

高知工科大学

基礎数学ワークブック

(2003年度版)

初級編

No. 2

解答

< 1 ページ. 関数の極限 >

問の解答

$$(1) \lim_{x \rightarrow 3} \sqrt{x+1} = \sqrt{4} = 2$$

$$(2) \lim_{x \rightarrow \frac{\pi}{3}} \cos x = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$(3) \lim_{x \rightarrow 0} \sin x = \sin 0 = 0$$

$$(4) \lim_{x \rightarrow \frac{1}{2}} \log_2 x = \log_2 \frac{1}{2} = -1$$

$$(5) \lim_{x \rightarrow 0} \frac{x^2 - 1}{x - 1} = \frac{0 - 1}{0 - 1} = 1$$

$$(6) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x + 1} = \frac{4 - 2 - 2}{2 + 1} = 0$$

$$(7) \lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x-3)}{x-1} = \lim_{x \rightarrow 1} (x-3) = -2$$

$$(8) \lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x + 2} = \lim_{x \rightarrow -2} \frac{(x+2)(x-3)}{x+2} = \lim_{x \rightarrow -2} (x-3) = -5$$

< 2 ページ. 左極限・右極限 1 >

問 1 の解答

(1) 10 の左表現 = $9.\dot{9}$, 10 の右表現 = $10.\dot{0}$

(2) 5.3 の左表現 = $5.2\dot{9}$, 5.3 の右表現 = $5.3\dot{0}$

問 2 の解答

(1) $\lim_{x \rightarrow 1-0} [x] = 0$

(2) $\lim_{x \rightarrow 1+0} [x] = 1$

(3) $\lim_{x \rightarrow 3-0} [x] = 2$

(4) $\lim_{x \rightarrow 3+0} [x] = 3$

< 3 ページ. 左極限・右極限 2 >

問 1 の解答

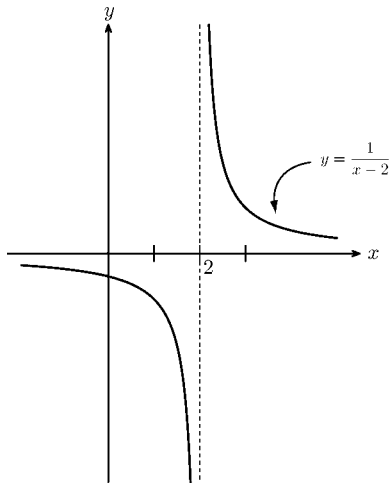
(1) $\lim_{x \rightarrow -0} [x] = -1$

(2) $\lim_{x \rightarrow +0} [x] = 0$

問 2 の解答

(1) $\lim_{x \rightarrow 2+0} \frac{1}{x-2} = +\infty$

(2) $\lim_{x \rightarrow 2-0} \frac{1}{x-2} = -\infty$



< 4 ページ. 左極限・右極限 3 >

問の解答

$$(1) \lim_{x \rightarrow -0} |x| = 0 \quad , \quad \lim_{x \rightarrow +0} |x| = 0 \quad \text{より} \quad \lim_{x \rightarrow 0} |x| = 0$$

$$(2) \lim_{x \rightarrow -0} \frac{|x|}{x} = -1 \quad , \quad \lim_{x \rightarrow +0} \frac{|x|}{x} = 1$$

< 5 ページ. 弧度法の復習 >

問 1 の解答

度数法	45°	60°	90°	120°	180°	360°
弧度法 θ	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	2π
弧の長さ l	$\frac{1}{4}\pi r$	$\frac{\pi}{3}r$	$\frac{\pi}{2}r$	$\frac{2\pi}{3}r$	πr	$2\pi r$
面積 S	$\frac{\pi r^2}{8}$	$\frac{\pi r^2}{6}$	$\frac{1}{4}\pi r^2$	$\frac{\pi r^2}{3}$	$\frac{\pi r^2}{2}$	πr^2

問 2 の解答

$$l = \theta r$$

$$S = \frac{1}{2}\theta r^2$$

< 6 ページ. 三角関数の極限 1 >

問 1 の解答

$$(1) \ell_1 = \sin \theta \quad , \quad \ell_2 = \tan \theta$$

問 2 の解答

$$\ell_2 = \theta$$

問 3 の解答

$$\sin \theta < \theta < \tan \theta$$

問 4 の解答

$$0 < \theta < \frac{\pi}{2} \text{ のとき } \boxed{\sin \theta} < \theta < \boxed{\frac{\sin \theta}{\cos \theta}}$$

< 7ページ. 三角関数の極限 2 >

問 以下の証明中の \square 内に適当な数または数式を記入せよ。

[定理の証明]

[1] $\lim_{\theta \rightarrow +0} \frac{\sin \theta}{\theta} = 1$ を示す。

前ページの結果より $0 < \theta < \frac{\pi}{2}$ のとき

$$(*) \quad \sin \theta < \theta < \frac{\sin \theta}{\cos \theta}$$

がわかる。 $\cos \theta > 0$, $\theta > 0$ より

$$\theta < \frac{\sin \theta}{\cos \theta} \quad \Rightarrow \quad \square < \frac{\sin \theta}{\theta} \quad \dots \textcircled{1}$$

また

$$\sin \theta < \theta \quad \Rightarrow \quad \frac{\sin \theta}{\theta} < \square \quad \dots \textcircled{2}$$

①, ②より

$$\square < \frac{\sin \theta}{\theta} < \square \quad \dots \textcircled{3}$$

ここで $\theta \rightarrow +0$ のとき $\cos \theta \rightarrow \cos 0 = 1$ より③から

$$\lim_{\theta \rightarrow +0} \frac{\sin \theta}{\theta} = 1$$

がわかる。

[2] $\lim_{\theta \rightarrow -0} \frac{\sin \theta}{\theta} = 1$ を示す。

$\theta \rightarrow -0$ のとき $\theta = -\theta_1$ ($\theta_1 > 0$) とおくと, $\theta_1 \rightarrow +0$ より

$$\lim_{\theta \rightarrow -0} \frac{\sin \theta}{\theta} = \lim_{\theta_1 \rightarrow +0} \frac{\sin(-\theta_1)}{-\theta_1} = \lim_{\theta_1 \rightarrow +0} \frac{\square}{-\theta_1} = \lim_{\theta_1 \rightarrow +0} \frac{\sin \theta_1}{\theta_1} = 1$$

[3] [1] と [2] より右極限值と左極限值が一致するので定理の極限が証明された。

(証明終)

