3}{2}$...	-1	...	0	...
y'	-	0	+	+	+	0	+
y''	+	+	+	0	-	0	+
y	↩	$-\frac{1}{16}$	↪	0	↩	1	↪

変曲点は $(-1, 0)$ と $(0, 1)$



< 9 ページ. 微分係数と極限值 >

問の解答

(1) $f(x) = e^x$ とおくと $f'(x) = e^x$ より

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{e^x - e^0}{x - 0} = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = f'(0) = e^0 = 1$$

(2) $f(x) = x^6$ とおくと $f'(x) = 6x^5$ より

$$\lim_{x \rightarrow 2} \frac{x^6 - 64}{x - 2} = \lim_{x \rightarrow 2} \frac{x^6 - 2^6}{x - 2} = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = f'(2) = 6 \times 2^5 = 192$$

(3) $f(x) = \sin x$ とおくと $f'(x) = \cos x$ より

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - \sin(\frac{\pi}{2})}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{f(x) - f(\frac{\pi}{2})}{x - \frac{\pi}{2}} = f'(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) = 0$$

< 10 ページ. ロピタルの定理 1 >

問の解答

$$(1) \lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2} = \lim_{x \rightarrow 2} \frac{4x^3 - 0}{1 - 0} = 4 \times 2^3 = 32$$

$$(2) \lim_{x \rightarrow 1} \frac{e^x - e}{x - 1} = \lim_{x \rightarrow 1} \frac{e^x - 0}{1 - 0} = e^1 = e$$

$$(3) \lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \cos 0 = 1$$

< 11 ページ. ロピタルの定理 2 >

問の解答

$$(1) \lim_{x \rightarrow 1} \frac{x^5 - 1 - 5(x-1)}{(x-1)^2} = \lim_{x \rightarrow 1} \frac{5x^4 - 5}{2(x-1)} = \lim_{x \rightarrow 1} \frac{20x^3}{2} = 10$$

$$(2) \lim_{x \rightarrow 2} \frac{x^5 - 2^5 - 5 \times 2^4(x-2)}{(x-2)^2} = \lim_{x \rightarrow 2} \frac{5x^4 - 80}{2(x-2)} = \lim_{x \rightarrow 2} \frac{20x^3}{2} = 80$$

$$(3) \lim_{x \rightarrow 1} \frac{x^4 - 1 - 4(x-1) - 6(x-1)^2}{(x-1)^3} = \lim_{x \rightarrow 1} \frac{4x^3 - 4 - 12(x-1)}{3(x-1)^2} \\ = \lim_{x \rightarrow 1} \frac{12x^2 - 12}{6(x-1)} = \lim_{x \rightarrow 1} \frac{24x}{6} = 4$$

$$(4) \lim_{x \rightarrow 1} \frac{x^5 - 1 - 5(x-1) - 10(x-1)^2 - 10(x-1)^3}{(x-1)^4} \\ = \lim_{x \rightarrow 1} \frac{5x^4 - 5 - 20(x-1) - 30(x-1)^2}{4(x-1)^3} \\ = \lim_{x \rightarrow 1} \frac{20x^3 - 20 - 60(x-1)}{12(x-1)^2} = \lim_{x \rightarrow 1} \frac{60x^2 - 60}{24(x-1)} = \lim_{x \rightarrow 1} \frac{120x}{24} = 5$$

< 12 ページ. 微分記号 1 >

問の解答

(1) 0

(3) 0

(5) a^4

(7) $4a(ax + b)^3$

(9) $3a^3(x - a)^2$

(11) $2x - 2a$

(12) $3x^2 - 3a^2 - 12a(x - a)$
($= 3x^2 - 12ax + 9a^2$)

(2) $b^4 + 2c^5x$

(4) $(a - b)^2$

(6) $2a^3(x + c)$

(8) $5(x - a)^4$

(10) $16a^3(x - b)^3$

< 13 ページ. 微分記号 2 >

問 1 の解答

(1) $12t^2 + 10t - 2$

(2) $30y^5 - 21y^2 + 32y^3$

(3) $5(t + 4)^4$

(4) $18(3y + 1)^5$

(5) $60(t - 5)^5$

(6) $120(y - 4)^7$

問 2 の解答

(1) $(a - b)^2$

(2) a^4

(3) $2a(at + b)$

(4) $3a(ay - b)^2$

(5) $2a^4(t - 1)$

(6) $3a^5(y - b)^2$

(7) $4a^5(t - a)^3$

(8) $15a^2(y + a)^4$

< 14 ページ. ロピタルの定理 3 >

問の解答

$$(1) \lim_{x \rightarrow a} \frac{x^2 - a^2 - 2a(x - a)}{(x - a)^2} = \lim_{x \rightarrow a} \frac{2x - 2a}{2(x - a)} = 1$$

$$(2) \lim_{x \rightarrow a} \frac{x^4 - a^4 - 4a^3(x - a)}{(x - a)^2} = \lim_{x \rightarrow a} \frac{4x^3 - 4a^3}{2(x - a)} = \lim_{x \rightarrow a} \frac{12x^2}{2} = 6a^2$$

$$(3) \lim_{x \rightarrow a} \frac{x^5 - a^5 - 5a^4(x - a)}{(x - a)^2} = \lim_{x \rightarrow a} \frac{5x^4 - 5a^4}{2(x - a)} = \lim_{x \rightarrow a} \frac{20x^3}{2} = 10a^3$$

< 15 ページ. ロピタルの定理 4 >

問の解答

$$(1) \lim_{y \rightarrow a} \frac{\log y - \log a}{y - a} = \lim_{y \rightarrow a} \frac{\frac{1}{y} - 0}{1} = \frac{1}{a}$$

$$(2) \lim_{t \rightarrow b} \frac{\cos t - \cos b}{t - b} = \lim_{t \rightarrow b} \frac{-\sin t}{1} = -\sin b$$

$$(3) \lim_{\beta \rightarrow a} \frac{a \sin(b\beta) - \beta \sin(ab)}{a^3 - a\beta^2} = \lim_{\beta \rightarrow a} \frac{ab \cos(b\beta) - \sin(ab)}{-2a\beta}$$
$$= \frac{ab \cos(ab) - \sin(ab)}{-2a^2} = \frac{\sin(ab)}{2a^2} - \frac{b \cos(ab)}{2a}$$

