

高知工科大学

基礎数学ワークブック

(2003年度版)

番外編

フーリエ級数  
(改訂版)

解答

## &lt; 1 ページ. 三角関数 &gt;

## 問 1 の解答

度数法	-720°	-540°	-360°	-180°	-90°	0°	30°	45°	60°	90°	180°	270°	360°	540°	720°	900°
弧度法	-4π	-3π	-2π	-π	$-\frac{\pi}{2}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π	3π	4π	5π
sin θ	0	0	0	0	-1	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0	0	0	0
cos θ	1	-1	1	-1	0	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1	-1	1	-1

## 問 2 の解答

$$(1) \sin\left(-\frac{3}{2}\pi\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$(2) \cos\left(\frac{\pi}{2} + \frac{5}{6}\pi\right) = -\sin\left(\frac{5}{6}\pi\right) = -\frac{1}{2}$$

$$(3) \sin\left(\frac{\pi}{6} + 8\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$(4) \cos\left(\frac{2}{3}\pi - 4\pi\right) = \cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$$

$$(5) \sin\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$(6) \cos\left(\frac{\pi}{4} + 3\pi\right) = \cos\left(\frac{\pi}{4} + \pi\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

## &lt; 2 ページ. 加法定理 1 &gt;

## 問 1 の解答

$$(1) \cos\left(\theta - \frac{\pi}{2}\right) = \cos\theta \cos\left(\frac{\pi}{2}\right) + \sin\theta \sin\left(\frac{\pi}{2}\right) = \sin\theta$$

$$(2) \sin\left(\theta - \frac{\pi}{2}\right) = \sin\theta \cos\left(\frac{\pi}{2}\right) - \cos\theta \sin\left(\frac{\pi}{2}\right) = -\cos\theta$$

## 問 2 の解答

$$(1) \sin\left(\theta + \frac{\pi}{6}\right) = \sin\theta \cos\left(\frac{\pi}{6}\right) + \cos\theta \sin\left(\frac{\pi}{6}\right) \\ = \frac{\sqrt{3}}{2} \sin\theta + \frac{1}{2} \cos\theta$$

$$(2) \sin\left(\theta + \frac{\pi}{4}\right) = \sin\theta \cos\left(\frac{\pi}{4}\right) + \cos\theta \sin\left(\frac{\pi}{4}\right) \\ = \frac{\sqrt{2}}{2} (\sin\theta + \cos\theta)$$

$$(3) \sin\left(\theta + \frac{2}{3}\pi\right) = \sin\theta \cos\left(\frac{2\pi}{3}\right) + \cos\theta \sin\left(\frac{2\pi}{3}\right) \\ = -\frac{1}{2} \sin\theta + \frac{\sqrt{3}}{2} \cos\theta$$

$$(4) \sin\left(\theta - \frac{\pi}{6}\right) = \sin\theta \cos\left(\frac{\pi}{6}\right) - \cos\theta \sin\left(\frac{\pi}{6}\right) \\ = \frac{\sqrt{3}}{2} \sin\theta - \frac{1}{2} \cos\theta$$

## &lt; 3 ページ. 加法定理 2 &gt;

## 問の解答

$$(1) \sin \theta + \cos \theta = \sqrt{2} \sin \left( \theta + \frac{\pi}{4} \right)$$

$$(2) \sqrt{3} \cos \theta + \sin \theta = 2 \sin \left( \theta + \frac{\pi}{3} \right)$$

$$(3) \cos \theta - \sin \theta = \sqrt{2} \sin \left( \theta + \frac{3}{4}\pi \right)$$

$$(4) -4 \cos \theta - 4\sqrt{3} \sin \theta = 8 \sin \left( \theta + \frac{7}{6}\pi \right)$$