< 8 ページ. 三角関数の極限 3 >

問の解答

$$(1) \lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \times \frac{1}{\cos x} \right) = 1$$

$$(2) \lim_{x \rightarrow 0} \frac{\sin(2x)}{3x} = \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \times \frac{2}{3} = \frac{2}{3}$$

$$(3) \lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(5x)} = \lim_{x \rightarrow 0} \frac{\frac{\sin(3x)}{3x}}{\frac{\sin(5x)}{5x}} \times \frac{3}{5} = \frac{3}{5}$$

$$(4) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x} = \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x)}{(x \sin x)(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x \sin x(1 + \cos x)}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 x}{x \sin x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \frac{1}{1 + \cos x} = \frac{1}{2}$$

$$(5) \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = \lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{x(\cos x + 1)} = \lim_{x \rightarrow 0} \frac{-\sin^2 x}{x(\cos x + 1)} = \lim_{x \rightarrow 0} \left(\frac{-\sin x}{\cos x + 1} \times \frac{\sin x}{x} \right) = 0$$

< 9 ページ. 三角関数の極限 4 >

問の解答

$$(1) \lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{3} + h) - \sin \frac{\pi}{3}}{h} = \frac{1}{2}$$

$$(2) \lim_{h \rightarrow 0} \frac{\cos(\pi + h) - \cos \pi}{h} = 0$$

$$(3) \lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h} = \cos x$$

$$(4) \lim_{h \rightarrow 0} \frac{\cos(x + h) - \cos x}{h} = -\sin x$$

< 10 ページ. 関数の極限の練習 >

問 1 の解答

$$\lim_{x \rightarrow +0} f(x) = +\infty, \quad \lim_{x \rightarrow -0} f(x) = -\infty, \quad \lim_{x \rightarrow 0} f(x) = \text{存在しない}$$

$$\lim_{x \rightarrow a+0} f(x) = 0, \quad \lim_{x \rightarrow a-0} f(x) = 0, \quad \lim_{x \rightarrow a} f(x) = 0$$

$$\lim_{x \rightarrow b+0} f(x) = q, \quad \lim_{x \rightarrow b-0} f(x) = s, \quad \lim_{x \rightarrow b} f(x) = \text{存在しない}$$

$$\lim_{x \rightarrow c+0} f(x) = p, \quad \lim_{x \rightarrow c-0} f(x) = p, \quad \lim_{x \rightarrow c} f(x) = p$$

問 2 の解答

$$(1) \quad \lim_{x \rightarrow +0} \frac{1}{x} = +\infty \qquad (2) \quad \lim_{x \rightarrow -0} \frac{1}{x} = -\infty \qquad (3) \quad \lim_{x \rightarrow 0} \frac{1}{x} = \text{存在しない}$$

$$(4) \quad \lim_{x \rightarrow +0} \frac{1}{x^2} = +\infty \qquad (5) \quad \lim_{x \rightarrow -0} \frac{1}{x^2} = +\infty \qquad (6) \quad \lim_{x \rightarrow 0} \frac{1}{x^2} = +\infty$$

$$(7) \quad \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \frac{1}{2} \qquad (8) \quad \lim_{x \rightarrow 1} \frac{\frac{1}{x} - 1}{x - 1} = -1 \qquad (9) \quad \lim_{x \rightarrow 0} \frac{\sin(\frac{2}{3}x)}{x} = \frac{2}{3}$$

$$(10) \quad \lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180} \qquad (11) \quad \lim_{x \rightarrow 0} \frac{\tan(3x)}{\sin(2x)} = \frac{3}{2} \qquad (12) \quad \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin(x^2)} = 1$$

$$(13) \quad \lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} = 2$$

< 11 ページ. 関数の連続性 >

問の解答

$$(1) \lim_{x \rightarrow \frac{\pi}{2}-0} \tan x = -\infty, \quad \lim_{x \rightarrow \frac{\pi}{2}+0} \tan x = +\infty$$

より極限 $\lim_{x \rightarrow \frac{\pi}{2}} f(x)$ は存在しない.

従って不連続

$$(2) \lim_{x \rightarrow +0} |x| = \lim_{x \rightarrow +0} x = 0, \quad \lim_{x \rightarrow -0} |x| = \lim_{x \rightarrow -0} (-x) = 0$$

より $\lim_{x \rightarrow 0} |x| = 0 = |0|$

よって $\lim_{x \rightarrow 0} f(x) = f(0)$ 従って連続

$$(3) \left. \begin{array}{l} \lim_{x \rightarrow 1+0} f(x) = \lim_{x \rightarrow 1+0} (x - [x]) = 1 - 1 = 0 \\ \lim_{x \rightarrow 1-0} f(x) = \lim_{x \rightarrow 1-0} (x - [x]) = 1 - 0 = 1 \end{array} \right\} \text{より極限 } \lim_{x \rightarrow 1} f(x) \text{ は存在しない.}$$

従って不連続

< 12 ページ. 微分可能性 >

問の解答

$$\lim_{h \rightarrow +0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow +0} (2+h) = 2$$

$$\lim_{h \rightarrow -0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow -0} \frac{-(2+h)h}{h} = \lim_{h \rightarrow -0} (-2-h) = -2$$

より $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$ は存在しないから $x = 1$ で微分可能ではない.

< 13 ページ. 導関数 1 >

問の解答

$$\begin{aligned} (1) f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h+1})^2 - (\sqrt{x+1})^2}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}} \end{aligned}$$

$$(2) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{-h}{(x+h)h} = -\frac{1}{x^2}$$

< 14 ページ. 導関数 2 >

問の解答

(1) $y' = 5x^4$

(2) $y' = 6x^5$

(3) $y' = -12x^3$

(4) $y' = 5x^4 + 8x^3$

(5) $y' = 8x^3 - 15x^4$

(6) $y' = 3x^2 - 2x + 1$

(7) $y' = 3x^2 - 6x - 4$

(8) $y' = 4x^3 + 3x^2 - 1$

< 15 ページ. 積の微分 >

問 1 の解答

(1) $y' = 3x^2 - 2x + 1$

(2) $y' = 3x^2 - 6x - 4$

(3) $y' = 4x^3 + 3x^2 - 1$

(4) $y' = 4x^3 + 12x^2 + 12x + 4$

問 2 の解答

$$\begin{aligned}\{f(x)g(x)h(x)\}' &= (f(x)g(x))' \times h(x) + (f(x)g(x)) \times h'(x) \\ &= \{f'(x)g(x) + f(x)g'(x)\} \times h(x) + f(x)g(x)h'(x) \\ &= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)\end{aligned}$$