< 16 ページ. ロピタルの定理 5 >

問の解答

$$\begin{aligned} (1) \lim_{x \rightarrow a} \frac{x^4 - a^4 - 4a^3(x-a) - 6a^2(x-a)^2}{(x-a)^3} \\ = \lim_{x \rightarrow a} \frac{4x^3 - 4a^3 - 12a^2(x-a)}{3(x-a)^2} = \lim_{x \rightarrow a} \frac{12x^2 - 12a^2}{6(x-a)} = \lim_{x \rightarrow a} \frac{24x}{6} = 4a \end{aligned}$$

$$\begin{aligned} (2) \lim_{x \rightarrow a} \frac{x^6 - a^6 - 6a^5(x-a) - 15a^4(x-a)^2}{(x-a)^3} \\ = \lim_{x \rightarrow a} \frac{6x^5 - 6a^5 - 30a^4(x-a)}{3(x-a)^2} = \lim_{x \rightarrow a} \frac{30x^4 - 30a^4}{6(x-a)} = \lim_{x \rightarrow a} \frac{120x^3}{6} = 20a^3 \end{aligned}$$

$$\begin{aligned} (3) \lim_{x \rightarrow a} \frac{x^7 - a^7 - 7a^6(x-a) - 21a^5(x-a)^2}{(x-a)^3} \\ = \lim_{x \rightarrow a} \frac{7x^6 - 7a^6 - 42a^5(x-a)}{3(x-a)^2} = \lim_{x \rightarrow a} \frac{42x^5 - 52a^5}{6(x-a)} = \lim_{x \rightarrow a} \frac{210x^4}{6} = 35a^4 \end{aligned}$$

< 17 ページ. ロピタルの定理 6 >

問の解答

$$\begin{aligned}
 (1) \quad & \lim_{x \rightarrow a} \frac{x^5 - a^5 - 5a^4(x-a) - 10a^3(x-a)^2 - 10a^2(x-a)^3}{(x-a)^4} \\
 &= \lim_{x \rightarrow a} \frac{5x^4 - 5a^4 - 20a^3(x-a) - 30a^2(x-a)^2}{4(x-a)^3} \\
 &= \lim_{x \rightarrow a} \frac{20x^3 - 20a^3 - 60a^2(x-a)}{12(x-a)^2} = \lim_{x \rightarrow a} \frac{60x^2 - 60a^2}{24(x-a)} \\
 &= \lim_{x \rightarrow a} \frac{120x}{24} = 5a
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \lim_{x \rightarrow a} \frac{x^7 - a^7 - 7a^6(x-a) - 21a^5(x-a)^2 - 35a^4(x-a)^3}{(x-a)^4} \\
 &= \lim_{x \rightarrow a} \frac{7x^6 - 7a^6 - 42a^5(x-a) - 105a^4(x-a)^2}{4(x-a)^3} \\
 &= \lim_{x \rightarrow a} \frac{42x^5 - 42a^5 - 210a^4(x-a)}{12(x-a)^2} = \lim_{x \rightarrow a} \frac{210x^4 - 210a^4}{24(x-a)} \\
 &= \lim_{x \rightarrow a} \frac{840x^3}{24} = 35a^3
 \end{aligned}$$

< 18 ページ. ロピタルの定理 7 >

問の解答

(1) $\lim_{x \rightarrow 1} \frac{5x^4}{1} = 5$

(2) $\lim_{x \rightarrow a} \frac{e^x}{1} = e^a$

(3) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin x}{1}$
 $= -\sin\left(\frac{\pi}{2}\right) = -1$

(4) $\lim_{x \rightarrow 1} \frac{\frac{1}{x}}{1} = 1$

(5) $\lim_{x \rightarrow 2} \frac{4x^3 - 32}{2(x-2)} = \lim_{x \rightarrow 2} \frac{12x^2}{2} = 24$

(6) $\lim_{x \rightarrow 1} \frac{e^x - e}{2(x-1)} = \lim_{x \rightarrow 1} \frac{e^x}{2} = \frac{e}{2}$

(7) $\lim_{x \rightarrow 1} \frac{\frac{1}{x} - 1}{2(x-1)} = \lim_{x \rightarrow 1} \frac{-\frac{1}{x^2}}{2} = -\frac{1}{2}$

(8) $\lim_{x \rightarrow 4} \frac{\frac{1}{2\sqrt{x}} - \frac{1}{4}}{2(x-4)} = \lim_{x \rightarrow 4} \frac{-\frac{1}{4x\sqrt{x}}}{2} = -\frac{1}{8 \times 4\sqrt{4}} = -\frac{1}{64}$

(9) $\lim_{x \rightarrow a} \frac{e^x - e^a - e^a(x-a)}{3(x-a)^2}$
 $= \lim_{x \rightarrow a} \frac{e^x - e^a}{6(x-a)} = \lim_{x \rightarrow a} \frac{e^x}{6} = \frac{e^a}{6}$

(10) $\lim_{x \rightarrow a} \frac{\cos x - \cos a + (\sin a)(x-a)}{3(x-a)^2}$
 $= \lim_{x \rightarrow a} \frac{-\sin x + \sin a}{6(x-a)} = \lim_{x \rightarrow a} \frac{-\cos x}{6} = -\frac{\cos a}{6}$

(11) $\lim_{x \rightarrow a} \frac{8x^7 - 8a^7 - 56a^6(x-a) - 168a^5(x-a)^2}{4(x-a)^3} = \lim_{x \rightarrow a} \frac{56x^6 - 56a^6 - 336a^5(x-a)}{12(x-a)^2}$
 $= \lim_{x \rightarrow a} \frac{336x^5 - 336a^5}{24(x-a)} = \lim_{x \rightarrow a} \frac{336 \times 5x^4}{24} = \lim_{x \rightarrow a} 70x^4 = 70a^4$