## &lt; 4 ページ. 加法定理 3 &gt;

## 問 1 の解答

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos\theta}{2}}$$

## 問 2 の解答

$$\begin{aligned}\cos(3\theta) &= \cos(2\theta)\cos\theta - \sin(2\theta)\sin\theta \\ &= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta\cos\theta \\ &= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta \\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \\ &= 4\cos^3\theta - 3\cos\theta\end{aligned}$$

## &lt; 5 ページ. 積和公式 &gt;

## 問 1 の解答

$$(1) \sin \theta \cos \theta = \frac{1}{2} \sin (2\theta)$$

$$(2) \sin^2 \theta = \frac{1}{2} \{1 - \cos (2\theta)\}$$

## 問 2 の解答

$$(1) \sin (5t) \cos (4t) = \frac{1}{2} \{\sin (9t) + \sin (t)\}$$

$$(2) \sin (5t) \sin (4t) = \frac{1}{2} \{\cos (t) - \cos (9t)\}$$

$$(3) \cos (5t) \cos (4t) = \frac{1}{2} \{\cos (9t) + \cos (t)\}$$

$$(4) \sin (mt) \cos (nt) = \frac{1}{2} \{\sin ((m+n)t) + \sin ((m-n)t)\}$$

$$(5) \sin (mt) \sin (nt) = \frac{1}{2} \{\cos ((m-n)t) - \cos ((m+n)t)\}$$

$$(6) \cos (mt) \cos (nt) = \frac{1}{2} \{\cos ((m+n)t) + \cos ((m-n)t)\}$$

## &lt; 6 ページ. 偶関数と奇関数 1 &gt;

## 問の解答

(1)  $f(x) = x^4$  偶関数

(2)  $f(x) = x^5$  奇関数

(3)  $f(x) = x^6$  偶関数

(4)  $f(x) = x^7$  奇関数

(5)  $f(x) = \cos(2x)$  偶関数

(6)  $f(x) = \sin(2x)$  奇関数

(7)  $f(x) = \cos(3x)$  偶関数

(8)  $f(x) = \sin(3x)$  奇関数

(9)  $f(x) = \sin^2(x)$  偶関数

## &lt; 7 ページ. 偶関数と奇関数 2 &gt;

**問 1 の解答**

- (1)  $x^2 \times x^4$       偶関数
- (2)  $x^2 \times x^5$       奇関数
- (3)  $x^3 \times x^7$       偶関数
- (4)  $x \sin(2x)$       偶関数
- (5)  $x^2 \cos(3x)$       偶関数
- (6)  $x^3 \cos(5x)$       奇関数
- (7)  $\sin(2x) \sin(3x)$       偶関数
- (8)  $\sin(4x) \cos(3x)$       奇関数
- (9)  $\cos(2x) \cos(5x)$       偶関数

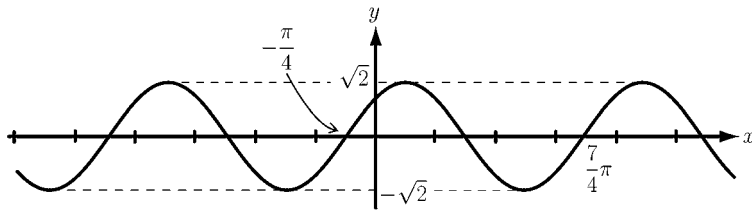
**問 2 の解答**

- (1) 奇関数  $\times$  奇関数 = 偶関数
- (2) 偶関数  $\times$  偶関数 = 偶関数
- (3) 奇関数  $\times$  偶関数 = 奇関数

## &lt; 8 ページ. 正弦波のグラフ &gt;

## 問の解答

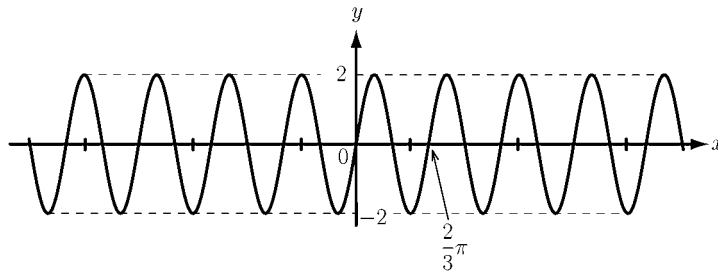
(1)