< 16 ページ. 商の微分 >

問 1 の解答

$$\begin{aligned}\left\{\frac{f(x)}{g(x)}\right\}' &= (f(x))' \times \frac{1}{g(x)} + f(x) \times \left(\frac{1}{g(x)}\right)' = f'(x) \times \frac{1}{g(x)} + f(x) \times \left\{-\frac{g'(x)}{(g(x))^2}\right\} \\ &= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{\{g(x)\}^2} = \frac{f'(x)g(x)}{\{g(x)\}^2} - \frac{f(x)g'(x)}{\{g(x)\}^2} = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}\end{aligned}$$

問 2 の解答

$$(1) \left(\frac{1}{x^2}\right)' = -\frac{(x^2)'}{x^4} = -\frac{2}{x^3}$$

$$(2) \left(\frac{1}{2x^2}\right)' = -\frac{(2x^2)'}{(2x^2)^2} = -\frac{1}{x^3}$$

$$(3) \left(\frac{x+1}{x^2}\right)' = \frac{(x+1)'x^2 - (x+1)(x^2)'}{(x^2)^2} = -\frac{x+2}{x^3}$$

$$(4) \left(\frac{x^3}{x+1}\right)' = \frac{(x^3)'(x+1) - x^3(x+1)'}{(x+1)^2} = \frac{2x^3 + 3x^2}{(x+1)^2}$$

< 17 ページ. 三角関数の微分 >

問 1 の解答

(1) $(3 \sin x + 4 \cos x)' = 3 \cos x - 4 \sin x$

(2) $(-3 \cos x + 5 \tan x)' = 3 \sin x + \frac{5}{\cos^2 x}$

(3) $(\sin x \cos x)' = \cos^2 x - \sin^2 x$

(4) $(\sin^2 x)' = 2 \sin x \cos x$

(5) $(\cos^2 x)' = -2 \sin x \cos x$

(6) $(x \tan x)' = \tan x + \frac{x}{\cos^2 x}$

(7) $\left(\frac{\sin x}{x}\right)' = \frac{x \cos x - \sin x}{x^2}$

(8) $\left(\frac{\cos x}{x}\right)' = \frac{-x \sin x - \cos x}{x^2}$

問 2 の解答

(1) $(\operatorname{cosec} x)' = \left(\frac{1}{\sin x}\right)' = -\frac{(\sin x)'}{(\sin x)^2} = -\frac{\cos x}{\sin^2 x}$

(2) $(\sec x)' = \left(\frac{1}{\cos x}\right)' = -\frac{(\cos x)'}{(\cos x)^2} = \frac{\sin x}{\cos^2 x}$

(3) $(\cot x)' = \left(\frac{1}{\tan x}\right)' = -\frac{(\tan x)'}{(\tan x)^2} = -\frac{1}{\sin^2 x}$

< 18 ページ. 微分の練習 1 >

問 1 の解答

$$(1) f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} = \frac{1}{2\sqrt{x+3}}$$

$$(2) f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = -\frac{1}{(x+1)^2}$$

問 2 の解答

$$(1) \left((x^2 - 2x)(3x + 1) \right)' = 9x^2 - 10x - 2$$

$$(2) \left((2x^3 + 1)(3x^2 - 1) \right)' = 30x^4 - 6x^2 + 6x$$

$$(3) \left((x+1)(x+2)(x+3) \right)' = 3x^2 + 12x + 11$$

$$(4) \left(\frac{2x}{x+1} \right)' = \frac{2}{(x+1)^2}$$

$$(5) \left(\frac{x-1}{x^2+1} \right)' = \frac{-x^2 + 2x + 1}{(x^2+1)^2}$$

$$(6) \left(\frac{3x}{(x+1)^2} \right)' = \frac{-3x+3}{(x+1)^3}$$

$$(7) (4 \sin x - 5 \cos x)' = 4 \cos x + 5 \sin x$$

$$(8) (x^2 \sin x)' = 2x \sin x + x^2 \cos x$$

$$(9) (x^3 \cos x)' = 3x^2 \cos x - x^3 \sin x$$

$$(10) \left(\frac{\tan x}{x} \right)' = \frac{x - \sin x \cos x}{x^2 \cos^2 x}$$

$$(11) (2 \sec x + 3 \cot x)' = \frac{2 \sin x}{\cos^2 x} - \frac{3}{\sin^2 x}$$

$$(12) \left(\frac{\cos x}{3 + \sin x} \right)' = \frac{-1 - 3 \sin x}{(3 + \sin x)^2}$$

問 3 の解答

$$(1) (x^{-3})' = -3x^{-4}$$

$$(2) (x^{-4})' = -4x^{-5}$$

$$(3) (x^{-n})' = -nx^{-n-1}$$

< 19 ページ. 微分記号 >

問の解答

(1) $\frac{dy}{dx} = 2x + 1$

(2) $\frac{dy}{dt} = -9.8$

(3) $\frac{d\ell}{dt} = 6t - 2$

(4) $\frac{dS}{dr} = 2\pi r$

(5) $\frac{dV}{dr} = 4\pi r^2$

< 20 ページ. 増分記号 Δ (デルタ) >

問の解答

$$(1) \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^5 - x^5}{\Delta x} = (x^5)' = 5x^4$$

$$(2) \lim_{\Delta t \rightarrow 0} \frac{\sin(t + \Delta t) - \sin(t)}{\Delta t} = (\sin t)' = \cos t$$

$$(3) \lim_{\Delta u \rightarrow 0} \frac{\cos(u + \Delta u) - \cos(u)}{\Delta u} = (\cos u)' = -\sin u$$

< 21 ページ. 合成関数の微分 1 >

問 1 の解答

$$\begin{aligned} \frac{dy}{dx} &= \left(\lim_{\Delta u \rightarrow 0} \frac{\cos(u + \Delta u) - \cos u}{\Delta u} \right) \times \left(\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x} \right) \\ &= (\cos u)' \times (x^4)' \\ &= -\sin u \times 4x^3 = -4x^3 \sin u = -4x^3 \sin(x^4) \end{aligned}$$

