高知工科大学

基礎数学ワークブック

(2003年度版)

入門編

No. 4

解答

＜ 1 ページ. 面積関数 1 ＞

問 1 の解答

(1) $S(t) = t$

(2) $S(t) = \dfrac{1}{2}t^2$

(3) $S(t) = \dfrac{1}{2}\{1 + (1 + 2t)\}t = t + t^2$

(4) $S(t) = \dfrac{1}{2}\left\{3 + (3 + \dfrac{1}{2}t)\right\}t = 3t + \dfrac{1}{4}t^2$

問 2 の解答

(1) $\displaystyle\int_0^t 1dx = [x]_0^t = t$

(2) $\displaystyle\int_0^t xdx = \left[\frac{1}{2}x^2\right]_0^t = \frac{1}{2}t^2$

(3) $\displaystyle\int_0^t (1 + 2x)dx = \left[x + x^2\right]_0^t = t + t^2$

(4) $\displaystyle\int_0^t (3 + \frac{1}{2}x)dx = \left[3x + \frac{1}{4}x^2\right]_0^t = 3t + \frac{1}{4}t^2$

< 2ページ. 面積関数 2 >

問の解答

(1) $\displaystyle\lim_{h\to 0}\frac{S(t+h)-S(t)}{h}=t^2$

(2) $S'(t)=t^2$

(3) $\displaystyle S(t)=\int S'(t)dt=\int t^2dt=\frac{1}{3}t^3+C$

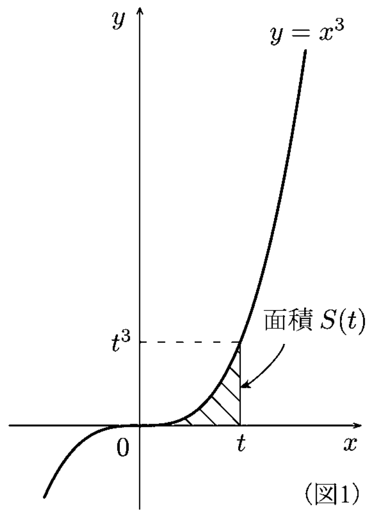
(4) $C=0$, $\displaystyle S(t)=\frac{1}{3}t^3$

＜ 3ページ. 面積関数 3 ＞

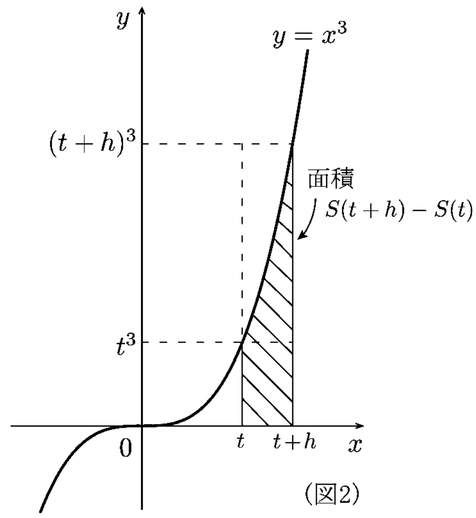
問の解答

問 正の数 t に対し，3 次曲線 $y = x^3$ と x 軸, y 軸および直線 $x = t$ で囲まれた部分の面積を $S(t)$ とおく (図 1)。

$S(t)$ を t の式で表したい。前ページを参考にして，次の文章中の □ 内に適当な文字式を記入せよ。



(図1)



(図2)

「 正の数 h に対し，

$$S(t+h) - S(t)$$

は図 2 の斜線部分の面積である。

そこで x 軸の t から $t+h$ までの部分を底辺とする図 2 の 2 つの長方形の面積を考えると

$$\boxed{t^3h} \leqq S(t+h) - S(t) \leqq \boxed{(t+h)^3h}$$

が成り立つ。すなわち

$$\boxed{t^3} \leqq \frac{S(t+h) - S(t)}{h} \leqq \boxed{(t+h)^3} \cdots$$

となる。

ここで，h を限りなく 0 に近づけたとき，$(t+h)^3$ は限りなく t^3 に近づく。

従って①式より

$$\lim_{h \to 0} \frac{S(t+h) - S(t)}{h} = \boxed{t^3}$$

であることがわかる。すなわち $S(t)$ の導関数は $S'(t) = \boxed{t^3}$ である。

ゆえに $S(t)$ は $\boxed{t^3}$ の不定積分

$$S(t) = \int \boxed{t^3}\, dt = \boxed{\frac{1}{4}t^4} + C$$

である。$S(0) = 0$ より $C = \boxed{0}$ であるから $S(t) = \boxed{\frac{1}{4}t^4}$ である。 」

＜ 4 ページ. 面積 1 ＞

問 1 の解答

$$S = S(3) - S(1) = \frac{1}{3} \times 3^3 - \frac{1}{3} \times 1^3 = \frac{26}{3}$$

問 2 の解答

$$S = S(b) - S(a) = \frac{1}{3}b^3 - \frac{1}{3}a^3$$

問 3 の解答

$$S = S(b) - S(a) = [S(x)]_a^b = \left[\frac{1}{3}x^3\right]_a^b = \int_a^b x^2 dx$$

問 4 の解答

$$S = S(5) - S(1) = \frac{1}{4} \times 5^4 - \frac{1}{4} \times 1^4 = \frac{624}{4} = 156$$

問 5 の解答

$$S = S(b) - S(a) = \frac{1}{4}b^4 - \frac{1}{4}a^4$$

問 6 の解答

$$S = S(b) - S(a) = [S(x)]_a^b = \left[\frac{1}{4}x^4\right]_a^b = \int_a^b x^3 dx$$

＜ 5ページ. 面積 2 ＞

問の解答

(1) $S = \displaystyle\int_1^3 x^2 dx = \left[\frac{1}{3}x^3\right]_1^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$

(2) $$S = \int_0^2 (x+1)^2 dx = \int_0^2 (x^2+2x+1)dx = \left[\frac{1}{3}x^3 + x^2 + x\right]_0^2$$

$$= \frac{8}{3} + 4 + 2 = \frac{8+18}{3} = \frac{26}{3}$$

＜ 6ページ. 面積 3 ＞

問の解答

(1) $S = \int_{-1}^{1}(-x^2+1)dx = \left[-\frac{1}{3}x^3 + x\right]_{-1}^{1} = \frac{4}{3}$

(2) $S = \int_{0}^{4}(-x^2+4x)dx = \left[-\frac{1}{3}x^3 + 2x^2\right]_{0}^{4} = \frac{32}{3}$

(3) $S = \int_{2}^{3}\{-(x-2)(x-3)\}dx = \int_{2}^{3}\{-x^2+5x-6\} = \left[-\frac{x^3}{3} + \frac{5}{2}x^2 - 6x\right]_{2}^{3} = \frac{1}{6}$

(4) $y = -x^2 + 2x + 8 = -(x-4)(x+2)$

$$S = \int_{-2}^{4}(-x^2+2x+8)dx = \left[-\frac{x^3}{3} + x^2 + 8x\right]_{-2}^{4} = 36$$

＜7ページ. 面積 4＞

問の解答

$$\begin{cases} y = -x^2 + 2x + 4 & \cdots ① \\ y = x^2 & \cdots ② \end{cases}$$

②＝① より

$x^2 = -x^2 + 2x + 4 \Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$

$$S = \int_{-1}^{2}(-x^2 + 2x + 4)dx - \int_{-1}^{2} x^2 dx$$

$$= \int_{-1}^{2}(-2x^2 + 2x + 4)dx = \left[-\frac{2}{3}x^3 + x^2 + 4x\right]_{-1}^{2}$$

$$= 9$$

＜ 8 ページ. 面積 5 ＞

問 1 の解答

$$S = \int_a^b \{f(x) + C\}dx - \int_a^b \{g(x) + C\}dx$$

$$= \int_a^b \{f(x) - g(x)\}dx$$

問 2 の解答

$$S = \int_1^2 \{(-x^2 + 3x) - (x^2 - 2x)\}dx$$

$$= \int_1^2 \{-2x^2 + 5x\}dx = \left[-\frac{2}{3}x^3 + \frac{5}{2}x^2\right]_1^2$$

$$= \frac{17}{6}$$

問 3 の解答

$$S = \int_1^2 \{(-x^2 + 4x - 1) - (x^2 - 3)\}dx = \int_1^2 (-2x^2 + 4x + 2)dx$$

$$= \left[-\frac{2}{3}x^3 + 2x^2 + 2x\right]_1^2$$

$$= \frac{10}{3}$$

＜ 9 ページ. 面積 6 ＞

問 1 の解答

(1) $S=\int_1^2\left\{-(x-1)(x-2)\right\}dx=\int_1^2(-x^2+3x-2)dx=\left[-\frac{x^3}{3}+\frac{3}{2}x^2-2x\right]_1^2=\frac{1}{6}$

(2) $S=\int_{-3}^0(-x^2-3x)dx=\left[-\frac{1}{3}x^3-\frac{3}{2}x^2\right]_{-3}^0=\frac{9}{2}$