周期  $2\pi$  , 振幅  $\sqrt{2}$  , 初期位相  $-\frac{\pi}{4}$

$$y = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$$

(2)



周期  $\frac{2\pi}{3}$  , 振幅  $2$  , 初期位相  $0$

$$y = 2 \sin(3x)$$

## &lt; 9 ページ. 同周期正弦波の和 &gt;

## 問の解答

$$(1) \sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$$

周期  $2\pi$  , 振幅  $\sqrt{2}$  , 初期位相  $-\frac{\pi}{4}$

$$(2) \sqrt{3} \sin(2x) + \cos(2x) = 2 \sin\left(2x + \frac{\pi}{6}\right)$$

周期  $\pi$  , 振幅  $2$  , 初期位相  $-\frac{\pi}{12}$

$$(3) \sin(3x) - \cos(3x) = \sqrt{2} \sin\left(3x - \frac{\pi}{4}\right)$$

周期  $\frac{2\pi}{3}$  , 振幅  $\sqrt{2}$  , 初期位相  $\frac{\pi}{12}$

## &lt; 10 ページ. 異周期正弦波の和 &gt;

## 問の解答

(1)  $\cos(x) + \cos(3x)$  周期  $2\pi$

(2)  $\sin(2x) + \cos(5x)$  周期  $2\pi$

(3)  $\cos(3x) + \sin(6x)$  周期  $\frac{2}{3}\pi$

## &lt; 11 ページ. 三角多項式 1 &gt;

## 問の解答

$$\begin{aligned}(1) \quad f(x) &= 2 + \sin(2x) + \cos(2x) \\ &= 2 + \sqrt{2} \sin\left(2x + \frac{\pi}{4}\right)\end{aligned}$$

の基本周期は  $\pi$

(この関数は偶関数でも奇関数でもない)

$$\begin{aligned}(1) \quad f(x) &= \sin^2 x \\ &= \frac{1}{2} - \frac{1}{2} \cos(2x)\end{aligned}$$

の基本周期は  $\pi$

$f(x) = \sin^2 x$  は偶関数である。

## &lt; 12 ページ. 三角多項式 2 &gt;

## 問の解答

図 4 : (3)  $y = 3 + \cos(2x) + \sin(3x)$

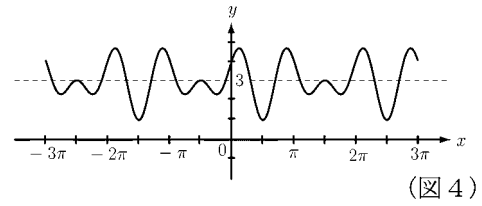


図 5 : (4)  $y = 2 + \sin(2x) + \cos(2x)$

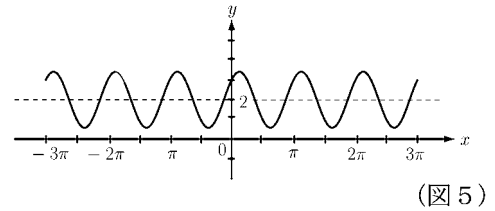
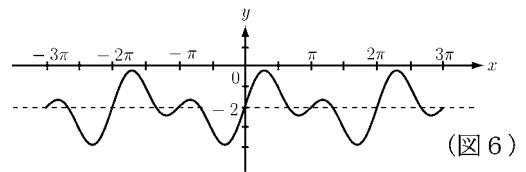