問 2 の解答

$$\begin{aligned} u &= x^3 + 2x^2 \\ \Delta u &= (x + \Delta x)^3 + 2(x + \Delta x)^2 - x^3 - 2x^2 \\ \Delta y &= \sin(u + \Delta u) - \sin u \\ \frac{dy}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{\sin\{(x + \Delta x)^3 + 2(x + \Delta x)^2\} - \sin(x^3 + x^2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sin(u + \Delta u) - \sin u}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sin(u + \Delta u) - \sin u}{\Delta u} \times \frac{\Delta u}{\Delta x} \\ &= \lim_{\Delta u \rightarrow 0} \frac{\sin(u + \Delta u) - \sin u}{\Delta u} \times \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 + 2(x + \Delta x)^2 - x^3 - 2x^2}{\Delta x} \\ &= (\sin u)' \times (x^3 + 2x^2)' = \cos u \times (3x^2 + 4x) = (3x^2 + 4x) \cos(x^3 + 2x^2) \end{aligned}$$

< 22 ページ. 合成関数の微分 2 >

問 1 の解答

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$

問 2 の解答

$$(1) \frac{dy}{dx} = 6(x-1)(x^2 - 2x + 5)^2$$

$$(2) \frac{dy}{dx} = -2 \sin(2x - 3)$$

$$(3) \frac{dy}{dx} = (5x^4 - 4x) \cos(x^5 - 2x^2)$$

< 23 ページ. 合成関数の微分 3 >

問 1 の解答

$$(1) \left((3x + 5)^7 \right)' = 21(3x + 5)^6$$

$$(2) \left((4x^2 + 5x)^8 \right)' = 8(8x + 5)(4x^2 + 5x)^7$$

問 2 の解答

$$\left((f(x))^n \right)' = n(f(x))^{n-1} \times f'(x)$$

問 3 の解答

$$(1) \left((3x + 4)^5 \right)' = 15(3x + 4)^4$$

$$(2) \left((4x^2 + 9x)^6 \right)' = 6(8x + 9)(4x^2 + 9x)^5$$

$$(3) \left((x^4 - 2x^3)^{10} \right)' = 10(4x^3 - 6x^2)(x^4 - 2x^3)^9$$

$$(4) \left((3 + 4 \sin x)^5 \right)' = 20 \cos x (3 + 4 \sin x)^4$$

$$(5) \left((x - 3 \cos x)^7 \right)' = 7(1 + 3 \sin x)(x - 3 \cos x)^6$$

< 24 ページ. 合成関数の微分 4 >

問 1 の解答

$$(1) \left(\sin(5x - 4) \right)' = 5 \cos(5x - 4)$$

$$(2) \left(\sin(x^6 + 7x^2 - 3) \right)' = (6x^5 + 14x) \cos(x^6 + 7x^2 - 3)$$

$$(3) \left(\cos(4x + 3) \right)' = -4 \sin(4x + 3)$$

$$(4) \left(\cos(x^5 - 2x + 1) \right)' = -(5x^4 - 2) \sin(x^5 - 2x + 1)$$

問 2 の解答

$$\left(\cos(f(x)) \right)' = -\sin(f(x)) \times f'(x)$$

問 3 の解答

$$(1) \left(\sin(x^6 + 7x^5 - 3x^2 + 4x) \right)' \\ = (6x^5 + 35x^4 - 6x + 4) \cos(x^6 + 7x^5 - 3x^2 + 4x)$$

$$(2) \left(\sin(x^7 - 8x^5 + 4x^3 - 6x + 1) \right)' \\ = (7x^6 - 40x^4 + 12x^2 - 6) \cos(x^7 - 8x^5 + 4x^3 - 6x + 1)$$

< 26 ページ. 対数関数の導関数 2 >

問 1 の解答

$$\begin{aligned} (1) f'(3) &= \lim_{\Delta x \rightarrow 0} \frac{f(3 + \Delta x) - f(3)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\log_{10}(3 + \Delta x) - \log_{10} 3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \log_{10} \left(\frac{3 + \Delta x}{3} \right) = \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \log_{10} \left(1 + \frac{\Delta x}{3} \right) \end{aligned}$$

ここで $\frac{\Delta x}{3} = h$ とおくと $\Delta x \rightarrow 0$ のとき $h \rightarrow 0$ より

$$f'(3) = \lim_{h \rightarrow 0} \frac{1}{3h} \log_{10}(1 + h) = \lim_{h \rightarrow 0} \frac{1}{3} \log_{10}(1 + h)^{\frac{1}{h}} = \frac{1}{3} \log_{10} e$$

$$\begin{aligned} (2) f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\log_{10}(x + \Delta x) - \log_{10} x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \log_{10} \left(\frac{x + \Delta x}{x} \right) = \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \log_{10} \left(1 + \frac{\Delta x}{x} \right) \end{aligned}$$

ここで $\frac{\Delta x}{x} = h$ とおくと $\Delta x \rightarrow 0$ のとき $h \rightarrow 0$ より

$$f'(x) = \lim_{h \rightarrow 0} \frac{1}{xh} \log_{10}(1 + h) = \lim_{h \rightarrow 0} \frac{1}{x} \log_{10}(1 + h)^{\frac{1}{h}} = \frac{1}{x} \log_{10} e$$

問 2 の解答

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\log_a(x + \Delta x) - \log_a(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \log_a \left(1 + \frac{\Delta x}{x} \right)$$

ここで $\frac{\Delta x}{x} = h$ とおくと $\Delta x \rightarrow 0$ のとき $h \rightarrow 0$ より

$$f'(x) = \lim_{h \rightarrow 0} \frac{1}{xh} \log_a(1 + h) = \frac{1}{x} \log_a e$$

< 27 ページ. 自然対数 >

問 1 の解答

(1) $(\log_{10} x)' = \frac{1}{x} \log_{10} e$

(2) $(\log_a x)' = \frac{1}{x} \log_a e$

問 2 の解答

(答) $(\log_e x)' = \frac{1}{x} \log_e e = \frac{1}{x}$

問 3 の解答

(1) $\log e = 1$ (2) $\log(\sqrt[3]{e}) = \frac{1}{3}$ (3) $\log\left(\frac{1}{e}\right) = -1$ (4) $\log 1 = 0$

(5) $\ln\left(\frac{1}{e}\right) = -1$ (6) $\ln(\sqrt[4]{e}) = \frac{1}{4}$ (7) $\ln(e) = 1$ (8) $\ln(e\sqrt{e}) = \frac{3}{2}$