問 2 の解答

(1) 放物線 $y=x^2$ と直線 $y=-x+6$ との交点の x 座標は

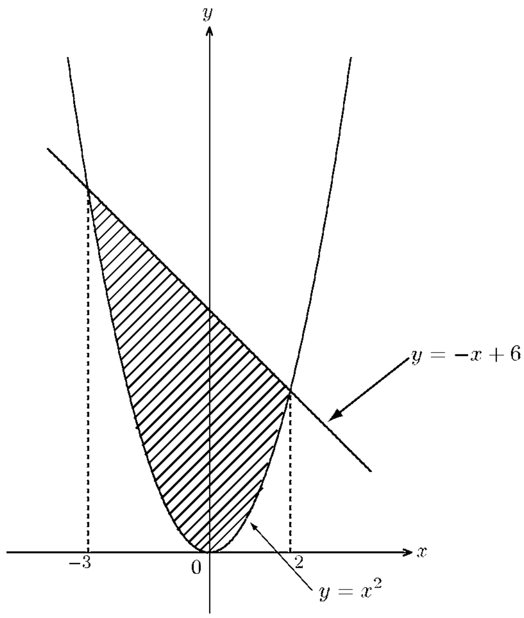
$x^2=-x+6 \Rightarrow (x-2)(x+3)=0$

より $x=2,\ -3$

$S=\int_{-3}^2(-x+6-x^2)dx$

$=\left[-\frac{1}{3}x^3-\frac{x^2}{2}+6x\right]_{-3}^2$

$=\frac{125}{6}$



(2) 放物線 $y=-x^2+3$ と直線 $y=2x$ との交点の x 座標は

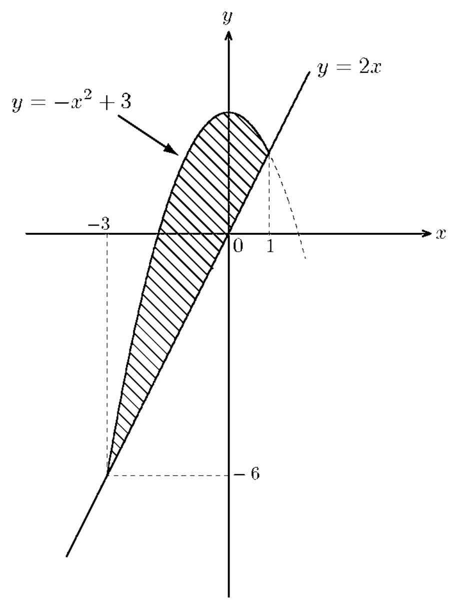
$-x^2+3=2x \Rightarrow -(x-1)(x+3)=0$

より $x=1,\ -3$

$S=\int_{-3}^1(-x^2+3-2x)dx$

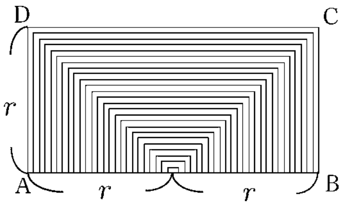
$=\left[-\frac{x^3}{3}-x^2+3x\right]_{-3}^1$

$=\frac{32}{3}$



＜ 10 ページ. 線と面 ＞

問の解答



$$\ell(x) = \mathrm{B'C'} + \mathrm{C'D'} + \mathrm{D'A'} = 4x$$

$$\left(\int_0^r \ell(x)dx = \int_0^r 4xdx = \left[2x^2\right]_0^r = 2r^2 = S \right)$$

＜ 11 ページ. 面積の計算 ＞

問 1 の解答

(1) $S = \int_{-1}^{2} (-x^2 + x + 2)dx = \dfrac{9}{2}$

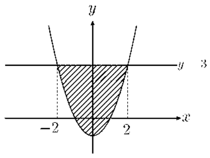
(2) $S = \int_{-4}^{1} (-x^2 - 3x + 4)dx = \dfrac{125}{6}$

(3) $S = \int_{1}^{3} (-x^2 + 4x - 3)dx = \dfrac{4}{3}$

(4) $S = \int_{-4}^{2} (-x^2 - 2x + 8)dx = 36$

問 2 の解答

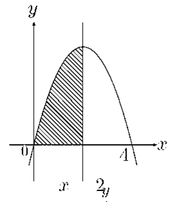
(1) 放物線 $y = x^2 - 1$ と直線 $y = 3$ で囲まれた図形



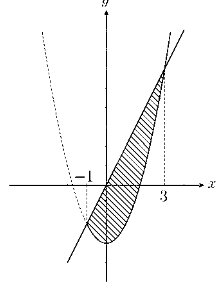
$$\int_{-2}^{2} \left\{3 - (x^2 - 1)\right\}dx = \int_{-2}^{2} (4 - x^2)dx = \left[4x - \frac{x^2}{3}\right]_{-2}^{2} = \frac{32}{3}$$

(2) 放物線 $y = -x^2 + 4x$ と x 軸および直線 $x = 2$ で囲まれた左側 $(x \leqq 2)$ の図形

$$\int_{0}^{2} (-x^2 + 4x)dx = \left[-\frac{x^3}{3} + 2x^2\right]_{0}^{2} = \frac{16}{3}$$



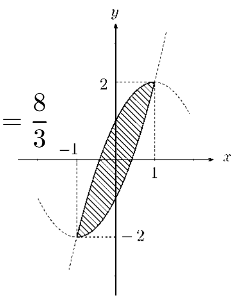
(3) 放物線 $y = x^2 - 3$ と直線 $y = 2x$ で囲まれた図形



$$\int_{-1}^{3} \left\{2x - (x^2 - 3)\right\}dx = \int_{-1}^{3} (-x^2 + 2x + 3)dx = \left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^{3} = \frac{32}{3}$$

(4) 放物線 $y = x^2 + 2x - 1$ と $y = -x^2 + 2x + 1$ で囲まれた図形

$$\int_{-1}^{1} \left\{(-x^2 + 2x + 1) - (x^2 + 2x + 1)\right\}dx = \int_{-1}^{1} (-2x^2 + 2)dx = \left[-\frac{2}{3}x^3 + 2x\right]_{-1}^{1} = \frac{8}{3}$$

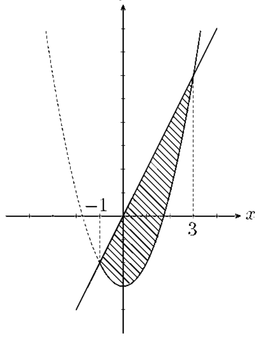


問 3 の解答

(1) $\ell(x) = \dfrac{1}{8} \times 2\pi x = \dfrac{\pi}{4}x$

(2) $S = \int_{a}^{b} \ell(x)dx$

(3) $S = \int_{a}^{b} \dfrac{\pi}{4}xdx = \left[\dfrac{\pi}{8}x^2\right]_{a}^{b} = \dfrac{\pi}{8}b^2 - \dfrac{\pi}{8}a^2$