図 6 : (5)  $y = -2 + \sin x + \sin(2x)$

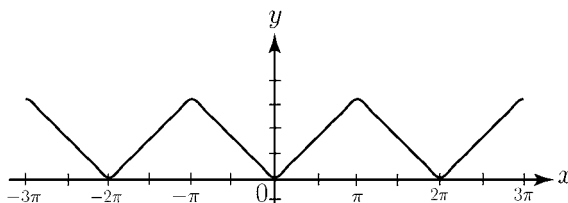


## &lt; 13 ページ. 三角多項式 3 &gt;

## 問の解答

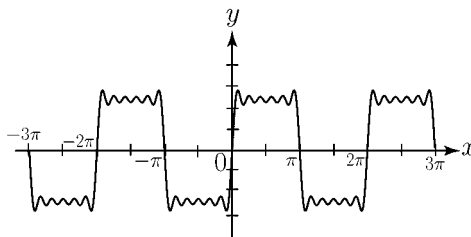
$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \cos x - \frac{4}{9\pi} \cos(3x) - \frac{4}{25\pi} \cos(5x) - \frac{4}{49\pi} \cos(7x)$$

$f(x)$  のグラフ... 図 6



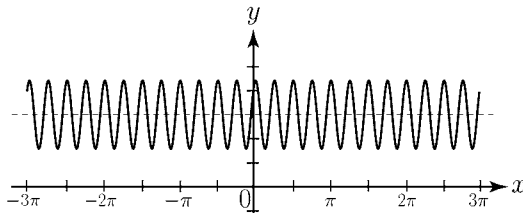
$$g(x) = 3 \sin x + \sin(3x) + \frac{3}{5} \sin(5x) + \frac{3}{7} \sin(7x) + \frac{1}{3} \sin(9x) + \frac{3}{11} \sin(11x)$$

$g(x)$  のグラフ... 図 5



$$h(x) = 3 + \cos(8x) + \sin(8x)$$

$h(x)$  のグラフ... 図 4

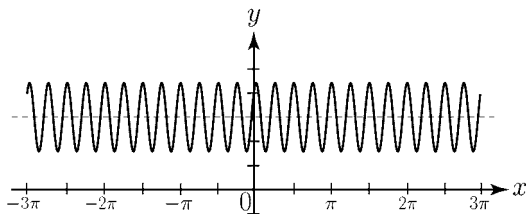


## &lt; 13 ページ. 三角多項式 3 &gt;

## 問の解答

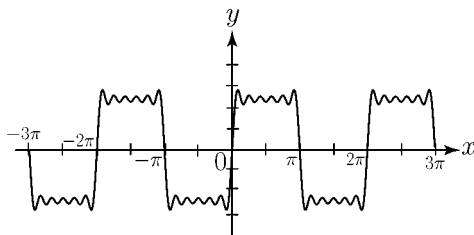
$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \cos x - \frac{4}{9\pi} \cos(3x) - \frac{4}{25\pi} \cos(5x) - \frac{4}{49\pi} \cos(7x)$$

$f(x)$  のグラフ... 図 6



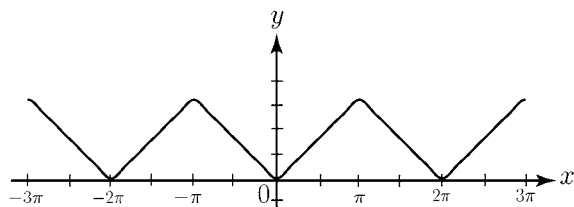
$$g(x) = 3 \sin x + \sin(3x) + \frac{3}{5} \sin(5x) + \frac{3}{7} \sin(7x) + \frac{1}{3} \sin(9x) + \frac{3}{11} \sin(11x)$$