問 4 の解答

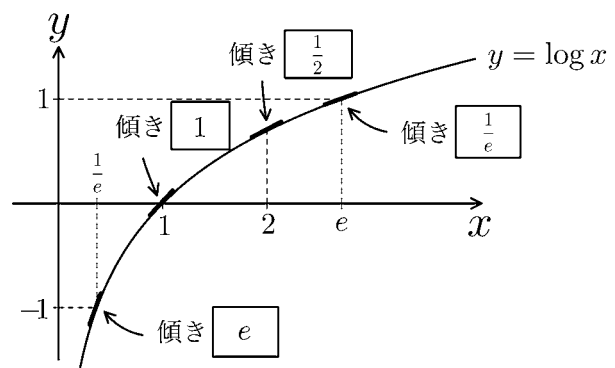
(1) $(\log x)' = \frac{1}{x}$

(2) $(\ln x)' = \frac{1}{x}$

問 5 の解答

$f'\left(\frac{1}{e}\right) = e, \quad f'(1) = 1$

$f'(2) = \frac{1}{2}, \quad f'(e) = \frac{1}{e}$



< 28 ページ.log $f(x)$ の導関数 >

問 1 の解答

$$(1) \frac{dy}{dx} = \frac{3x^2 + 2}{x^3 + 2x - 5}$$

$$(2) \frac{dy}{dx} = \frac{\cos x}{1 + \sin x}$$

$$(3) \frac{dy}{dx} = \frac{\sin x}{5 - \cos x}$$

問 2 の解答

$$\left(\log(f(x)) \right)' = \frac{f'(x)}{f(x)}$$

問 3 の解答

$$(1) \left(\log(x^2 + 2x) \right)' = \frac{2x + 2}{x^2 + 2x}$$

$$(2) \left(\log(x^6 + 3x^4) \right)' = \frac{6x^2 + 12}{x^3 + 3x}$$

$$(3) \left(\log(\sin x) \right)' = \frac{\cos x}{\sin x} \quad (= \cot x)$$

< 29 ページ. 逆関数の微分 1 >

問 1 の解答

$$y = \cos^{-1} x \iff x = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\cos y)'} = \frac{1}{-\sin y} = -\frac{1}{\sqrt{1 - \cos^2 y}} = -\frac{1}{\sqrt{1 - x^2}}$$

問 2 の解答

$$y = \tan^{-1} x \iff x = \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\tan y)'} = \frac{1}{\frac{1}{\cos^2 y}} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

< 30 ページ. 逆関数の微分 2 >

問 1 の解答

(1) $y = x^{\frac{1}{4}} \iff x = y^4$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(y^4)'} = \frac{1}{4y^3} = \frac{1}{4(x^{\frac{1}{4}})^3} = \frac{1}{4}x^{-\frac{3}{4}}$$

(2) $y = x^{\frac{1}{n}} \iff x = y^n$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(y^n)'} = \frac{1}{ny^{n-1}} = \frac{1}{n(x^{\frac{1}{n}})^{n-1}} = \frac{1}{n}x^{\frac{1}{n}-1}$$

問 2 の解答

(1) $y = 2^x \iff x = \log_2 y$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\log_2 y)'} = \frac{1}{\frac{1}{y} \log_2 e} = y \log_e 2 = 2^x \log_e 2$$

(2) $y = a^x \iff x = \log_a y$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\log_a y)'} = \frac{1}{\frac{1}{y} \log_a e} = y \log_e a = a^x \log_e a$$

< 31 ページ. 指数関数の微分 >

問 1 の解答

$$(1) \left(e^{3x}\right)' = 3e^{3x}$$

$$(2) \left(e^{x^2+3}\right)' = 2xe^{x^2+3}$$

$$(3) \left(e^{-x^2+2x}\right)' = (-2x + 2)e^{-x^2+2x}$$

問 2 の解答

$$\left(e^{f(x)}\right)' = e^{f(x)} \times f'(x)$$

問 3 の解答

$$(1) \left(e^{-3x}\right)' = -3e^{-3x}$$

$$(2) \left(e^{-\frac{x^2}{2}}\right)' = -xe^{-\frac{x^2}{2}}$$

< 32 ページ. 対数微分法 1 >

問 1 の解答

$$(解) \log y = x \log 3$$

$$\frac{y'}{y} = \log 3 \Rightarrow y' = y \times \log 3 = 3^x \log 3$$

$$(答) \underline{(3^x)' = 3^x \log 3}$$

問 2 の解答

$$(解) \log y = x \log a$$

$$\frac{y'}{y} = \log a \Rightarrow y' = a^x \log a$$

$$(答) \underline{(a^x)' = a^x \log a}$$

問 3 の解答

$$(答) (e^x)' = e^x \log e = e^x$$

< 33 ページ. 対数微分法 2 >

問 1 の解答

$$\text{(解)} \log y = \frac{4}{3} \log x$$

$$\frac{y'}{y} = \frac{4}{3x} \Rightarrow y' = y \times \frac{4}{3x} = x^{\frac{4}{3}} \times \frac{4}{3x} = \frac{4}{3} x^{\frac{1}{3}}$$

$$\text{(答)} \left(x^{\frac{4}{3}}\right)' = \frac{4}{3} x^{\frac{1}{3}}$$

問 2 の解答

$$\text{(解)} \log y = r \log x$$

$$\frac{y'}{y} = \frac{r}{x} \Rightarrow y' = y \times \frac{r}{x} = x^{r-1} \times r$$

$$\text{(答)} (x^r)' = r x^{r-1}$$

< 34 ページ x^r の導関数 >

問 1 の解答

(1) $(\sqrt[4]{x^5})' = \frac{5}{4}\sqrt[4]{x}$

(2) $(\sqrt[5]{x^7})' = \frac{7}{5}\sqrt[5]{x^2}$

(3) $(\sqrt{x^3})' = \frac{3}{2}\sqrt{x}$

問 2 の解答

(1) $(\frac{1}{x^3})' = -\frac{3}{x^4}$

(2) $(\frac{1}{x^4})' = -\frac{4}{x^5}$

(3) $(\frac{1}{x})' = -\frac{1}{x^2}$

問 3 の解答

(1) $(\sqrt[4]{x})' = \frac{1}{4\sqrt[4]{x^3}}$

(2) $(\sqrt[5]{x^4})' = \frac{4}{5\sqrt[5]{x}}$

(3) $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

問 4 の解答

(1) $(\frac{1}{\sqrt[3]{x^2}})' = -\frac{2}{3x\sqrt[3]{x^2}}$

(2) $(\frac{1}{\sqrt[4]{x}})' = -\frac{1}{4x\sqrt[4]{x}}$

(3) $(\frac{1}{\sqrt{x}})' = -\frac{1}{2x\sqrt{x}}$

< 35 ページ $\log |x|$ の導関数 >

問 1 の解答

$$(1) \frac{dy}{dx} = \frac{\frac{1}{\cos^2 x}}{\tan x} = \frac{1}{\cos x \sin x}$$

$$(2) \frac{dy}{dx} = \frac{2x + 3}{x^2 + 3x}$$

$$(3) \frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

< 36 ページ. 微分の練習 2 >

問 1 の解答

$$\begin{array}{ll}
 (1) (k)' = 0 & (2) (x^n)' = nx^{n-1} \\
 (3) (\sin x)' = \cos x & (4) (\cos x)' = -\sin x \\
 (5) (\log x)' = \frac{1}{x} & (6) (e^x)' = e^x \\
 (7) (\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}} & (8) (\tan^{-1} x)' = \frac{1}{1+x^2}
 \end{array}$$