問 3 の別解

(3) 半径 x, 中心角 45° の扇形の面積を $S(x)$ とする。

$S(x)$ は円の面積 πx^2 の $\frac{1}{8}$ であるから, $S(x) = \dfrac{\pi x^2}{8}$ である。よって

$S = S(b) - S(a) = \dfrac{\pi b^2}{8} - \dfrac{\pi a^2}{8}$

＜ 12 ページ. 累乗の指数 ＞

問 1 の解答

1 京 $= 10^4$ 兆 $= 10^4 \times 10^{12} = 10^{16}$

問 2 の解答

1 メガ $= 1000$ キロ $= 10^3 \times 10^3 = 10^6$

1 ギガ $= 10^3$ メガ $= 10^9$

1 テラ $= 10^3$ ギガ $= 10^{12}$

問 3 の解答

(1) $10^4 \times 10^6 = 10^{10}$

(2) $10^8 \times 10^9 = 10^{17}$

(3) $10^{15} \times 10^{27} = 10^{42}$

(4) $10^5 \div 10^2 = 10^3$

(5) $10^9 \div 10^5 = 10^4$

(6) $10^{23} \div 10^{15} = 10^8$

＜ 13 ページ. 指数法則 ＞

問 1 の解答

(1) $a^2 \times a^5 = a^7$

(2) $a^6 \times a^3 = a^9$

(3) $a^8 \div a^3 = a^5$

問 2 の解答

(1) $(a^5)^3 = a^{15}$

(2) $(ab^2)^3 = a^3b^6$

(3) $(a^2b^3)^4 = a^8b^{12}$

問 3 の解答

(1) $a^4 \times a^5 \times a^3 = a^{4+5+3} = a^{12}$

(2) $(a^3)^2 \times a^4 = a^{6+4} = a^{10}$

(3) $a^{10} \div (a^3)^3 = a$

(4) $(a^2b)^3 \times (ab^2)^2 = a^6b^3 \times a^2b^4 = a^8b^7$

< 14ページ. 負の指数 >

問の解答

(1) $2^0 = 1$

(2) $1^{-1} = 1$

(3) $2^{-5} = \dfrac{1}{32}$

(4) $3^0 = 1$

(5) $3^{-2} = \dfrac{1}{9}$

(6) $4^{-3} = \dfrac{1}{64}$

(7) $(2^{-3})^2 = \dfrac{1}{64}$

(8) $(3^2)^{-2} = \dfrac{1}{81}$

(9) $(2^2)^{-3} = \dfrac{1}{64}$

< 15 ページ. 整数指数 1 >

問 1 の解答

(1) $a^3 \times a^{-5} = a^{-2}$

(2) $a^{-3} \times a^2 = a^{-1}$

(3) $a^4 \times a^{-7} = a^{-3}$

(4) $a^{-8} \times a^5 = a^{-3}$

(5) $a^{-3} \times a^{-4} = a^{-7}$

(6) $a^{-5} \times a^{-6} = a^{-11}$

問 2 の解答

(1) $a^4 \div a^6 = a^{-2}$

(2) $a^3 \div a^{-2} = a^5$

(3) $a^{-2} \div a^3 = a^{-5}$

(4) $a^4 \div a^{-5} = a^9$

(5) $a^{-7} \div a^{-4} = a^{-3}$

(6) $a^{-7} \div a^{-9} = a^2$

＜ 16 ページ. 整数指数 2 ＞

問 1 の解答

(1) $(a^4)^{-2} = a^{-8}$

(2) $(a^{-2})^4 = a^{-8}$

(3) $(a^{-5})^{-2} = a^{-10}$

(4) $(a^{-3})^4 = a^{-12}$

(5) $(a^{-3})^{-3} = a^9$

(6) $(a^{-6})^{-5} = a^{30}$

問 2 の解答

(1) $(ab)^4 = a^4b^4$

(2) $(a^2b)^3 = a^6b^3$

(3) $(ab^{-1})^2 = a^2b^{-2}$

(4) $(ab)^{-3} = a^{-3}b^{-3}$

(5) $(a^{-1}b^2)^3 = a^{-3}b^6$

(6) $(a^{-2}b^3)^{-2} = a^4b^{-6}$

(7) $(ab^2)^{-3} \times (a^2b)^2 = a^{-3}b^{-6} \times a^4b^2 = ab^{-4}$

(8) $(a^3b^2)^3 \div (ab^3)^4 = (a^9b^6) \div (a^4b^{12}) = a^5b^{-6}$

＜ 17ページ. 整数指数 3 ＞

問 1 の解答

(1) $10^3 \times 10^{-20} = 10^{-17}$ (2) $10^5 \div 10^{-6} = 10^{11}$ (3) $10^{-7} \div 10^{-8} = 10$

(4) $1 \div 10^{-20} = 10^{20}$ (5) $(10^{-2})^3 = 10^{-6}$ (6) $(10^{-3})^{-1} = 10^3$

問 2 の解答

(1) $a^{-3} \times a^{-1} = a^{-4}$ (2) $a^{-3} \times a^{-5} = a^{-8}$ (3) $a^4 \div a^{-2} = a^6$

(4) $a^{-4} \div a^{-2} = a^{-2}$ (5) $(a^3)^{-3} = a^{-9}$ (6) $(a^{-4})^{-1} = a^4$

問 3 の解答

(1) $4^7 \times 4^{-4} = 4^3 = 64$ (2) $5^{-7} \div 5^{-6} = 5^{-1} = \dfrac{1}{5}$

(3) $2^{-2} \div 2^{-5} = 2^3 = 8$ (4) $(3^{-2})^{-2} = 3^4 = 81$

問 4 の解答

(1) $a^3 \times a^{-6} = \dfrac{1}{a^3}$ (2) $a^{-4} \div a^{-10} = a^6$ (3) $a^{-5} \div a^{-5} = 1$

(4) $(ab^{-1})^2 = \dfrac{a^2}{b^2}$ (5) $(a^{-1}b)^{-3} = \dfrac{a^3}{b^3}$ (6) $(a^{-2}b^3)^2 = \dfrac{b^6}{a^4}$

＜ 18 ページ. 整数指数 4 ＞

問 1 の解答

1 マイクロ $= 10^{-3}$ ミリ $= 10^{-3} \times 10^{-3} = 10^{-6}$

1 ナノ $= 10^{-3}$ マイクロ $= 10^{-3} \times 10^{-6} = 10^{-9}$

1 ピコ $= 10^{-3}$ ナノ $= 10^{-3} \times 10^{-9} = 10^{-12}$

問 2 の解答

(1) $43000 = 4.3 \times 10^4$

(2) $2730000000 = 2.73 \times 10^9$

(3) $0.000045 = 4.5 \times 10^{-5}$

(4) $0.000000000000368 = 3.68 \times 10^{-13}$

問 3 の解答

(1) $(1.5 \times 10^4) \times (4 \times 10^8) = 6.0 \times 10^{12}$

(2) $(4.2 \times 10^{-2}) \times (2.5 \times 10^{-7}) = 4.2 \times 2.5 \times 10^{-9} = 1.05 \times 10^{-10}$

(3) $(6.8 \times 10^{-2}) \times (5 \times 10^3) = \dfrac{6.8}{5} \times 10^{-5} = 1.36 \times 10^{-5}$

(4) $(4.8 \times 10^2) \times (1.2 \times 10^{-10}) = \dfrac{4.8}{1.2} \times 10^{12} = 4.0 \times 10^{12}$

(5) $1 \div (2.5 \times 10^{-15}) = \dfrac{1}{2.5} \times 10^{15} = \dfrac{4}{10} \times 10^{15} = 0.4 \times 10^{15} = 4 \times 10^{14}$

問 4 の解答

$$
\begin{aligned}
1 \div (4.5 \times 10^{-23}) &= \frac{1}{4.5} \times 10^{23} = \frac{2}{9} \times 10^{23} \\
&= 0.222 \times 10^{23} \\
&\fallingdotseq 2.2 \times 10^{22}
\end{aligned}
$$

＜ 19 ページ. 累乗根 1 ＞

問の解答

(1) $\sqrt{169} = 13$

(2) $\sqrt[3]{8} = 2$

(3) $\sqrt[3]{125} = 5$

(4) $\sqrt[4]{256} = 4$

(5) $\sqrt[4]{\dfrac{81}{625}} = \dfrac{3}{5}$

(6) $\sqrt[5]{3125} = 5$

＜ 20 ページ. 累乗根 2 ＞

問の解答

(1) $\sqrt[3]{3} \times \sqrt[3]{5} = \sqrt[3]{15}$

(2) $\sqrt[4]{2} \times \sqrt[4]{4} = \sqrt[4]{8}$

(3) $\dfrac{\sqrt[3]{3}}{\sqrt[3]{15}} = \sqrt[3]{\dfrac{1}{5}} \ \left(= \dfrac{1}{\sqrt[3]{5}}\right)$

(4) $\dfrac{\sqrt[5]{128}}{\sqrt[5]{4}} = \sqrt[5]{\dfrac{128}{4}} = \sqrt[5]{32} = 2$

＜ 21 ページ. 累乗根 3 ＞

問 1 の解答

(1) $\sqrt[3]{54} = 3\sqrt[3]{2}$

(2) $\sqrt[4]{112} = 2\sqrt[4]{7}$

(3) $\sqrt[5]{64} = 2\sqrt[5]{2}$

問 2 の解答

(1) $(\sqrt[4]{25})^2 = 5$

(2) $(\sqrt[6]{4})^3 = 2$

(3) $\sqrt[4]{16^2} = 4$

(4) $\sqrt[3]{27^2} = 9$

< 22 ページ. 分数指数 1 >

問 1 の解答

(1) $121^{\frac{1}{2}} = 11$

(2) $27^{\frac{1}{3}} = 3$

(3) $25^{\frac{3}{2}} = 125$

(4) $343^{\frac{2}{3}} = 49$

(5) $81^{\frac{5}{4}} = 243$

(6) $32^{\frac{4}{5}} = 16$

(7) $16^{-\frac{1}{2}} = \dfrac{1}{\sqrt{16}} = \dfrac{1}{4}$

(8) $27^{-\frac{4}{3}} = \dfrac{1}{81}$

(9) $64^{-\frac{2}{3}} = \dfrac{1}{16}$

問 2 の解答

(1) $\sqrt[3]{a} = a^{\frac{1}{3}}$

(2) $\sqrt[3]{a^2} = a^{\frac{2}{3}}$

(3) $(\sqrt[4]{a})^5 = a^{\frac{5}{4}}$

(4) $(\sqrt[4]{a})^{-3} = a^{-\frac{3}{4}}$

＜ 23 ページ. 分数指数 2 ＞

問 1 の解答

(1) $\sqrt[6]{4^3} = 2$

(2) $\sqrt[12]{7^4} = \sqrt[3]{7}$

(3) $\sqrt[3]{5^9} = 125$

(4) $\sqrt[6]{27^4} = 9$

問 2 の解答

(1) $\sqrt[2]{10} \times \sqrt[4]{100} = 10$

(2) $\dfrac{\sqrt[3]{9}}{\sqrt[6]{9}} = \dfrac{\sqrt[3]{9}}{\sqrt[3]{3}} = \sqrt[3]{3}$

(3) $\sqrt{\sqrt[3]{9}} = \sqrt[3]{3}$

(4) $(\sqrt[3]{\sqrt{27}})^2 = (((3^3)^{\frac{1}{2}})^{\frac{1}{3}})^2 = 3^{3\times\frac{1}{2}\times\frac{1}{3}\times 2} = 3$

＜ 24 ページ. 指数法則の拡張 ＞

問１の解答

正の数 a と b、および有理数 p と q に対して

$1^\circ : a^p \times a^q = a^{\boxed{p+q}}$, $2^\circ : a^p \div a^q = a^{\boxed{p\text{-}q}}$

$3^\circ : (a^p)^q = a^{\boxed{pq}}$, $4^\circ : (ab)^p = a^p b^p$

問２の解答

(1) $\sqrt[4]{a} \times \sqrt[4]{a^3} = a$

(2) $\sqrt[3]{a^4} \div \sqrt[3]{a} = a$

(3) $(\sqrt[3]{a})^4 \times \sqrt[3]{a^2} = a^{\frac{4}{3}+\frac{2}{3}} = a^2$

(4) $\sqrt[3]{a^7} \div (\sqrt[3]{a})^4 = a^{\frac{7}{3}-\frac{4}{3}} = a$

(5) $(\sqrt[4]{a})^{\frac{8}{3}} = a^{\frac{1}{4}\times\frac{8}{3}} = a^{\frac{2}{3}} = \sqrt[3]{a^2}$