< 19 ページ. 接線の方程式 >

問の解答

- (1) $y' = e^x = e^0 = 1$ $y = 1(x - 0) + e^0$ (答) $y = x + 1$
- (2) $y' = \frac{1}{x} = \frac{1}{1} = 1$ $y = 1(x - 1) + \log 1$ (答) $y = x - 1$
- (3) $y' = -\sin x = -\sin \frac{\pi}{2} = -1$ $y = -1(x - \frac{\pi}{2}) + \cos \frac{\pi}{2}$ (答) $y = -x + \frac{\pi}{2}$
- (4) $y' = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{4}}$ $y = \frac{1}{4}(x - 4) + \sqrt{4}$ (答) $y = \frac{1}{4}x + 1$
- (5) $y = e^a(x - a) + e^a$ (答) $y = e^a x + (1 - a)e^a$
- (6) $y = \cos a(x - a) + \sin a$ (答) $y = (\cos a)x - a \cos a + \sin a$
- (7) $y = \frac{1}{a}(x - a) + \log a$ (答) $y = \frac{1}{a}x - 1 + \log a$
- (8) $y = \frac{1}{2\sqrt{a}}(x - a) + \sqrt{a}$ (答) $y = \frac{1}{2\sqrt{a}}x + \frac{1}{2}\sqrt{a}$
- (9) $y = \frac{1}{3\sqrt[3]{a^2}}(x - a) + \sqrt[3]{a}$ (答) $y = \frac{1}{3\sqrt[3]{a^2}}x + \frac{2}{3}\sqrt[3]{a}$

< 20 ページ. 関数の 1 次近似 1 >

問の解答

$$(1) f'(a) = 6a^5 \text{ より } x \doteq a \text{ のとき} \quad x^6 \doteq a^6 + 6a^5(x - a) = 6a^5x - 5a^6$$

$$(2) f'(a) = -\frac{1}{a^2} \text{ より } x \doteq a \text{ のとき} \quad \frac{1}{x} \doteq \frac{1}{a} - \frac{1}{a^2}(x - a) = -\frac{1}{a^2}x + \frac{2}{a}$$

$$(3) f'(a) = \frac{1}{2\sqrt{a}} \text{ より } x \doteq a \text{ のとき} \quad \sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a) = \frac{1}{2\sqrt{a}}x + \frac{\sqrt{a}}{2}$$

$$(4) f'(a) = \frac{1}{4\sqrt[4]{a^3}} \text{ より } x \doteq a \text{ のとき} \quad \sqrt[4]{x} \doteq \sqrt[4]{a} + \frac{1}{4\sqrt[4]{a^3}}(x - a) \\ = \frac{1}{4\sqrt[4]{a^3}}x + \frac{3}{4}\sqrt[4]{a}$$

< 21 ページ. 関数の 1 次近似 2 >

問 1 の解答

(1) $x \doteq a$ のとき $\cos x \doteq \cos a - (\sin a)(x - a)$

(2) $x \doteq a$ のとき $\tan x \doteq \tan a + \frac{1}{\cos^2 a}(x - a)$

(3) $x \doteq a$ のとき $\log x \doteq \log a + \frac{1}{a}(x - a)$

(4) $x \doteq a$ のとき $e^x \doteq e^a + e^a(x - a)$

問 2 の解答

(1) $x \doteq 0$ のとき $\sin x \doteq x$

(2) $x \doteq \frac{\pi}{2}$ のとき $\cos x \doteq -(x - \frac{\pi}{2}) = -x + \frac{\pi}{2}$

(3) $x \doteq 0$ のとき $e^x \doteq 1 + x$

(4) $x \doteq 1$ のとき $\log x \doteq x - 1$

(5) $x \doteq 1$ のとき $\sqrt{x} \doteq 1 + \frac{1}{2}(x - 1) = \frac{1}{2}x + \frac{1}{2}$

(6) $x \doteq 1$ のとき $\sqrt[3]{x} \doteq 1 + \frac{1}{3}(x - 1) = \frac{1}{3}x + \frac{2}{3}$

< 22 ページ.1 次近似値 >

問の解答

$$(1) \sqrt[3]{x} \doteq \sqrt[3]{a} + \frac{1}{3\sqrt[3]{a^2}}(x-a) \quad \text{で } x = 1.1, a = 1 \text{ とおくと}$$

$$\sqrt[3]{1.1} \doteq \sqrt[3]{1} + \frac{1}{3\sqrt[3]{1^2}}(1.1-1) = 1 + \frac{1}{3} \times 0.1 = 1 + \frac{1}{30} \doteq \underline{1.03}$$

$$(2) \sqrt[4]{x} \doteq \sqrt[4]{a} + \frac{1}{4\sqrt[4]{a^3}}(x-a) \quad \text{で } x = 1.1, a = 1 \text{ とおくと}$$

$$\sqrt[4]{1.1} \doteq \sqrt[4]{1} + \frac{1}{4\sqrt[4]{1^3}}(1.1-1) = 1 + \frac{1}{4} \times 0.1 = 1 + \frac{1}{40} = \underline{1.025}$$

$$(3) \log x \doteq \log a + \frac{1}{a}(x-a) \quad \text{で } x = 1.1, a = 1 \text{ とおくと}$$

$$\log 1.1 \doteq \log 1 + \frac{1}{1}(1.1-1) = 0 + 0.1 = \underline{0.1}$$

$$(4) e^x \doteq e^a + e^a(x-a) \quad \text{で } x = 0.1, a = 0 \text{ とおくと}$$

$$e^{0.1} \doteq e^0 + e^0(0.1-0) = 1 + 1 \times 0.1 = \underline{1.1}$$

$$(5) \sin x \doteq \sin a + \cos a(x-a) \quad \text{で } x = 0.1, a = 0 \text{ とおくと}$$