$g(x)$  のグラフ... 図 5



$$h(x) = 3 + \cos(8x) + \sin(8x)$$

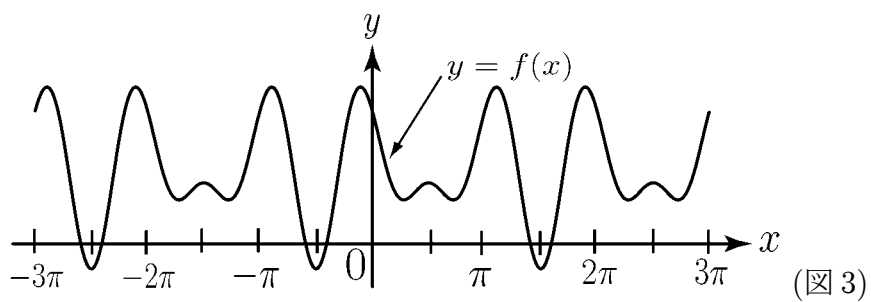
$h(x)$  のグラフ... 図 4



## &lt; 14 ページ. 三角多項式 4 &gt;

## 問の解答

$$f(x) = 2.1 + 1.6 \cos(2x) - 1.2 \sin(3x)$$



## &lt; 15 ページ. 積分 1 &gt;

## 問 1 の解答

$$(1) \int \cos(4x) dx = \frac{1}{4} \sin(4x) + C$$

$$(2) \int \sin(5x) dx = -\frac{1}{5} \cos(5x) + C$$

$$(3) \int \cos(nx) dx = \frac{1}{n} \sin(nx) + C$$

$$(4) \int \sin(nx) dx = -\frac{1}{n} \cos(nx) + C$$

## 問 2 の解答

$$(1) \int \sin^2 x dx = \int \frac{1}{2}(1 - \cos(2x)) dx = \frac{x}{2} - \frac{1}{4} \sin(2x) + C$$

$$(2) \int \cos^2(2x) dx = \int \frac{1}{2}(1 + \cos(4x)) dx = \frac{x}{2} + \frac{1}{8} \sin(4x) + C$$

$$(3) \int \sin^2(3x) dx = \int \frac{1}{2}(1 - \cos(6x)) dx = \frac{x}{2} - \frac{1}{12} \sin(6x) + C$$

$$(4) \int \sin(3x) \sin(2x) dx \\ = \int \frac{1}{2} \{ \cos x - \cos(5x) \} dx = \frac{1}{2} \sin x - \frac{1}{10} \sin(5x) + C$$

$$(5) \int \cos(2x) \cos(4x) dx \\ = \int \frac{1}{2} \{ \cos(6x) + \cos(2x) \} dx = \frac{1}{12} \sin(6x) + \frac{1}{4} \sin(2x) + C$$

$$(6) \int \sin(4x) \cos(5x) dx \\ = \int \frac{1}{2} \{ \sin(9x) - \sin x \} dx = -\frac{1}{18} \cos(9x) + \frac{1}{2} \cos x + C$$

## &lt; 16 ページ. 積分 2 &gt;

## 問の解答

$$(1) \int_{-\pi}^{\pi} \sin^2 x dx = \int_{-\pi}^{\pi} \left\{ \frac{1}{2} - \frac{1}{2} \cos(2x) \right\} dx = \left[ \frac{1}{2}x - \frac{1}{4} \sin(2x) \right]_{x=-\pi}^{x=\pi} = \pi$$

$$(2) \int_{-\pi}^{\pi} \cos^2(3x) dx = \int_{-\pi}^{\pi} \left\{ \frac{1}{2} + \frac{1}{2} \cos(6x) \right\} dx$$

$$= \left[ \frac{1}{2}x + \frac{1}{12} \sin(6x) \right]_{x=-\pi}^{x=\pi}$$

$$= \pi$$

$$(3) \int_{-\pi}^{\pi} \sin^2(4x) dx = \int_{-\pi}^{\pi} \left\{ \frac{1}{2} - \frac{1}{2} \cos(8x) \right\} dx = \left[ \frac{1}{2}x - \frac{1}{16} \sin(8x) \right]_{x=-\pi}^{x=\pi} = \pi$$

$$(4) \int_{-\pi}^{\pi} \sin x \sin(4x) dx = \int_{-\pi}^{\pi} \frac{1}{2} \{ \cos(3x) - \cos(5x) \} dx$$