(6) $\left(\sqrt[5]{\sqrt[4]{a^{-3}}}\right)^{-2} = \left(\left((a^{-3})^{\frac{1}{4}}\right)^{\frac{1}{5}}\right)^{-2} = a^{-3\times\frac{1}{4}\times\frac{1}{5}\times(-2)} = a^{\frac{3}{10}} = \sqrt[10]{a^3}$

問３の解答

(1) $(3^3 \times 5^2)^{\frac{1}{7}} \times (3^4 \times 5^5)^{\frac{1}{7}} = 3^{\frac{3}{7}+\frac{4}{7}} \times 5^{\frac{2}{7}\times\frac{5}{7}} = 3 \times 5 = 15$

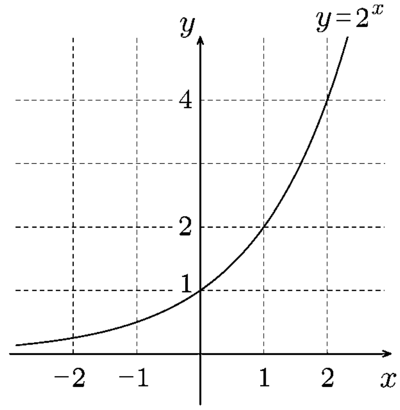
(2) $\sqrt[4]{18} \times \sqrt[4]{72} = (2 \times 3^2)^{\frac{1}{4}} \times (2^3 \times 3^2)^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} \times (3^4)^{\frac{1}{4}} = 6$

< 25 ページ. 指数関数 >

問の解答

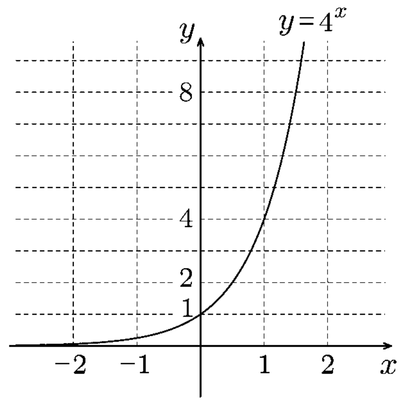
(1) $y = 2^x$

x	-2	-1	0	$\frac{1}{2}$	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	$\sqrt{2}$	2	4



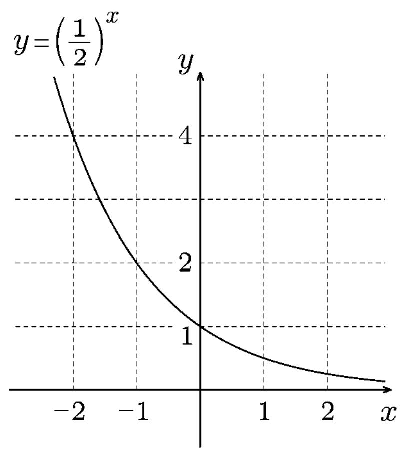
(2) $y = 4^x$

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



(3) $y = \left(\frac{1}{2}\right)^x$

x	-2	-1	0	1	2
y	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



< 26 ページ. 指数方程式 >

問の解答

(1) $x = 0$	(2) $x = 1$	(3) $x = 2$	(4) $x = -1$
(5) $x = \frac{1}{2}$	(6) $x = 0$	(7) $x = 2$	(8) $x = \frac{1}{3}$
(9) $x = -1$	(10) $x = -2$	(11) $x = 0$	(12) $x = 2$
(13) $x = 5$	(14) $x = \frac{1}{4}$	(15) $x = \frac{3}{2}$	(16) $x = -1$
(17) $x = -3$	(18) $x = -2$	(19) $x = -\frac{1}{2}$	(20) $x = 0$
(21) $x = 1$	(22) $x = 3$	(23) $x = 2$	(24) $x = -1$
(25) $x = -2$	(26) $x = -\frac{1}{2}$	(27) $x = 0$	(28) $x = 2$
(29) $x = \frac{1}{2}$	(30) $x = \frac{3}{2}$	(31) $x = -1$	(32) $x = \frac{1}{4}$

＜ 27 ページ. 対数 1 ＞

問 1 の解答

(1) $\dfrac{1}{2} = \log_2 \sqrt{2}$

(2) $-1 = \log_5 \dfrac{1}{5}$

(3) $27 = 3^3$

(4) $27 = 9^{\frac{3}{2}}$

問 2 の解答

(1) $\log_2 64 = 6$

(2) $\log_3 243 = 5$

(3) $\log_{10} 1000 = 3$

(4) $\log_5 625 = 4$

＜ 28 ページ. 対数 2 ＞

問 1 の解答

(1) $\log_2 64 = 6$ (2) $\log_2 \sqrt{2} = \dfrac{1}{2}$ (3) $\log_2 0.5 = -1$

(4) $\log_2(2\sqrt{2}) = \dfrac{3}{2}$ (5) $\log_4 64 = 3$ (6) $\log_4 1 = 0$

(7) $\log_6 \sqrt[3]{6} = \dfrac{1}{3}$ (8) $\log_5 0.2 = -1$ (9) $\log_{10} 0.01 = -2$

(10) $\log_7 \sqrt[3]{49} = \dfrac{2}{3}$ (11) $\log_2\left(\dfrac{1}{\sqrt{2}}\right) = -\dfrac{1}{2}$ (12) $\log_4 8 = \dfrac{3}{2}$

問 2 の解答

$\log_2(M \times N) = \log_2(2^{\alpha} \times 2^{\beta}) = \log_2(2^{\alpha+\beta}) = \alpha + \beta$

$\log_2 M + \log_2 N = \log_2 2^{\alpha} + \log_2 2^{\beta} = \alpha + \beta$

よって $\log_2(M \times N) = \log_2 M + \log_2 N$

< 29 ページ. 対数 3 >

問 1 の解答

$$\log_2\left(\frac{M}{N}\right) = \log_2\left(\frac{2^\alpha}{2^\beta}\right) = \log_2(2^{\alpha-\beta}) = \alpha - \beta$$

$$\log_2 M - \log_2 N = \log_2(2^\alpha) - \log_2(2^\beta) = \alpha - \beta$$

より $\log_2\left(\frac{M}{N}\right) = \log_2 M - \log_2 N$

問 2 の解答

$$\log_2(M^r) = \log_2\left((2^\alpha)^r\right) = \log_2(2^{\alpha r}) = \alpha r$$

$$r \times \log_2 M = r \times \log_2(2^\alpha) = r \times \alpha = \alpha r$$

よって $\log_2(M^r) = r \times \log_2 M$

＜ 30 ページ. 対数 4 ＞

問 1 の解答

$$\log_a(\frac{M}{N}) = \log_a M - \log_a N$$

問 2 の解答

$$\log_a(M^r) = r \times \log_a M$$

問 3 の解答

(1) $\log_2 12 + \log_2(\frac{1}{3}) = \log_2 4 = 2$

(2) $\log_3 108 - \log_3 4 = \log_3 \frac{108}{3} = \log_3 27 = 3$

(3)
$$\begin{aligned}\log_6 12 + \log_6 2 + 2\log_6 3 &= \log_6(12 \times 2 \times 3^2) \\ &= \log_6(2^3 \times 3^3) \\ &= \log_6(6^3) = 3\end{aligned}$$

(4) $\log_{10} 4 + \log_{10} 25 + \log_{10} 0.1 = \log_{10}(\frac{4 \times 25}{0.1}) = \log_{10}(\frac{100 \times 10}{1}) = 3$

< 31 ページ. 底の変換 >

問 1 の解答

(1) $\log_8 16 = \dfrac{\log_2 16}{\log_2 8} = \dfrac{4}{3}$

(2) $\log_{16} 64 = \dfrac{\log_4 64}{\log_4 16} = \dfrac{3}{2}$

(3) $\log_{27} 81 = \dfrac{\log_3 81}{\log_3 27} = \dfrac{4}{3}$

(4) $\log_8 2 = \dfrac{\log_2 2}{\log_2 8} = \dfrac{1}{3}$

(5) $\log_4 \sqrt{2} = \dfrac{\log_2 \sqrt{2}}{\log_2 4} = \dfrac{\frac{1}{2}}{2} = \dfrac{1}{4}$

(6) $\log_{27} \sqrt{3} = \dfrac{\log_3 \sqrt{3}}{\log_3 27} = \dfrac{\frac{1}{2}}{3} = \dfrac{1}{6}$

問 2 の解答

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$$

問 3 の解答

(1) $\log_4 32 + \log_{16} 64 = \dfrac{\log_2 32}{\log_2 4} + \dfrac{\log_4 64}{\log_4 16} = \dfrac{5}{2} + \dfrac{3}{2} = \dfrac{8}{2} = 4$

(2) $(\log_3 4) \times (\log_4 9) = \log_3 4 \times \dfrac{\log_3 9}{\log_3 4} = 2$

(3) $(\log_2 3) \times (\log_3 4) \times (\log_4 2) = \log_2 3 \times \dfrac{\log_2 4}{\log_2 3} \times \dfrac{1}{\log_2 4} = 1$