$$\sin 0.1 \doteq \sin 0 + \cos 0(0.1-0) = 0 + 1 \times 0.1 = \underline{0.1}$$

$$(6) \cos x \doteq \cos a + \sin a(x-a) \quad \text{で } x = 0.1, a = 0 \text{ とおくと}$$

$$\cos 0.1 \doteq \cos 0 + \sin 0(0.1-0) = 1 - 0 \times 0.1 = \underline{1}$$

< 23 ページ. 関数の高次近似 1 >

問の解答

(1) $f'(x) - f'(a)$

(2) $f''(x) - f''(a)$

(3) $f'(x) - f'(a) - f''(a)(x - a)$

(4) $f''(x) - f''(a) - f'''(a)(x - a)$

(5) $f'(x) - f'(a) - f''(a)(x - a) - \frac{1}{2}f'''(a)(x - a)^2$

< 24 ページ. 関数の高次近似 2 >

問の解答

$$\begin{aligned}
 (1) \quad & \lim_{x \rightarrow a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2}{(x-a)^3} \\
 &= \lim_{x \rightarrow a} \frac{f'(x) - f'(a) - f''(a)(x-a)}{3(x-a)^2} \\
 &= \lim_{x \rightarrow a} \frac{f''(x) - f''(a)}{6(x-a)} = \lim_{x \rightarrow a} \frac{f'''(x)}{6} = \frac{f'''(a)}{6}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \lim_{x \rightarrow a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2 - \frac{1}{6}f'''(a)(x-a)^3}{(x-a)^4} \\
 &= \lim_{x \rightarrow a} \frac{f'(x) - f'(a) - f''(a)(x-a) - \frac{1}{2}f'''(a)(x-a)^2}{4(x-a)^3} \\
 &= \lim_{x \rightarrow a} \frac{f''(x) - f''(a) - f'''(a)(x-a)}{12(x-a)^2} \\
 &= \lim_{x \rightarrow a} \frac{f'''(x) - f'''(a)}{24(x-a)} = \lim_{x \rightarrow a} \frac{f''''(x)}{24} = \frac{f''''(a)}{24}
 \end{aligned}$$

< 25 ページ. 関数の高次近似 3 >

問の解答

$$f(x) \doteq f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3 + \frac{1}{24}f''''(a)(x-a)^4$$

< 26 ページ. 高次微分係数 >

問の解答

$$(1) f^{(4)}(x) = e^x \quad , \quad f^{(4)}(0) = e^0 = 1$$

$$(2) f^{(n)}(x) = e^x \quad , \quad f^{(n)}(0) = e^0 = 1$$

$$(3) f^{(1)}(x) = \cos x \quad , \quad f^{(2)}(x) = -\sin x \quad , \quad f^{(3)}(x) = -\cos x \quad , \quad f^{(4)}(x) = \sin x$$

$$f^{(5)}(x) = \cos x \quad , \quad f^{(6)}(x) = -\sin x \quad , \quad f^{(7)}(x) = -\cos x \quad , \quad f^{(8)}(x) = \sin x$$

$$f^{(1)}(0) = \cos 0 = 1 \quad , \quad f^{(2)}(0) = -\sin 0 = 0 \quad , \quad f^{(3)}(0) = -\cos 0 = -1 \quad , \quad f^{(4)}(0) = \sin 0 = 0$$

$$f^{(5)}(0) = \cos 0 = 1 \quad , \quad f^{(6)}(0) = -\sin 0 = 0 \quad , \quad f^{(7)}(0) = -\cos 0 = -1 \quad , \quad f^{(8)}(0) = \sin 0 = 0$$

< 27 ページ. 関数の n 次近似 1 >

問の解答

$$f(x) \doteq f(a) + \frac{1}{1!}f^{(1)}(a)(x-a) + \frac{1}{2!}f^{(2)}(a)(x-a)^2 + \frac{1}{3!}f^{(3)}(a)(x-a)^3 + \frac{1}{4!}f^{(4)}(a)(x-a)^4$$

< 28 ページ. 関数の n 次近似 2 >

問の解答

$$(1) e^x \doteq e^a + e^a(x-a) + \frac{e^a}{2!}(x-a)^2 + \frac{e^a}{3!}(x-a)^3 + \cdots + \frac{1}{n!}e^a(x-a)^n$$

$$(2) e^x \doteq e + e(x-1) + \frac{e}{2!}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \cdots + \frac{e}{n!}(x-1)^n$$

$$(3) e^x \doteq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

< 29 ページ. テーラー展開 >

問 1 の解答

$$e^x = e^a + e^a(x-a) + \frac{1}{2!}e^a(x-a)^2 + \frac{1}{3!}e^a(x-a)^3 + \cdots + \frac{1}{n!}e^a(x-a)^n + \cdots$$

問 2 の解答

$$(1) e^x = e + e(x-1) + \frac{1}{2!}e(x-1)^2 + \frac{1}{3!}e(x-1)^3 + \cdots + \frac{1}{n!}e(x-1)^n + \cdots$$

$$(2) e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$$

< 30 ページ. マクローリン展開 1 >

問の解答

$$\begin{aligned}\sin x &= \sin 0 + \frac{\cos 0}{1!}x - \frac{\sin 0}{2!}x^2 - \frac{\cos 0}{3!}x^3 + \frac{\sin 0}{4!}x^4 \\ &\quad + \frac{\cos 0}{5!}x^5 - \frac{\sin 0}{6!}x^6 - \frac{\cos 0}{7!}x^7 + \frac{\sin 0}{8!}x^8 + \cdots \\ &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} + \cdots\end{aligned}$$