問 2 の解答

$$\begin{array}{ll}
 (1) (f(x) + g(x))' = f'(x) + g'(x) & (2) (f(x) - g(x))' = f'(x) - g'(x) \\
 (3) (kf(x))' = kf'(x) & (4) (f(x) \times g(x))' = f'(x)g(x) + f(x)g'(x) \\
 (5) \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}
 \end{array}$$

問 3 の解答

$$\begin{array}{ll}
 (1) ((f(x))^n)' = n(f(x))^{n-1} \times f'(x) & (2) (\sin(f(x)))' = \cos(f(x)) \times f'(x) \\
 (3) (\cos(f(x)))' = -\sin(f(x)) \times f'(x) & (4) (\log |f(x)|)' = \frac{f'(x)}{f(x)} \\
 (5) (e^{f(x)})' = e^{f(x)} \times f'(x)
 \end{array}$$

問 4 の解答

$$\begin{array}{ll}
 (1) (x^4 - 5x^3 + 6x^2 - 7x + 8)' & (2) (\sqrt{x})' = \frac{1}{2\sqrt{x}} \\
 = 4x^3 - 15x^2 + 12x - 7 & \\
 (3) (x\sqrt{x})' = \frac{3}{2}\sqrt{x} & (4) \left(\frac{\sin x}{x}\right)' = \frac{x \cos x - \sin x}{x^2} \\
 (5) (\sin x \cos x)' = \cos^2 x - \sin^2 x (= \cos(2x)) & (6) (\tan x)' = \frac{1}{\cos^2 x} \\
 (7) (x \log x - x)' = \log x & (8) (-\log |\cos x|)' = \tan x \\
 (9) (e^{2x} \sin(3x))' = 2e^{2x} \sin(3x) + 3e^{2x} \cos(3x) & (10) \left(\log(x + \sqrt{x^2 + 1})\right)' \\
 & = \frac{1 + \frac{2x}{2\sqrt{x^2+1}}}{x + \sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}
 \end{array}$$