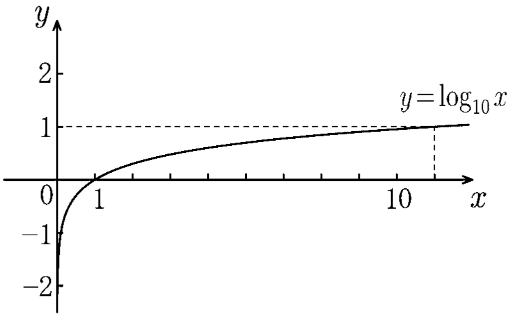
＜ 32 ページ. 対数関数 ＞

問の解答

(1) $y = \log_{10} x \quad (x > 0)$

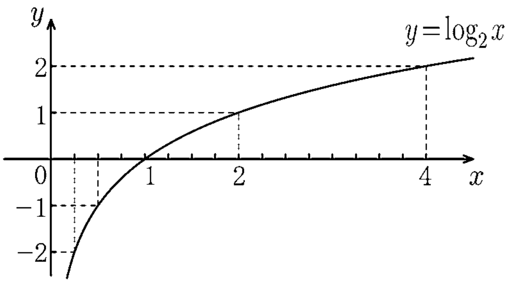
x	0.1	1	$\sqrt{10}$	10
y	-1	0	$\frac{1}{2}$	1

注) $\sqrt{10} \fallingdotseq 3.16$



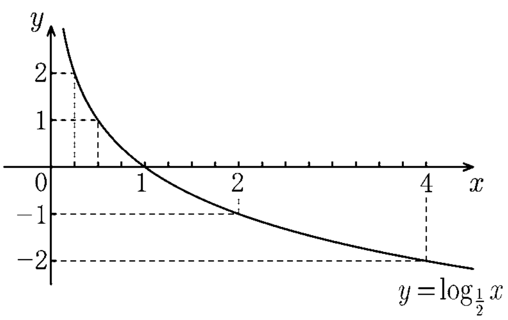
(2) $y = \log_2 x \quad (x > 0)$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
y	-2	-1	0	1	2



(3) $y = \log_{\frac{1}{2}} x \quad (x > 0)$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
y	2	1	0	-1	-2



< 33ページ.常用対数 1 >

問 1 の解答

(1) $\log_{10}(1410) = \log_{10} 1.41 + \log_{10} 1000 = 0.1492 + 3 = 3.1492$

(2) $\log_{10}(203000) = \log_{10} 2.03 + \log_{10} 100000 = 0.3075 + 5 = 5.3075$

(3) $\log_{10} 0.00302 = \log_{10}(3.02 \times 10^{-3}) = 0.48 - 3 = -2.5200$

問 2 の解答

$x = 3^{50}$ とおくと

$\log_{10} x = \log_{10} 3^{50} = 50 \times \log_{10} 3 = 50 \times 0.4771 = 23.855$

$\Rightarrow x = 10^{23.855} \Rightarrow 10^{23} < x < 10^{24}$

(答) 24 桁

< 34 ページ. 常用対数 2 >

問 1 の解答

$$\log_{10}\sqrt[4]{5} = \frac{1}{4}\log_{10}5 = \frac{1}{4}\log_{10}\frac{10}{2} = \frac{1}{4}(\log_{10}10 - \log_{10}2)$$

$$= \frac{1}{4}(1 - 0.301) = \frac{1}{4} \times 0.699 = 0.17475$$

問 2 の解答

$$\log_2 3 = \frac{\log_{10}3}{\log_{10}2} = \frac{0.4771}{0.301} \fallingdotseq 1.5850$$

問 3 の解答

$$x = 0.5^{20} \Rightarrow \log_{10}x = 20\log_{10}0.5 = 20 \times \log_{10}\frac{1}{2} = -20 \times \log_{10}2 = -20 \times 0.301 = -6.02$$

$$x = 10^{-6.02} \Rightarrow 10^{-7} < x < 10^{-6} \Rightarrow 0.0000001 < x < 0.000001$$

(答) 小数第 7 位

問 4 の解答

時間	$\cdots$	倍率
30 分	$\cdots$	$2 = 2^1$
1 時間	$\cdots$	$4 = 2^2$
1.5 時間	$\cdots$	$8 = 2^3$
2 時間	$\cdots$	$16 = 2^4$
2.5 時間	$\cdots$	$32 = 2^5$
3 時間	$\cdots$	$64 = 2^6$
x 時間	$\cdots$	2^{2x}

より求める時間を x とすると

$$2^{2x} = 1000$$

$$\log_{10}(2^{2x}) = \log_{10}1000$$

$$2x\log_{10}2 = 3$$

$$x = \frac{3}{2\log_{10}2} = \frac{3}{0.602} = \frac{3000}{602} = 4.983$$

$$x = 4.983\mathrm{h}$$

$$0.983\mathrm{h} = 0.983 \times 60\mathrm{min} = 58.98\mathrm{min}$$

(答) 約 4 時間 59 分後

< 35 ページ. 指数・対数の練習 1 >

問 1 の解答

(1) $27^{-\frac{2}{3}} = 3^{-2} = \frac{1}{9}$ (2) $(0.25)^{0.5} = \left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1}{2}$ (3) $\frac{1}{(0.2)^{-2}} = \frac{1}{(\frac{1}{5})^{-2}} = \frac{1}{5^2} = \frac{1}{25}$

(4) $\log_9 27 = \frac{3}{2}$ (5) $\log_{\frac{1}{2}} 16 = \frac{4}{-1} = -4$

(6) $\log_{0.2} 125 = \log_{\frac{1}{5}} 125 = \frac{\log_5 125}{\log_5 \frac{1}{5}} = \frac{3}{-1} = -3$

問 2 の解答

(1) $36^{1.5} \times 32^{-0.2} = (2^2 \times 3^2)^{\frac{3}{2}} \times (2^5)^{-\frac{1}{5}} = 2^3 \times 3^3 \times 2^{-1} = 108$

(2) $\frac{(\frac{1}{4})^2 \times (\frac{1}{8})^{-4} \div 16}{(\frac{1}{2})^3} = (2^{-2})^2 \times (2^{-3})^{-4} \div (2^4 \times 2^{-3}) = 128$

(3) $(a^3b)^{\frac{1}{6}} \times (ab^2)^{\frac{2}{3}} \div (ab^{-3})^{\frac{1}{6}} = (a^{\frac{1}{2}}b^{\frac{1}{6}}) \times (a^{\frac{2}{3}}b^{\frac{4}{3}}) \div (a^{\frac{1}{6}}b^{-\frac{1}{2}}) = a^{\frac{1}{2}+\frac{2}{3}-\frac{1}{6}} \times b^{\frac{1}{6}+\frac{4}{3}+\frac{1}{2}} = ab^2$

(4) $\sqrt[6]{a^5b} \times \sqrt{ab} \div \sqrt[3]{a^4b^{-1}} = a^{\frac{5}{6}}b^{\frac{1}{6}} \times a^{\frac{1}{2}}b^{\frac{1}{2}} \div a^{\frac{4}{3}}b^{-\frac{1}{3}} = a^{\frac{5}{6}+\frac{1}{2}-\frac{4}{3}}b^{\frac{1}{6}+\frac{1}{2}+\frac{1}{3}} = b$

(5) $108^{0.2} \times 72^{0.2} = (2^2 \times 3^3)^{\frac{1}{5}} \times (2^3 \times 3^2)^{\frac{1}{5}} = 2 \times 3 = 6$

(6) $\sqrt[14]{800} \times \sqrt[14]{12500} = (2^5 5^2)^{\frac{1}{14}} \times (2^2 \times 5^5)^{\frac{1}{14}} = \sqrt{10}$

問 3 の解答

(1) $\log_5 20 + \log_5 100 - 2\log_5 4 = \log_5\left(\frac{20 \times 100}{16}\right) = 3$

(2) $4\log_3 \sqrt{3} - \frac{1}{2}\log_3 2 + \log_3 \frac{\sqrt{2}}{3} = \log_3\left(\frac{9 \times \sqrt{2}}{\sqrt{2} \times 3}\right) = 1$

(3) $\frac{1}{2}\log_5 3 + 3\log_5 \sqrt{2} - \log_5 \sqrt{24} = \log_5\left(\frac{\sqrt{3} \times 2\sqrt{2}}{\sqrt{24}}\right) = \log_5 1 = 0$

(4) $(\log_4 5)(\log_5 6)(\log_6 8) = \frac{\log_2 5}{\log_2 4} \times \frac{\log_2 6}{\log_2 5} \times \frac{\log_2 8}{\log_2 6} = \frac{\log_2 8}{\log_2 4} = \frac{3}{2}$

(5) $\log_3 \frac{3}{2} + \log_9 \frac{81}{4} + \log_{27} \frac{64}{27} = \log_3 \frac{3}{2} + \frac{\log_3 \frac{81}{4}}{\log_3 9} + \frac{\log_3 \frac{64}{27}}{\log_3 27}$

$= \log_3 \frac{3}{2} + \frac{1}{2}\log_3(\frac{81}{4}) + \frac{1}{3}\log_3(\frac{64}{27}) = \log_3\left(\frac{3}{2} \times \frac{9}{2} \times \frac{4}{3}\right)$

$= \log_3 9 = 2$

(6) $(\log_2 3 + \log_4 9)(\log_3 4 + \log_9 2) = \log_2 3\log_3 4 + \log_2 3\log_9 2 + \log_4 9\log_3 4 + \log_4 9\log_9 2$