< 32 ページ. マクローリン展開 3 >

問の解答

$$e \doteq 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$$

$$= 2 + \frac{12 + 4 + 1}{24}$$

$$= \frac{65}{24}$$

$$\doteq 2.708$$

< 33 ページ. 近似の練習 1 >

問 1 の解答

$$(1) x \doteq a \text{ のとき } \cos x \doteq \cos a - (\sin a)(x - a) - \frac{1}{2}(\cos a)(x - a)^2$$

$$(2) x \doteq a \text{ のとき } e^x \doteq e^a + e^a(x - a) + \frac{1}{2}e^a(x - a)^2$$

$$(3) x \doteq a \text{ のとき } \log x \doteq \log a + \frac{1}{a}(x - a) - \frac{1}{2a^2}(x - a)^2$$

$$(4) x \doteq a \text{ のとき } \sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a) - \frac{1}{8a\sqrt{a}}(x - a)^2$$

$$(5) x \doteq a \text{ のとき } e^{-x} \doteq e^{-a} - e^{-a}(x - a) + \frac{1}{2}e^{-a}(x - a)^2$$

問 2 の解答

$$(1) x \doteq \pi \text{ のとき } \cos x \doteq \cos \pi - (\sin \pi)(x - \pi) - \frac{1}{2}(\cos \pi)(x - \pi)^2 \\ = -1 + \frac{1}{2}(x - \pi)^2$$

$$(2) x \doteq 1 \text{ のとき } e^x \doteq e^1 + e^1(x - 1) + \frac{1}{2}e^1(x - 1)^2 \\ = \frac{e}{2}x^2 + \frac{e}{2}$$

$$(3) x \doteq 1 \text{ のとき } \log x \doteq \log 1 + \frac{1}{1}(x - 1) - \frac{1}{2 \times 1^2}(x - 1)^2 \\ = -\frac{1}{2}x^2 + 2x - \frac{3}{2}$$

$$(4) x \doteq 1 \text{ のとき } \sqrt{x} \doteq \sqrt{1} + \frac{1}{2\sqrt{1}}(x - 1) - \frac{1}{8 \times 1\sqrt{1}}(x - 1)^2 \\ = -\frac{1}{8}x^2 + \frac{3}{4}x + \frac{3}{8}$$

< 34 ページ. 近似の練習 2 (1) >

問 1 の解答

(1) $x \doteq 0$ のとき

$$e^{-x} \doteq e^{-0} - e^{-0}(x-0) + \frac{1}{2}e^{-0}(x-0)^2 = 1 - x + \frac{1}{2}x^2$$

(2) $f'(x) = -2xe^{-x^2}$, $f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$, $x \doteq 0$ のとき

$$\begin{aligned} e^{-x^2} &\doteq e^{-0^2} - 2 \times 0 \times e^{-0^2}(x-0) + \frac{-2e^{-0^2} + 4 \times 0^2 \times e^{-0^2}}{2}(x-0)^2 \\ &= 1 - x^2 \end{aligned}$$

(3) $f'(x) = \cos x - \sin x$, $f''(x) = -\sin x - \cos x$

$$f'(0) = 1 \text{ , } f''(0) = -1$$

 $x \doteq 0$ のとき

$$\sin x + \cos x \doteq 1 + 1(x-0) - \frac{1}{2}(x-0)^2 = -\frac{1}{2}x^2 + x + 1$$

(4) $f'(x) = \frac{1}{1+x}$, $f''(x) = -\frac{1}{(1+x)^2}$

$$f'(0) = 1 \text{ , } f''(0) = -1$$

$$\log(1+x) \doteq \log 1 + 1(x-0) - 1 \times \frac{1}{2}(x-0)^2 = -\frac{1}{2}x^2 + x$$

問 2 の解答

$$\sqrt{x} \doteq \sqrt{1} + \frac{1}{2\sqrt{1}}(x-1) - \frac{1}{8 \times 1\sqrt{1}}(x-1)^2 = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$$

 $x = 1.1$ とおくと

$$\sqrt{1.1} \doteq 1 + \frac{1}{2} \times 0.1 - \frac{1}{8} \times (0.1)^2 = 1 + 0.05 - 0.00125 = 1.04875$$

問 3 の解答

$$\log x \doteq \log 1 + \frac{1}{1}(x-1) - \frac{1}{2 \times 1^2}(x-1)^2 = (x-1) - \frac{1}{2}(x-1)^2$$

 $x = 1.1$ とおくと

$$\log 1.1 \doteq 0.1 - \frac{1}{2} \times (0.1)^2 = 0.1 - 0.005 = 0.095$$

< 34 ページ. 近似の練習 2 (2) >

問 4 の解答

$$\cos x \doteq \cos 0 - \sin 0(x - 0) - \frac{1}{2} \cos 0(x - 0)^2 = 1 - \frac{1}{2}x^2$$

$$\cos(0.1) \doteq 1 - \frac{1}{2} \times (0.1)^2 = 1 - 0.005 = 0.995$$

問 5 の解答

$$e^x \doteq e^0 + e^0 x + \frac{1}{2!} e^0 x^2 + \frac{1}{3!} e^0 x^3 = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

$$\sqrt{e} = e^{\frac{1}{2}} \doteq 1 + \frac{1}{2} + \frac{1}{2} \times \left(\frac{1}{2}\right)^2 + \frac{1}{6} \times \left(\frac{1}{2}\right)^3 = \frac{79}{48} \doteq 1.6458$$

問 6 の解答

$$e^x \doteq 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!} + \frac{1}{4!}x^4$$

$$\frac{1}{e} = e^{-1} \doteq 1 - 1 + \frac{1}{2} \times (-1)^2 + \frac{1}{6} \times (-1)^3 + \frac{1}{24} \times (-1)^4 = \frac{3}{8} = 0.375$$