< 37 ページ. 微分係数 >

問 1 の解答

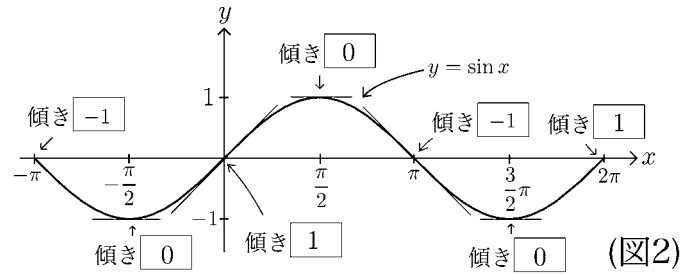
$$f'(x) = \cos x$$

$$f'(-\pi) = -1 \quad f'(-\frac{\pi}{2}) = 0$$

$$f'(0) = 1 \quad f'(\frac{\pi}{2}) = 0$$

$$f'(\pi) = -1 \quad f'(\frac{3}{2}\pi) = 0$$

$$f'(2\pi) = 1$$



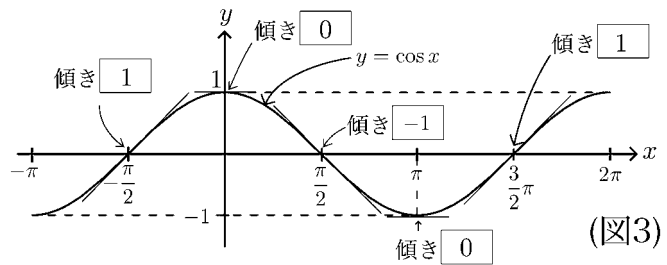
問 2 の解答

$$f'(x) = -\sin x$$

$$f'(-\frac{\pi}{2}) = 1 \quad f'(0) = 0$$

$$f'(\frac{\pi}{2}) = -1 \quad f'(\pi) = 0$$

$$f'(\frac{3}{2}\pi) = 1$$



問 3 の解答

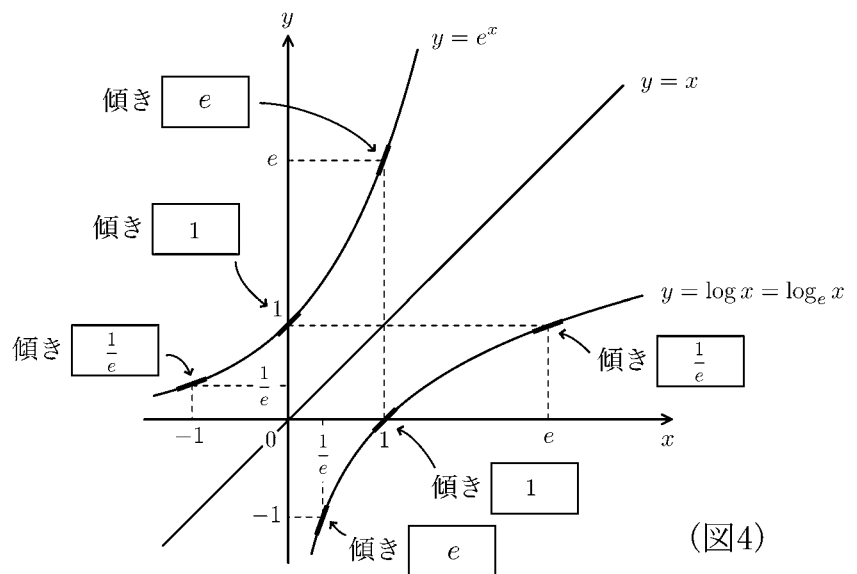
(1) $f^{-1}(x) = \log_e x$

(2) $f'(x) = e^x \quad g'(x) = \frac{1}{x}$

$$f'(-1) = \frac{1}{e} \quad g'(\frac{1}{e}) = e$$

$$f'(0) = 1 \quad g'(1) = 1$$

$$f'(1) = e \quad g'(e) = \frac{1}{e}$$



< 38 ページ. 接線の方程式 1 >

問の解答

(1) $y = x + 1$

(2) $y = x - 1$

(3) $y = x$

(4) $y = \frac{1}{4}x + 1$

(5) $y = -x + 2$

< 39 ページ. 接線の方程式 2 >

問の解答

(1) $y = 3x + \sqrt{3} - \frac{\pi}{3}$

(2) $y = -5\sqrt{3}x + \frac{5\sqrt{3}\pi}{6} + \frac{5}{2}$

(3) $y = 4x$

(4) $y = -\frac{1}{2}x - 1$

(5) $y = \frac{1}{6}x + \frac{3}{2}$

(6) $y = \frac{2}{3}x + \frac{5}{3}$

(7) $y = -\frac{x}{16} + \frac{3}{4}$

(8) $y = -2x + 3$

(9) $y = 2x + 1$

(10) $y = 2ex - e$

(11) $y = \frac{1}{e}x$

(12) $y = x - 1 + \log 2$

< 40 ページ. 接線の方程式 3 >

問の解答

$x^2 + y^2 = 16$ の両辺を x で微分する

$$2x + 2yy' = 0$$

↓

$$y' = -\frac{x}{y} = -\frac{2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}} \text{ が接線の傾きである}$$

よって点 $(2, 2\sqrt{3})$ を通り傾き $-\frac{1}{\sqrt{3}}$ の直線の式は

$$y = -\frac{1}{\sqrt{3}}(x - 2) + 2\sqrt{3}$$

$$\underline{\underline{(\text{答}) } y = -\frac{\sqrt{3}}{3}x + \frac{8\sqrt{3}}{3}}$$

< 41 ページ. 接線の方程式 4 >

問の解答

$$\frac{2x}{8} + \frac{2yy'}{2} = 0$$

$$y' = -\frac{x}{4y} = -\frac{-2}{4 \times 1} = \frac{1}{2} \quad \text{が接線の傾きである. よって接線の式は}$$

$$y = \frac{1}{2}(x + 2) + 1 \text{ より}$$

$$\underline{\underline{(\text{答}) } y = \frac{1}{2}x + 2}$$

問 2 の解答

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y} = -\frac{-3}{-4} = -\frac{3}{4} \quad \text{が接線の傾きである. よって接線の式は}$$

$$y = -\frac{3}{4}(x + 3) - 4$$

$$\underline{\underline{(\text{答}) } y = -\frac{3}{4}x - \frac{25}{4}}$$

問 3 の解答

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y} = -\frac{r \cos \theta}{r \sin \theta} = -\frac{1}{\tan \theta} \quad \text{が接線の傾きである. よって接線の式は}$$

$$y = -\frac{1}{\tan \theta}(x - r \cos \theta) + r \sin \theta$$

$$= -\frac{x}{\tan \theta} + \frac{r(\cos^2 \theta + \sin^2 \theta)}{\sin \theta}$$

$$\underline{\underline{(\text{答}) } y = -\frac{x}{\tan \theta} + \frac{r}{\sin \theta}}$$

< 42 ページ.2 階導関数 >

問 1 の解答

(1) $f''(x) = 24x - 10$

(2) $f''(x) = -\sin x$

(3) $f''(x) = -\frac{1}{x^2}$

(4) $\frac{d^2y}{dx^2} = 20x^3 - 12x^2$

(5) $\frac{d^2y}{dx^2} = -\cos x$

(6) $\frac{d^2y}{dx^2} = 4e^{2x}$

問 2 の解答

(1) $\frac{d^2y}{dt^2} = -9.8$

(2) $\frac{d^2y}{dt^2} = -4\sin(2t)$

(3) $\frac{d^2y}{dt^2} = -9\cos(3t)$

< 43 ページ. 直線上の運動 >

問の解答

(1) $v(t) = 4 - 10t$

$a(t) = -10$

(2) $v(t) = -6 \sin(2t)$

$a(t) = -12 \cos(2t)$

(3) $v(t) = 2e^{2t} \sin(4t) + 4e^{2t} \cos(4t)$

$a(t) = -12e^{2t} \sin(4t) + 16e^{2t} \cos(4t)$

< 44 ページ. 平面上の運動 1 >

問の解答

$$\vec{v} = \left(\frac{dx}{dt}, \frac{dy}{dt} \right) = (2, -2t)$$

$$|\vec{v}| = \sqrt{2^2 + (-2t)^2} = \sqrt{4 + 4t^2} (= 2\sqrt{1 + t^2})$$

< 45 ページ. 平面上の運動 2 >

問の解答

$$(1) \begin{cases} v_x(t) = \frac{dx}{dt} = k_1 \\ v_y(t) = \frac{dy}{dt} = k_2 - gt \end{cases}$$

$$(2) \frac{v_y(t)}{v_x(t)} = \frac{k_2 - gt}{k_1}$$

$$(3) y = \frac{k_2}{k_1}x - \frac{g}{2k_1^2}x^2$$

$$(4) f(x) = \frac{k_2}{k_1}x - \frac{g}{2k_1^2}x^2 \text{ より}$$

$$f'(x) = \frac{k_2}{k_1} - \frac{g}{k_1^2} \times 2x = \frac{k_2}{k_1} - \frac{g}{k_1^2}x$$

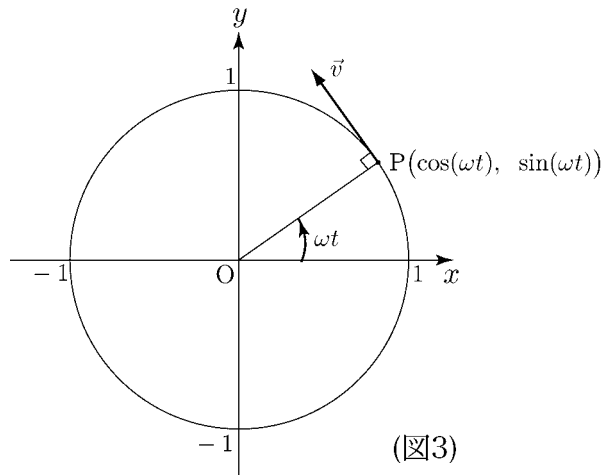
$$(5) f'(x(t)) = f'(k_1 t) = \frac{k_2}{k_1} - \frac{g}{k_1^2}(k_1 t) = \frac{k_2 - gt}{k_1} = \frac{v_y(t)}{v_x(t)}$$