＜ 36 ページ. 指数・対数の練習 2 ＞

問 1 の解答

(1) $x = 5.5 \ \left(= \dfrac{11}{2}\right)$ (2) $x = -\dfrac{1}{6}$

(3) $x = 2,\ -4$ (4) $x = 6$

問 2 の解答

(1) $x > -\dfrac{1}{2}$ (2) $\dfrac{5}{2} < x \leqq \dfrac{11}{4}$

問 3 の解答

(1) $(0.9)^{0.5} < 1 < (0.9)^{-\frac{1}{3}}$ (2) $3 < \dfrac{1}{2}\log_2 81 < \log_2 10$

(3) $\sqrt[6]{7} < \sqrt{2} < \sqrt[3]{3}$ (4) $\sqrt[3]{10} < \sqrt{7} < 3$

問 4 の解答

(1) $\log_{10}\sqrt{\dfrac{8}{9}} = \dfrac{1}{2}\log_{10} 8 - \dfrac{1}{2}\log_{10} 9 = \dfrac{3}{2}a - b$

(2) $\log_2 6 = \dfrac{\log_{10} 6}{\log_{10} 2} = \dfrac{a+b}{a} \ \left(= 1 + \dfrac{b}{a}\right)$

(3) $\log_9 \sqrt{5} = \dfrac{\log_{10}\sqrt{5}}{\log_{10} 9} = \dfrac{\log_{10} 10 - \log_{10} 2}{4\log_{10} 3} = \dfrac{1-a}{4b}$

問 5 の解答

(1) $x = 6^{40} \Rightarrow \log_{10} x = 40(\log_{10} 2 + \log_{10} 3) = 31.124$

$\Rightarrow x = 10^{31.124}$ (答) 32 桁

(2) $x = \left(\dfrac{1}{12}\right)^{10} \Rightarrow \log_{10} x = 10\log_{10}\dfrac{1}{12} = 10(-2\log_{10} 2 - \log_{10} 3) = -10.781$

$x = 10^{-10.781}$ (答) 少数第 11 位

問 6 の解答

x 時間後に 10 万倍になるとすると

$2^{\frac{x}{3}} = 100000 = 10^5$

$\log_{10} 2^{\frac{x}{3}} = 5$

$x = \dfrac{15}{\log_{10} 2} = \dfrac{15000}{301} = 49.833$

$0.834\text{h} = 50.04\text{min}$

(答) 49 時間 50 分

＜ 37 ページ. 三角関数の復習 1 ＞

問 1 の解答

$$1 + \tan^2\theta = \frac{1}{\cos^2\theta}$$

問 2 の解答

$$\cos\theta = -\sqrt{1 - \sin^2\theta}$$

問 3 の解答

$$\begin{aligned}(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 &= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta \\ &\quad + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta \\ &= 2(\sin^2\theta + \cos^2\theta) \\ &= 2\end{aligned}$$

＜ 38 ページ. 三角関数の復習 2 ＞

問 1 の解答

$a^2 = b^2 + c^2$

問 2 の解答

$b^2 = a^2 + c^2 - 2ac\cos 60^\circ = a^2 + c^2 - ac$

問 3 の解答

$c^2 = a^2 + b^2 - 2ab\cos 120^\circ = a^2 + b^2 + ab$

問 4 の解答

角度 θ	-90°	-60°	-45°	-30°	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	╳	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	╳	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

角度 θ	180°	210°	225°	240°	270°	300°	315°	330°	360°	390°	405°	420°	450°
$\sin\theta$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	╳	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	╳

< 39 ページ. 平面座標の三角関数表示 >

問の解答

(1) $(\sqrt{3}, 1) = (2\cos 30°,\ 2\sin 30°)$

(2) $(-2, 2) = (2\sqrt{2}\cos 135°,\ 2\sqrt{2}\sin 135°)$

(3) $(-\sqrt{3}, -3) = (2\sqrt{3}\cos 240°,\ 2\sqrt{3}\sin 240°)$

(4) $(3, -3) = \Big(3\sqrt{2}\cos(-45°),\ 3\sqrt{2}\sin(-45°)\Big) = (3\sqrt{2}\cos 315°,\ 3\sqrt{2}\sin 315°)$

＜ 40 ページ. 加法定理 1 ＞

問の解答

(1) $\mathrm{P}(\cos\beta,\ \sin\beta)$

(2) $\mathrm{Q}(\cos\alpha,\ -\sin\alpha)$

(3) $\mathrm{PQ}^2 = (\cos\beta - \cos\alpha)^2 + (\sin\beta + \sin\alpha)^2$

(4) $\mathrm{PQ}^2 = \cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta + 2\sin\beta\sin\alpha + \sin^2\alpha$

$= 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

(5) $\mathrm{PQ}^2 = 1^2 + 1^2 - 2\times 1\times 1\times\cos(\alpha+\beta)$

$= 2 - 2\cos(\alpha+\beta)$

(6) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

< 41 ページ. 加法定理 2 >

問の解答

$$
\begin{aligned}
\sin(\alpha+\beta) &= \cos(90^\circ-\alpha-\beta) \\
&= \cos\{(90^\circ-\alpha)+(-\beta)\} \\
&= \cos(90^\circ-\alpha)\cos(-\beta)-\sin(90^\circ-\alpha)\sin(-\beta) \\
&= \sin\alpha\cos\beta+\cos\alpha\sin\beta
\end{aligned}
$$

＜ 42 ページ. 加法定理 3 ＞

問 1 の解答

(1) $\cos 105^\circ = \cos(60^\circ + 45^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$$= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

(2) $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

問 2 の解答

(1) $\cos(\alpha - \beta) = \cos(\alpha + (-\beta)) = \cos\alpha \cos(-\beta) - \sin\alpha \sin(-\beta)$

$$= \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

(2) $\sin(\alpha - \beta) = \sin(\alpha + (-\beta)) = \sin\alpha \cos(-\beta) + \cos\alpha \sin(-\beta)$

$$= \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

問 3 の解答

(1) $\cos(-15^\circ) = \cos(30^\circ - 45^\circ) = \cos 30^\circ \cos 45^\circ + \sin 30^\circ \sin 45^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

(2) $\sin(-15^\circ) = \sin(30^\circ - 45^\circ) = \sin 30^\circ \cos 45^\circ - \cos 30^\circ \sin 45^\circ$

$$= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

< 43 ページ. 加法定理 4 >

問 1 の解答

(1) $\tan 105^\circ = \dfrac{\sin 105^\circ}{\cos 105^\circ} = \dfrac{\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ}{\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ}$

$$= \frac{\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}}{\frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}} = \frac{(\sqrt{6}+\sqrt{2})^2}{2-6} = \frac{6+2\sqrt{12}+2}{-4}$$

$$= -2-\sqrt{3}$$

問 2 の解答

$$\tan(\alpha+\beta) = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$= \frac{\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\beta}{\cos\beta}}{1 - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

問 3 の解答

$$\tan(\alpha-\beta) = \frac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha\tan(-\beta)} = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

＜ 44 ページ. 加法定理の応用 1 ＞

問 1 の解答

(1) $\cos(\theta + 360^\circ) = \cos\theta\cos 360^\circ - \sin\theta\sin 360^\circ = (\cos\theta)\times 1 - (\sin\theta)\times 0 = \cos\theta$

(2) $\tan(\theta + 360^\circ) = \dfrac{\tan\theta + \tan 360^\circ}{1 - \tan\theta\tan 360^\circ} = \dfrac{\tan\theta + 0}{1 - (\tan\theta)\times 0} = \dfrac{\tan\theta}{1} = \tan\theta$

(3) $\cos(-\theta) = \cos(360^\circ - \theta) = \cos 360^\circ\cos\theta + \sin 360^\circ\sin\theta = 1\times\cos\theta + 0\times\sin\theta = \cos\theta$

(4) $\tan(-\theta) = \tan(360^\circ - \theta) = \dfrac{\tan 360^\circ - \tan\theta}{1 + \tan 360^\circ\tan\theta} = \dfrac{0 - \tan\theta}{1 + 0\times\tan\theta} = -\tan\theta$

(5) $\sin(\theta + 180^\circ) = \sin\theta\cos 180^\circ + \cos\theta\sin 180^\circ = (\sin\theta)\times(-1) + (\cos\theta)\times 0 = -\sin\theta$

(6) $\cos(\theta + 180^\circ) = \cos\theta\cos 180^\circ - \sin\theta\sin 180^\circ = (\cos\theta)\times(-1) - (\sin\theta)\times 0 = -\cos\theta$

(7) $\sin(180^\circ - \theta) = \sin 180^\circ\cos\theta - \cos 180^\circ\sin\theta = 0\times\cos\theta - (-1)\times\sin\theta = \sin\theta$

(8) $\cos(180^\circ - \theta) = \cos 180^\circ\cos\theta + \sin 180^\circ\sin\theta = -1\times\cos\theta + 0\times\sin\theta = -\cos\theta$

(9) $\tan(180^\circ - \theta) = \dfrac{\tan 180^\circ - \tan\theta}{1 + \tan 180^\circ\tan\theta} = \dfrac{0 - \tan\theta}{1 + 0\times\tan\theta} = -\tan\theta$

(10) $\sin(\theta + 90^\circ) = \sin\theta\cos 90^\circ + \cos\theta\sin 90^\circ = (\sin\theta)\times 0 + (\cos\theta)\times 1 = \cos\theta$

(11) $\cos(\theta + 90^\circ) = \cos\theta\cos 90^\circ - \sin\theta\sin 90^\circ = (\cos\theta)\times 0 - (\sin\theta)\times 1 = -\sin\theta$