問 7 の解答

(1) e

(2) $e^{-1} = \frac{1}{e}$

(3) $\sin(1)$

(4) $\cos(1)$

< 35 ページ. 不定積分 1 >

問の解答

$$(1) \left(\frac{1}{\alpha+1} x^{\alpha+1} \right)' = x^\alpha \quad \Rightarrow \quad \int x^\alpha dx = \frac{1}{\alpha+1} x^{\alpha+1} + C$$

$$(2) (\log |x|)' = \frac{1}{x} \quad \Rightarrow \quad \int \frac{1}{x} dx = \log |x| + C$$

$$(3) (\sin x)' = \cos x \quad \Rightarrow \quad \int \cos x dx = \sin x + C$$

$$(4) (-\cos x)' = \sin x \quad \Rightarrow \quad \int \sin x dx = -\cos x + C$$

$$(5) (e^x)' = e^x \quad \Rightarrow \quad \int e^x dx = e^x + C$$

< 36 ページ. 不定積分 2 >

問の解答

$$(1) \frac{1}{7}x^7 + C$$

$$(2) \frac{4}{5}x^{\frac{5}{4}} + C = \frac{4}{5}x\sqrt[4]{x} + C$$

$$(3) 2x^{\frac{1}{2}} + C = 2\sqrt{x} + C$$

$$(4) \int x^{-3} dx = \frac{1}{-2}x^{-2} + C = -\frac{1}{2x^2} + C$$

$$(5) \int x^{\frac{2}{3}} dx = \frac{3}{5}x^{\frac{5}{3}} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$$

$$(6) \int x^{-\frac{1}{4}} dx = \frac{4}{3}x^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{x^3} + C$$

< 37 ページ. 不定積分 3 >

問の解答

$$(1) \int \left(\frac{1}{x} - \frac{4}{x^2} + \frac{1}{x^3} \right) dx = \log |x| + \frac{4}{x} - \frac{1}{2x^2} + C$$

$$(2) \int \frac{x^4 - 4x^2 + 3}{x^4} dx = \int \left(1 - \frac{4}{x^2} + \frac{3}{x^4} \right) dx = x + \frac{4}{x} - \frac{1}{x^3} + C$$

$$(3) \int \left(\sqrt{x} + \frac{2}{\sqrt{x}} \right) dx = \int \left(x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx = \frac{2}{3}x^{\frac{3}{2}} + 2 \times \frac{1}{\frac{1}{2}}x^{\frac{1}{2}} + C = \frac{2}{3}x\sqrt{x} + 4\sqrt{x} + C$$

$$(4) \int \frac{x - 2\sqrt{x} + 1}{x} dx = \int \left(1 - \frac{2}{\sqrt{x}} + \frac{1}{x} \right) dx = x - 4\sqrt{x} + \log |x| + C$$

< 38 ページ. 不定積分 4 >

問 1 の解答

$$(1) (\tan x)' = \frac{1}{\cos^2 x} \quad \Rightarrow \quad \int \frac{dx}{\cos^2 x} = \tan x + C$$

$$(2) \left(\frac{1}{\tan x}\right)' = -\frac{\frac{1}{\cos^2 x}}{\tan^2 x} = -\frac{1}{\sin^2 x} \quad \Rightarrow \quad \int \frac{dx}{\sin^2 x} = -\frac{1}{\tan x} + C \quad \left(= -\cot x + C \right)$$

$$(3) (a^x)' = a^x \log_e a \quad \Rightarrow \quad \int a^x dx = \frac{a^x}{\log_e a} + C$$

$$(4) (\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}} \quad \Rightarrow \quad \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$(5) (\tan^{-1} x)' = \frac{1}{1+x^2} \quad \Rightarrow \quad \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

問 2 の解答

$$(1) -4 \cos x - 3 \sin x + C$$

$$(2) \int \left(3 \cos x - \frac{1}{\cos^2 x} \right) dx = 3 \sin x - \tan x + C$$

$$(3) \int (2 \cos x - \sin x) dx = 2 \sin x + \cos x + C$$

$$(4) \int \frac{1}{-\cos^2 x} dx = -\tan x + C$$

$$(5) \int \left(\frac{1}{\sin^2 x} - 1 \right) dx = -\frac{1}{\tan x} - x + C = -\cot x - x + C$$

$$(6) \frac{3^x}{\log_e 3} - 2e^x + C$$

$$(7) 3 \sin^{-1} x + C$$

$$(8) 5 \tan^{-1} x + C$$

< 39 ページ. 積分記号 >

問の解答

(1) $10t - 4.9t^2 + C$

(2) $\frac{4}{3}\pi r^3 + C$

(3) $e^u + C$

(4) $\log |y| + C$

(5) $\sin u + C$

< 40 ページ. 置換積分法 1 >

問の解答

$$(1) \int u^3 \times \frac{1}{5} du = \frac{1}{20} u^4 + C = \frac{1}{20} (5x + 6)^4 + C$$

$$\left(\begin{array}{l} u = 5x + 6 \\ \frac{du}{dx} = 5 \Rightarrow dx = \frac{1}{5} du \end{array} \right)$$

$$(2) -\frac{1}{21(7x+5)^3} + C$$

$$(3) \frac{2}{15} (5x-3)\sqrt{5x-3} + C$$

$$(4) -\frac{1}{3} \cos(3x+2) + C$$

$$(5) -\frac{1}{3} e^{-3x+2} + C$$

$$(6) \frac{1}{4} \tan(4x+3) + C$$

< 41 ページ. 置換積分法 2 >

問の解答

(1) $\frac{1}{2}e^{x^2+1} + C$

(2) $\frac{1}{4}e^{x^4} + C$

(3) $\frac{1}{3}\sin(x^3 + 2) + C$

(4) $-\frac{1}{2}\cos(x^2 + 3) + C$

(5) $\frac{1}{2}\log(x^2 + 3) + C$

(6) $\frac{1}{12}(x^2 + 1)^6 + C$

< 42 ページ. 分数関数の積分 1 >

問の解答

(1) $\log|x+1| + C$

(2) $-\log|x-2| + C$

(3) $\frac{1}{2}\log|2x+3| + C$

(4) $-\frac{1}{4}\log|4x-3| + C$

(5) $\frac{1}{3-x} + C$

(6) $\frac{1}{4-x} + C$

(7) $-\frac{1}{3(3x-4)} + C$

(8) $\frac{1}{5(3-5x)} + C$

(9) $\tan^{-1}(x-2) + C$

(10) $\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$

(11) $\frac{1}{6}\tan^{-1}\left(\frac{2x+1}{3}\right) + C$

(12) $\frac{1}{6}\tan^{-1}\left(\frac{3x-5}{2}\right) + C$

< 43 ページ. 分数関数の積分 2 >

問の解答

$$(1) \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx = \log \left| \frac{x}{x+1} \right| + C$$