$$= \left[ \frac{1}{6} \sin(3x) - \frac{1}{10} \sin(5x) \right]_{x=-\pi}^{x=\pi}$$

$$= 0$$

$$(5) \int_{-\pi}^{\pi} \cos(3x) \cos(4x) dx = \int_{-\pi}^{\pi} \frac{1}{2} \{ \cos(7x) + \cos x \} dx$$

$$= \left[ \frac{1}{14} \sin(7x) + \frac{1}{2} \sin x \right]_{x=-\pi}^{x=\pi} = 0$$

$$(6) \int_{-\pi}^{\pi} \sin x \cos(4x) dx = \int_{-\pi}^{\pi} \frac{1}{2} \{ \sin(5x) - \sin(3x) \} dx$$

$$= \left[ -\frac{1}{10} \cos(5x) + \frac{1}{6} \cos(3x) \right]_{x=-\pi}^{x=\pi}$$

$$= 0$$

## &lt; 17 ページ. 積分 3 &gt;

## 問の解答

$$(1) \text{ (奇関数の積分より) } \int_{-\pi}^{\pi} x^2 \sin(3x) dx = 0$$

$$(2) \text{ (奇関数の積分より) } \int_{-\pi}^{\pi} \cos(x) \sin(2x) dx = 0$$

$$(3) \text{ (偶関数の積分より)}$$

$$\int_{-\pi}^{\pi} \sin(4x) \sin(3x) dx$$

$$= 2 \int_0^{\pi} \sin(4x) \sin(3x) dx$$

$$= 2 \int_0^{\pi} \frac{1}{2} \{ \cos x - \cos(7x) \} dx$$

$$= \left[ \sin x - \frac{1}{7} \sin(7x) \right]_0^{\pi}$$

$$= \left\{ \sin \pi - \frac{1}{7} \sin(7\pi) \right\} - \left\{ \sin 0 - \frac{1}{7} \sin 0 \right\}$$

$$= 0$$

## &lt; 18 ページ. 積分 4 &gt;

## 問 1 の解答

$$(1) \int_{-\pi}^{\pi} \cos^2(4x) dx = 2 \int_0^{\pi} \cos^2(4x) dx = \int_0^{\pi} (1 + \cos(8x)) dx = \left[ x + \frac{1}{8} \sin(8x) \right]_0^{\pi} = \pi$$

$$(2) \int_{-\pi}^{\pi} \sin^2(5x) dx = 2 \int_0^{\pi} \sin^2(5x) dx = \int_0^{\pi} (1 - \cos(10x)) dx = \left[ x - \frac{1}{10} \sin(10x) \right]_0^{\pi} = \pi$$

$$(3) \int_{-\pi}^{\pi} \sin(3x) \sin(4x) dx = 2 \int_0^{\pi} \sin(3x) \sin(4x) dx = \left[ \sin x - \frac{1}{7} \sin(7x) \right]_0^{\pi} = 0$$

$$(4) (\text{奇関数の積分より}) \int_{-\pi}^{\pi} \sin(3x) \cos(4x) dx = 0$$

$$(5) (\text{奇関数の積分より}) \int_{-\pi}^{\pi} \sin(4x) \cos(4x) dx = 0$$

$$(6) \int_{-\pi}^{\pi} \cos(4x) \cos(5x) dx = 2 \int_0^{\pi} \cos(4x) \cos(5x) dx \\ = \int_0^{\pi} (\cos(9x) + \cos x) dx = \left[ \frac{1}{9} \sin(9x) + \sin x \right]_0^{\pi} = 0$$