(12) $\sin(90^\circ - \theta) = \sin 90^\circ\cos\theta - \cos 90^\circ\sin\theta = 1\times\cos\theta - 0\times\sin\theta = \cos\theta$

(13) $\cos(90^\circ - \theta) = \cos 90^\circ\cos\theta + \sin 90^\circ\sin\theta = 0\times\cos\theta + 1\times\sin\theta = \sin\theta$

問 2 の解答

(1) $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

(2) $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha \ \left(= 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha\right)$

(3) $\tan(2\alpha) = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

＜ 45 ページ. 加法定理の応用 2 ＞

問の解答

(1) $\sin\theta + \sqrt{3}\cos\theta = 2\left(\dfrac{1}{2}\sin\theta + \dfrac{\sqrt{3}}{2}\cos\theta\right)$

$$= 2(\cos 60^\circ \sin\theta + \sin 60^\circ \cos\theta)$$

$$= 2\sin(\theta + 60^\circ)$$

(2) $-\sin\theta + \cos\theta = \sqrt{2}\left(-\dfrac{1}{\sqrt{2}}\sin\theta + \dfrac{1}{\sqrt{2}}\cos\theta\right)$

$$= \sqrt{2}(\cos 135^\circ \sin\theta + \sin 135^\circ \cos\theta)$$

$$= \sqrt{2}\sin(\theta + 135^\circ)$$

(3) $-\sqrt{3}\sin\theta - \cos\theta = 2\left(-\dfrac{\sqrt{3}}{2}\sin\theta - \dfrac{1}{2}\cos\theta\right)$

$$= 2(\cos 210^\circ \sin\theta + \sin 210^\circ \cos\theta)$$

$$= 2\sin(\theta + 210^\circ)$$

$$\bigl(= 2\sin(\theta - 150^\circ)\bigr)$$

(4) $\sin\theta - \cos\theta = \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\sin\theta - \dfrac{1}{\sqrt{2}}\cos\theta\right)$

$$= \sqrt{2}(\cos 315^\circ \sin\theta + \sin 315^\circ \cos\theta)$$

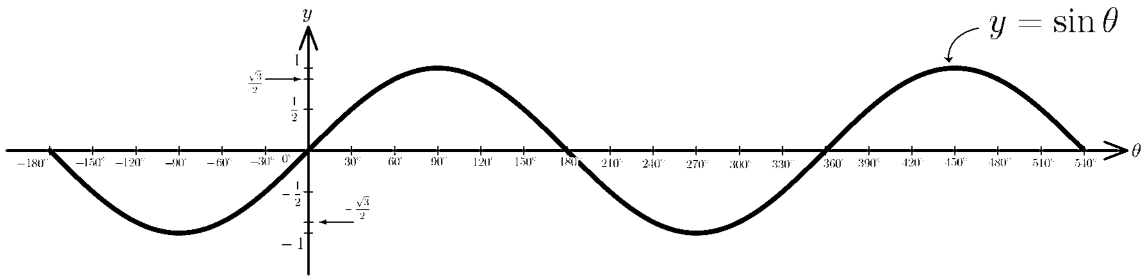
$$= \sqrt{2}\sin(\theta + 315^\circ)$$

$$\bigl(= \sqrt{2}\sin(\theta - 45^\circ)\bigr)$$

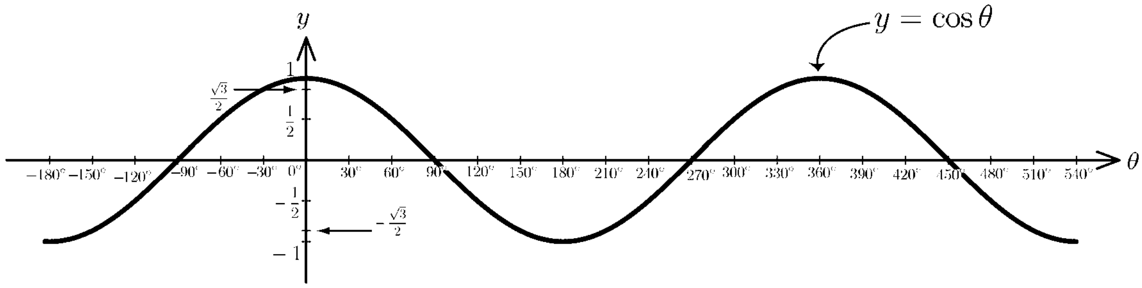
＜ 46 ページ. 三角関数のグラフ 1 ＞

問の解答

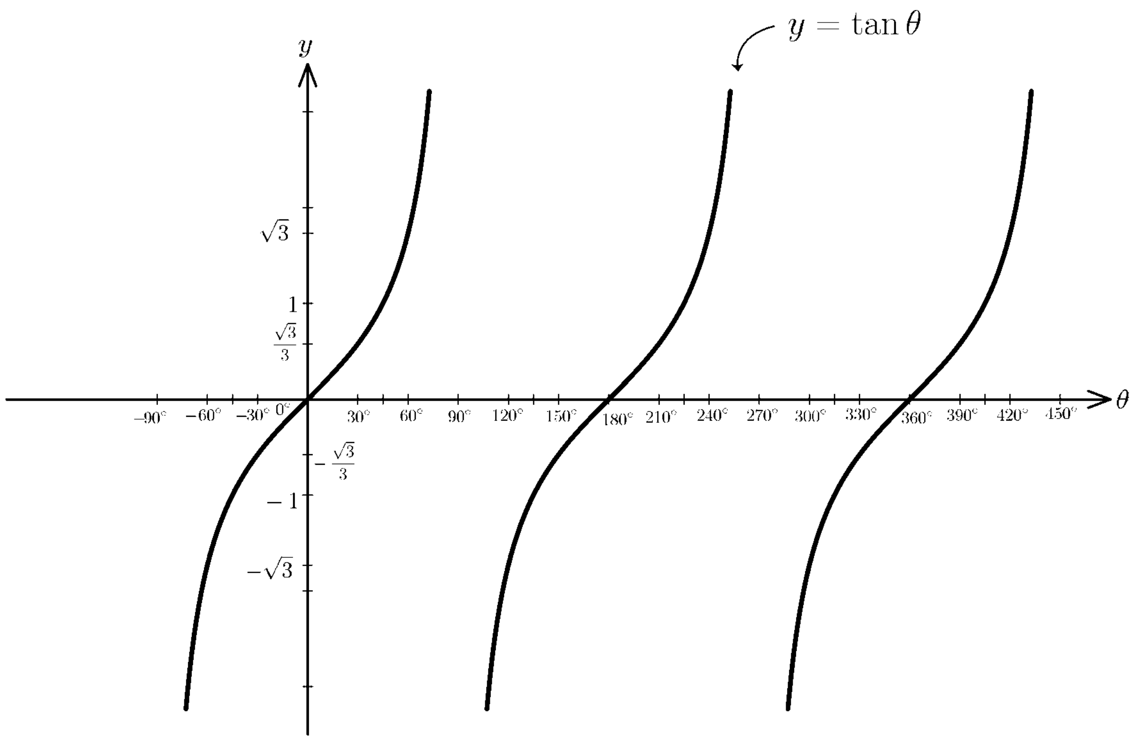
(1) $y = \sin\theta \ \ (-180^\circ \leqq \theta \leqq 540)$