$$(2) \int \frac{1}{2} \left\{ \frac{1}{x-1} - \frac{1}{x+1} \right\} dx = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$$

$$(3) \int \left(\frac{1}{x-3} - \frac{1}{x-2} \right) dx = \log \left| \frac{x-3}{x-2} \right| + C$$

$$(4) \int \frac{1}{7} \left\{ \frac{1}{x-3} - \frac{1}{x+4} \right\} dx = \frac{1}{7} \log \left| \frac{x-3}{x+4} \right| + C$$

$$\begin{aligned} (5) \int \frac{1}{(2x+1)(3x+4)} dx &= \int \left(\frac{\frac{2}{5}}{2x+1} - \frac{\frac{3}{5}}{3x+4} \right) dx \\ &= \frac{1}{5} \int \left(\frac{2}{2x+1} - \frac{3}{3x+4} \right) dx \\ &= \frac{1}{5} \left\{ \log |2x+1| - \log |3x+4| \right\} + C \\ &= \frac{1}{5} \log \left| \frac{2x+1}{3x+4} \right| + C \end{aligned}$$

< 44 ページ. 部分積分法 1 >

問の解答

$$(1) \int x \times (-\cos x)' dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$$

$$(2) \int x \times (e^x)' dx = x e^x - \int e^x dx = x e^x - e^x + C$$

$$(3) \int (\log x) \times \left(\frac{x^3}{3}\right)' dx = (\log x) \times \frac{x^3}{3} - \int \frac{1}{x} \times \frac{x^3}{3} dx = \frac{x^3}{3} \log x - \frac{x^3}{9} + C$$

$$(4) \int x \left(\frac{\sin(2x)}{2}\right)' dx = x \times \frac{\sin(2x)}{2} - \int \frac{\sin(2x)}{2} dx = \frac{x \sin(2x)}{2} + \frac{\cos(2x)}{4} + C$$

$$(5) \int x \times \left(\frac{e^{3x}}{3}\right)' dx = x \times \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx = \frac{x}{3} e^{3x} - \frac{1}{9} e^{3x} + C$$

< 45 ページ. 部分積分法 2 >

問 1 の解答

$$\begin{aligned}(1) \int \log(2x) \times x' dx &= (\log 2x) \times x - \int \frac{1}{x} \times x dx \\ &= x \log(2x) - x + C\end{aligned}$$

$$\begin{aligned}(2) \int \log(x^3) \times x' dx &= \log(x^3) \times x - \int \frac{3x^2}{x^3} \times x dx \\ &= x \log(x^3) - 3x + C \\ &\left(= 3(x \log x - x) + C \right)\end{aligned}$$

問 2 の解答

$$\begin{aligned}(1) \int x^2 \sin x dx &= -x^2 \cos x + \int 2x \cos x dx \\ &= -x^2 \cos x + 2x \sin x - \int 2 \sin x dx \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + C\end{aligned}$$

$$\begin{aligned}(2) \int x^2 e^x &= x^2 e^x - \int 2x e^x dx \\ &= x^2 e^x - \left\{ 2x e^x - \int 2e^x dx \right\} \\ &= x^2 e^x - 2x e^x + 2e^x + C\end{aligned}$$

< 46 ページ. 三角関数の不定積分 >

問の解答

$$(1) \int \left(\frac{1}{2} - \frac{1}{2} \cos(2x) \right) dx = \frac{1}{2}x - \frac{1}{4} \sin(2x) + C$$

$$(2) \int \frac{1}{2} \left\{ \cos(5x) + \cos x \right\} dx = \frac{1}{10} \sin(5x) + \frac{1}{2} \sin x + C$$

$$(3) \int \frac{1}{2} \left\{ \cos(3x) - \cos(5x) \right\} dx = \frac{1}{6} \sin(3x) - \frac{1}{10} \sin(5x) + C$$

$$(4) \int \frac{1}{2} \left\{ \sin(7x) + \sin x \right\} dx = -\frac{1}{14} \cos(7x) - \frac{1}{2} \cos x + C$$

$$(5) \int \frac{1}{2} \left\{ 1 + \cos(6x) \right\} dx = \frac{x}{2} + \frac{1}{12} \sin(6x) + C$$

$$(6) \int \frac{1}{2} \left\{ 1 - \cos(8x) \right\} dx = \frac{x}{2} - \frac{1}{16} \sin(8x) + C$$

< 47 ページ. 上半円の積分 >

問の解答

(1) $x = \sin \theta$ とおくと

$$\begin{aligned}
\int \sqrt{1-x^2} dx &= \int \sqrt{1-\sin^2 \theta} \cos \theta d\theta \\
&= \int \cos^2 \theta d\theta \\
&= \int \frac{1+\cos(2\theta)}{2} d\theta \\
&= \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C \\
&= \frac{1}{2}\theta + \frac{1}{2}\sin \theta \sqrt{1-\sin^2 \theta} + C \\
&= \frac{1}{2}\sin^{-1} x + \frac{1}{2}x\sqrt{1-x^2} + C
\end{aligned}$$

(2) $x = a \sin \theta$ とおくと

$$\begin{aligned}
\int \sqrt{a^2-x^2} dx &= \int \sqrt{a^2-a^2\sin^2 \theta} a \cos \theta d\theta \\
&= \int a^2 \cos^2 \theta d\theta \\
&= a^2 \int \cos^2 \theta d\theta \\
&= a^2 \left\{ \frac{1}{2}\theta + \frac{1}{2}\sin \theta \sqrt{1-\sin^2 \theta} \right\} + C \\
&= a^2 \left\{ \frac{1}{2}\sin^{-1} \left(\frac{x}{a} \right) + \frac{1}{2} \times \frac{x}{a} \sqrt{1-\frac{x^2}{a^2}} \right\} + C \\
&= \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + \frac{1}{2}x\sqrt{a^2-x^2} + C
\end{aligned}$$