< 46 ページ. 平面上の運動 3 >

問の解答

$$\vec{v} = (-\omega \sin(\omega t), \omega \cos(\omega t))$$

$$|\vec{v}| = \sqrt{(-\omega \sin(\omega t))^2 + (\omega \cos(\omega t))^2} = \sqrt{\omega^2 (\sin^2(\omega t) + \cos^2(\omega t))} = \omega$$

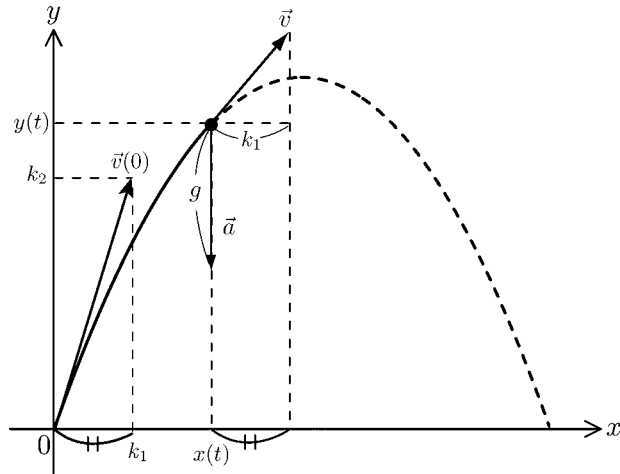


< 47 ページ. 平面上の運動 4 >

問の解答

(1) $\vec{v}(t) = (k_1, k_2 - gt)$

(2) $\vec{a}(t) = (0, -g)$



< 48 ページ. 平面上の運動 5 >

問 1 の解答

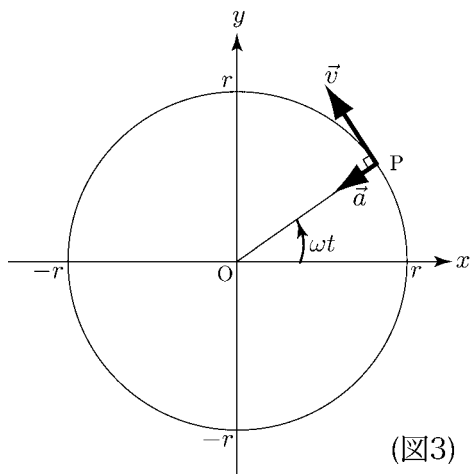
$$|\vec{v}| = \sqrt{(-r \sin t)^2 + (r \cos t)^2} = r$$

$$|\vec{a}| = \sqrt{(-r \cos t)^2 + (-r \sin t)^2} = r$$

問 2 の解答

$$\vec{v} = (-r\omega \sin(\omega t), r\omega \cos(\omega t)) , \quad |\vec{v}| = r|\omega|$$

$$\vec{a} = (-r\omega^2 \cos(\omega t), -r\omega^2 \sin(\omega t)) , \quad |\vec{a}| = r\omega^2$$



< 49 ページ. 微分の練習 3 >

問 1 の解答

$$(1) \lim_{x \rightarrow 0} \frac{\sin(3x)}{2x} = \frac{3}{2} \quad (2) \lim_{x \rightarrow 0} \frac{\tan(2x)}{3x} = \frac{2}{3}$$

$$(3) \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \quad (4) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

問 2 の解答

$$(1) f'(x) = \lim_{h \rightarrow 0} \frac{2\sqrt{x+h} - 2\sqrt{x}}{h} = \frac{1}{\sqrt{x}}$$

$$(2) f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = -\frac{1}{(x+1)^2}$$

問 3 の解答

$$(1) y' = \frac{1}{3\sqrt[3]{x^2}} \quad (2) y' = -\frac{3}{x^4} \quad (3) y' = -\frac{1}{2x\sqrt{x}}$$

$$(4) y' = 2\cos(2x) \quad (5) y' = -8\sin(4x) \quad (6) y' = \frac{5}{\cos^2(5x)}$$

$$(7) y' = \frac{1}{x} \quad (8) y' = \frac{3}{x} \quad (9) y' = -\tan x$$

$$(10) y' = 4e^{4x+1} \quad (11) y' = -xe^{-\frac{x}{2}} \quad (12) y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

$$(13) y' = \frac{3}{2}\sqrt{x} \quad (14) y' = \sin x + x \cos x \quad (15) y' = \cos^2 x - \sin^2 x (= \cos 2x)$$

$$(16) y' = e^x \sin x + e^x \cos x \quad (17) y' = -e^{-x} \cos x - e^{-x} \sin x \quad (18) y' = 3e^{3x} \sin(2x) + 2e^{3x} \cos(2x)$$

$$(19) y' = \frac{-x \sin x - \cos x}{x^2} \quad (20) y' = \frac{1-x}{2\sqrt{x}(1+x)^2} \quad (21) y' = -\frac{1}{2x\sqrt{x}} + \frac{1}{2\sqrt{x}} \left(= \frac{x-1}{2x\sqrt{x}} \right)$$

< 50 ページ. 微分の応用 >

問 1 の解答

(1) $y = \frac{1}{2}x + 1$

(2) $y = -2x + 3$

(3) $y = \frac{\sqrt{3}}{2}x - \frac{\sqrt{3}\pi}{12} + \frac{1}{2}$

(4) $y = 2x - \frac{\pi}{2} + 1$

(5) $y = -\frac{x}{\sqrt{e}} + \frac{2}{\sqrt{e}}$

(6) $y = x$

(7) $y = -\frac{\sqrt{3}}{6}x + \frac{2\sqrt{3}}{3}$

問 2 の解答

$$\begin{aligned}
 (1) \quad y &= -4t^2 + 8t + 5 \\
 &= -(2t)^2 + 4(2t) + 5 \\
 &= -x^2 + 4x + 5 \\
 &= -(x-2)^2 + 9 \quad \text{軌道は右図の放物線}
 \end{aligned}$$

(2) $\vec{v} = (2, -8t + 8)$

$$|\vec{v}| = \sqrt{2^2 + (-8t + 8)^2} = \sqrt{64t^2 - 128t + 68}$$

$$\left(= 2\sqrt{16t^2 - 32t + 17} \right)$$

(3) $\vec{a} = (0, -8)$

$$|\vec{a}| = 8$$

(4) 右図のベクトル