(2) $y = \cos\theta \ \ (-180^\circ \leqq \theta \leqq 540)$



(3) $y = \tan\theta \ \ (-90^\circ \leqq \theta \leqq 450)$



＜ 47 ページ. 三角関数のグラフ 2 ＞

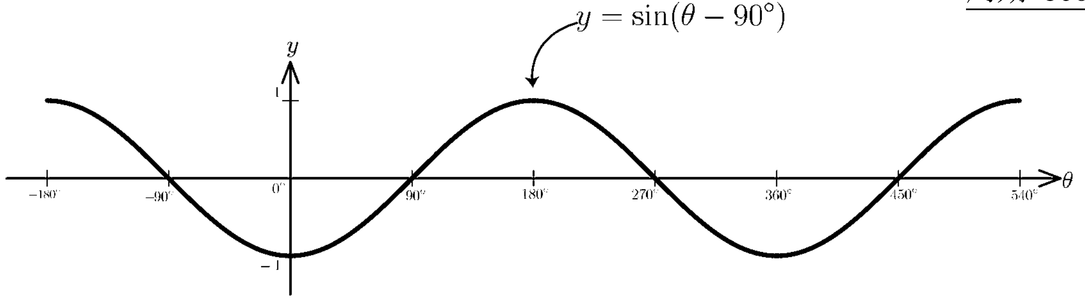
問 1 の解答

$\cos\theta$ の周期は $360°$, $\tan\theta$ の周期は $180°$

問 2 の解答

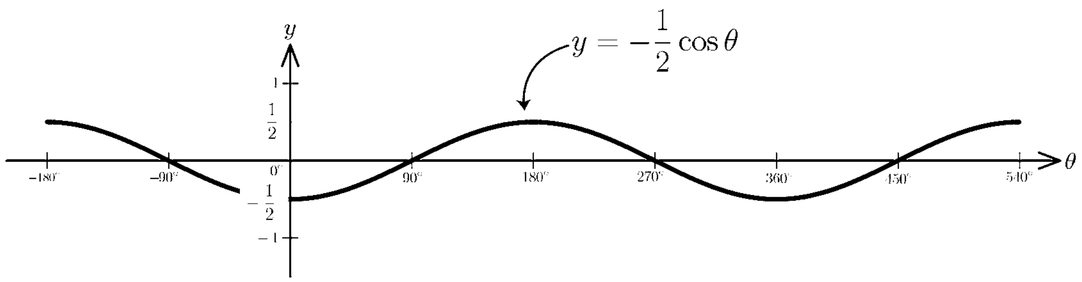
(1) $y = \sin(\theta - 90°)$

周期 $360°$



(2) $y = -\dfrac{1}{2}\cos\theta$

周期 $360°$

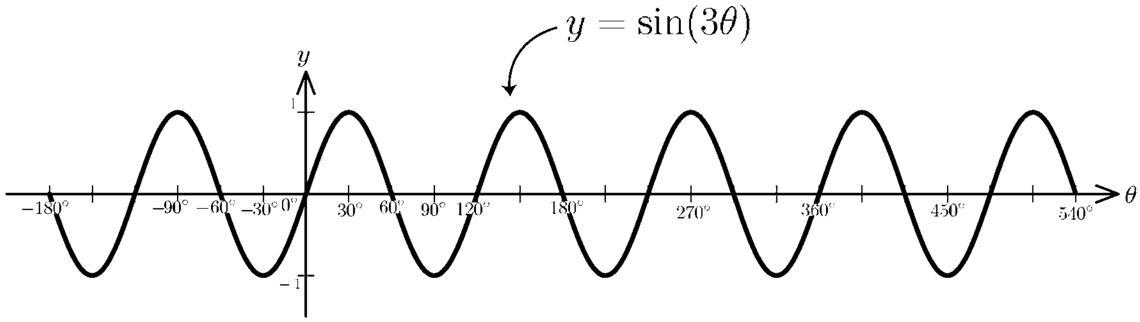


＜ 48 ページ. 三角関数のグラフ 3 ＞

問の解答

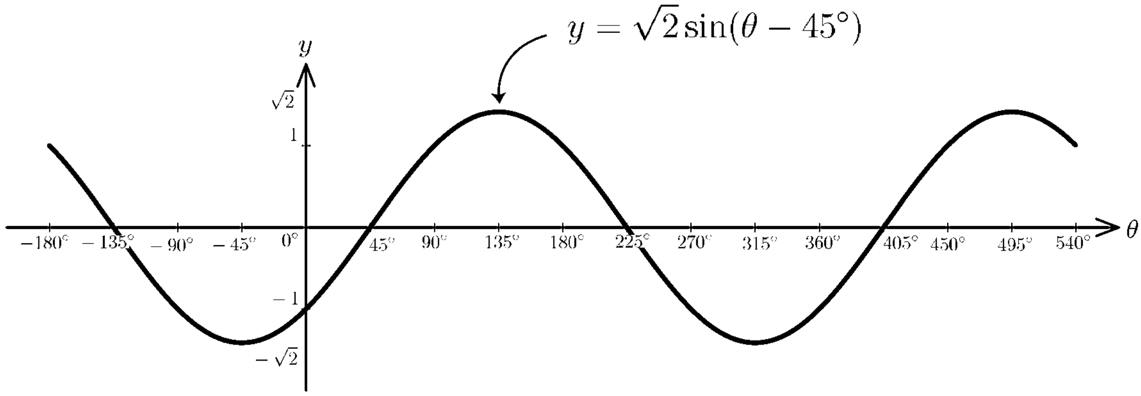
(1) $y = \sin(3\theta)$

周期 120°



(2) $y = \sqrt{2}\sin(\theta - 45°)$

周期 360°

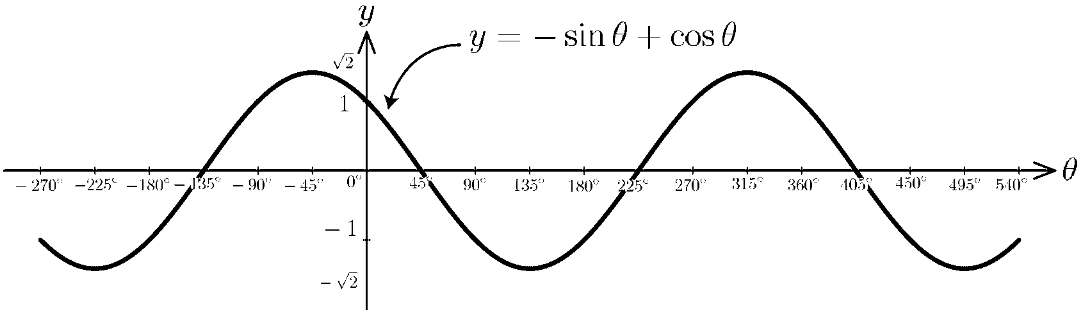


< 49 ページ. 三角関数のグラフ 4 >

問の解答

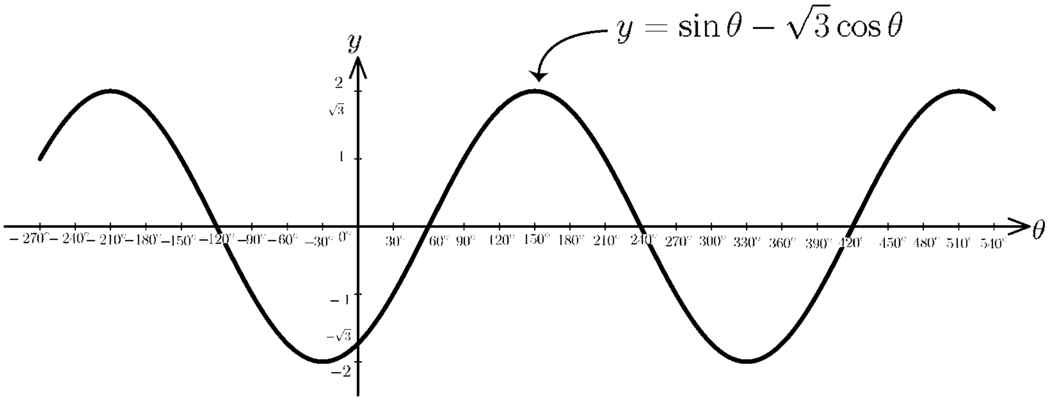
(1) $y = -\sin\theta + \cos\theta = \sqrt{2}\sin(\theta + 135°)$

y の最大値は $\sqrt{2}$ ，　y の最小値は $-\sqrt{2}$



(2) $y = \sin\theta - \sqrt{3}\cos\theta = 2\sin(\theta - 60°)$

y の最大値は 2 ，　y の最小値は -2



< 50ページ. 三角関数の練習 >

問 1 の解答

$\sin\theta = -\sqrt{1-\cos^2\theta}$

問 2 の解答

$$1+\tan^2\theta = \frac{1}{\cos^2\theta} \Rightarrow 1+a^2 = \frac{1}{\cos^2\theta} \Rightarrow \cos^2\theta = \frac{1}{1+a^2} \Rightarrow \cos\theta = -\frac{1}{\sqrt{1+a^2}}$$

問 3 の解答

(1) $\sin(\theta+180^\circ) = -\sin\theta$ (2) $\cos(180^\circ-\theta) = -\cos\theta$

(3) $\tan(\theta-180^\circ) = \tan\theta$ (4) $\sin(\theta+90^\circ) = \cos\theta$

(5) $\cos(\theta-90^\circ) = \sin\theta$ (6) $\tan(-\theta) = -\tan\theta$

問 4 の解答

(1) $(1,\ \sqrt{3}) = (2\cos 60^\circ, 2\sin 60^\circ)$

(2) $(-\sqrt{3},\ 1) = (2\cos 150^\circ, 2\sin 150^\circ)$

(3) $(-3,\ -\sqrt{3}) = (2\sqrt{3}\cos 210^\circ, 2\sqrt{3}\sin 210^\circ)$
$= (2\sqrt{3}\cos(-150^\circ),\ 2\sqrt{3}\sin(-150^\circ))$

(4) $(2,\ -2) = (2\sqrt{2}\cos 315^\circ, 2\sqrt{2}\sin 315^\circ)$
$= (2\sqrt{2}\cos(-45^\circ),\ 2\sqrt{2}\sin(-45^\circ))$

問 5 の解答

(1) $\sin 165^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ (2) $\cos 195^\circ = \dfrac{-\sqrt{6}-\sqrt{2}}{4}$

(3) $\tan 255^\circ = 2+\sqrt{3}$ (4) $\sin(-75^\circ) = \dfrac{-\sqrt{2}-\sqrt{6}}{4}$

問 6 の解答

$\cos^2\alpha = \dfrac{1+\cos(2\alpha)}{2}$

問 7 の解答

$\sin^2\alpha = \dfrac{1-\cos(2\alpha)}{2}$

問 8 の解答

(1) $\sqrt{3}\sin\theta + \cos\theta = 2\sin(\theta+30^\circ)$

(2) $-\sqrt{3}\sin\theta + 3\cos\theta = 2\sqrt{3}\sin(\theta+120^\circ)$

(3) $-2\sin\theta - 2\cos\theta = 2\sqrt{2}\sin(\theta-135^\circ)$
$= 2\sqrt{2}\sin(\theta+225^\circ)$

(4) $3\sin\theta - 3\cos\theta = 3\sqrt{2}\sin(\theta-45^\circ)$
$= 3\sqrt{2}\sin(\theta+315^\circ)$

問 9 の解答

(1) 周期 90° (2) 周期 120°

問 10 の解答

$y = \sin\theta + \sqrt{3}\cos\theta$
$= 2\sin(\theta+60^\circ)$

y の最大値は 2

y の最小値は -2