< 48 ページ. 不定積分の検証 >

問の解答

$$\begin{aligned} (1) \left(\frac{1}{4}(x^4 - 1)^4 \right)' &= \frac{1}{4} \times 4(x^4 - 1)^3 \times 4x^3 \\ &= 4x^3(x^4 - 1)^3 \quad \text{より正しくない} \end{aligned}$$

$$\begin{aligned} (2) \left(\frac{1}{2} \log |x^2 - 1| \right)' &= \frac{1}{2} \times \frac{2x}{x^2 - 1} \\ &= \frac{x}{x^2 - 1} \quad \text{より正しい} \end{aligned}$$

$$\begin{aligned} (3) (x^2 e^x - 2x e^x + 2e^x)' &= 2x e^x + x^2 e^x - 2e^x - 2x e^x + 2e^x \\ &= x^2 e^x \quad \text{より正しい} \end{aligned}$$

< 49 ページ. 不定積分の練習 1 >

問の解答

(1) $\frac{1}{6}x^6 + \frac{1}{8}x^8 + C$

(2) $-x^{-1} + C = -\frac{1}{x} + C$

(3) $\frac{3}{4}x^{\frac{4}{3}} + C = \frac{3}{4}x\sqrt[3]{x} + C$

(4) $\frac{1}{-2}x^{-2} + C = -\frac{1}{2x^2} + C$

(5) $\frac{2}{3}x^{\frac{3}{2}} + C = \frac{2}{3}x\sqrt{x} + C$

(6) $\frac{4}{5}x^{\frac{5}{4}} + C = \frac{4}{5}x\sqrt[4]{x} + C$

(7) $2\sqrt{x} + C$

(8) $\frac{x^2}{2} - 2x + \log|x| + C$

(9) $\log|x| + \frac{2}{x} - \frac{1}{2x^2} + C$

(10) $\int \left(2\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx = \frac{4}{3}x\sqrt{x} - 2\sqrt{x} + C$

(11) $\int \frac{x + 6\sqrt{x} + 9}{x} dx = x + 12\sqrt{x} + 9 \log|x| + C$

(12) $2 \sin x + 3 \cos x + C$

(13) $\int (2 \sin x + 3 \cos x) dx = -2 \cos x + 3 \sin x + C$

(14) $\int \left(\frac{1}{\cos^2 x} + 2 \right) dx = \tan x + 2x + C$

(15) $\int \frac{1}{\sin^2 x} dx = -\cot x + C$

(16) $\int \frac{1}{\cos^2 x} dx = \tan x + C$

(17) $\int \frac{1}{\sin^2 x} dx = -\cot x + C$

(18) $\int \frac{\cos^2 x}{\sin^2 x} dx = \int \left(\frac{1}{\sin^2 x} - 1 \right) dx = -\cot x - x + C$

(19) $3 \sin^{-1} x + C$

(20) $4 \tan^{-1} x + C$

(21) $\frac{4^x}{\log_e 4} - e^x + C$

(22) $\frac{t^3}{3} - 3t^2 + 5t + C$

(23) $\frac{1}{5}u^5 - u^3 + C$

(24) $-\cos t + C$

(25) $\sin u + C$

(26) $e^u + C$

(27) $\log|u| + C$

(28) $\tan \theta + C$

< 50 ページ. 不定積分の練習 2 >

問の解答

(1) $-\frac{1}{3}e^{-3x+1} + C$

(2) $\frac{1}{4}\sin(4x - 2) + C$

(3) $-\frac{1}{3}\cos(3x + 5) + C$

(4) $\frac{1}{5}\tan(5x + 6) + C$

(5) $\frac{1}{4}\log|4x + 3| + C$

(6) $\frac{1}{20}(5x - 2)^4 + C$

(7) $-\frac{1}{14(7x - 5)^2} + C$

(8) $\frac{2}{15}(5x + 3)\sqrt{5x + 3} + C$

(9) $\frac{2}{7}\sqrt{7x - 6} + C$

(10) $\frac{3}{20}(5x + 1)\sqrt[3]{5x + 1} + C$

(11) $-\frac{1}{2}e^{-x^2} + C$

(12) $-\frac{1}{3}e^{-x^3} + C$

(13) $\frac{1}{3}\sin(x^3 + 4) + C$

(14) $-\frac{1}{4}\cos(x^4) + C$

(15) $\frac{3}{2}\log(1 + x^2) + C$

(16) $\frac{4}{3}\log|x^3 + 2| + C$

(17) $\frac{1}{10}(x^2 + 3)^5 + C$

(18) $\frac{1}{3}(x^2 + 1)\sqrt{x^2 + 1} + C$

(19) $3x + 2\log|x + 1| + C$

(20) $\frac{x^2}{2} + 3x + 4\log|x - 1| + C$

(21) $-\frac{1}{x + 1} + C$

(22) $\frac{1}{2}\log\left|\frac{x}{x + 2}\right| + C$

(23) $\tan^{-1}(x + 1) + C$

(24) $\frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right| + C$

(25) $-xe^{-x} - e^{-x} + C$

(26) $\frac{x^2}{2}\log x - \frac{x^2}{4} + C$

(27) $\frac{x}{3}\sin(3x) + \frac{1}{9}\cos(3x) + C$

(28) $-\frac{1}{6}\cos(3x) + \frac{1}{2}\cos x + C$

(29) $\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x) + C$

(30) $\frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x) + C$

(31) $\frac{x}{2} - \frac{1}{8}\sin(4x) + C$

(32) $\frac{x}{2} + \frac{1}{8}\sin(4x) + C$