## 問 2 の解答

$$(1) \int_{-\pi}^{\pi} \cos^2(nx) dx = 2 \int_0^{\pi} \cos^2(nx) dx = \left[ x + \frac{1}{2n} \sin(2nx) \right]_0^{\pi} = \pi$$

$$(2) \int_{-\pi}^{\pi} \sin^2(nx) dx = 2 \int_0^{\pi} \sin^2(nx) dx = \left[ x - \frac{1}{2n} \sin(2nx) \right]_0^{\pi} = \pi$$

$$(3) \int_{-\pi}^{\pi} \sin(nx) \sin(mx) dx = 2 \int_0^{\pi} \sin(nx) \sin(mx) dx \\ = \left[ \frac{\sin((n-m)x)}{n-m} - \frac{\sin((n+m)x)}{n+m} \right]_0^{\pi} = 0$$

$$(4) (\text{奇関数の積分より}) \int_{-\pi}^{\pi} \sin(nx) \cos(nx) dx = 0$$

$$(5) (\text{奇関数の積分より}) \int_{-\pi}^{\pi} \sin(nx) \cos(mx) dx = 0$$

$$(6) \int_{-\pi}^{\pi} \cos(nx) \cos(mx) dx = 2 \int_0^{\pi} \cos(nx) \cos(mx) dx \\ = 2 \int_0^{\pi} \frac{1}{2} \{ \cos((n+m)x) + \cos((n-m)x) \} dx \\ = \left[ \frac{\sin((n+m)x)}{n+m} + \frac{\sin((n-m)x)}{n-m} \right]_0^{\pi} = 0$$

## &lt; 19 ページ. 積分 5 &gt;

## 問の解答

$$\begin{aligned}
 (1) \int_0^{\pi} x \cos(4x) dx &= \int_0^{\pi} x \times \left( \frac{1}{4} \sin(4x) \right)' dx = \left[ \frac{x}{4} \sin(4x) \right]_0^{\pi} - \int_0^{\pi} \frac{1}{4} \sin(4x) dx \\
 &= 0 - \left[ -\frac{1}{16} \cos(4x) \right]_0^{\pi} = \frac{1}{16} \cos(4\pi) - \frac{1}{16} \cos 0 = 0
 \end{aligned}$$

$$\begin{aligned}
 (2) \int_0^{\pi} x \sin(4x) dx &= \int_0^{\pi} x \times \left( -\frac{1}{4} \cos(4x) \right)' dx \\
 &= \left[ -\frac{x}{4} \cos(4x) \right]_0^{\pi} - \int_0^{\pi} \left( -\frac{1}{4} \cos(4x) \right) dx \\
 &= -\frac{\pi}{4} \cos(4\pi) - 0 + \left[ \frac{1}{16} \sin(4x) \right]_0^{\pi} = -\frac{\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 (3) \int_0^{\pi} x \cos(5x) dx &= \int_0^{\pi} x \times \left( \frac{1}{5} \sin(5x) \right)' dx = \left[ \frac{x}{5} \sin(5x) \right]_0^{\pi} - \int_0^{\pi} \frac{1}{5} \sin(5x) dx \\
 &= 0 - \left[ -\frac{1}{25} \cos(5x) \right]_0^{\pi} = \frac{1}{25} \cos(5\pi) - \frac{1}{25} \cos 0 = -\frac{2}{25}
 \end{aligned}$$

$$\begin{aligned}
 (4) \int_0^{\pi} x \sin(5x) dx &= \int_0^{\pi} x \times \left( -\frac{1}{5} \cos(5x) \right)' dx \\
 &= \left[ -\frac{x}{5} \cos(5x) \right]_0^{\pi} - \int_0^{\pi} \left( -\frac{1}{5} \cos(5x) \right) dx \\
 &= -\frac{\pi}{5} \cos(5\pi) - 0 + \left[ \frac{1}{25} \sin(5x) \right]_0^{\pi} = \frac{\pi}{5}
 \end{aligned}$$

## &lt; 20 ページ. 積分 6 &gt;

## 問の解答

$$(1) I_n = \int_0^\pi \sin(nx) dx = \left[ -\frac{1}{n} \cos(nx) \right]_0^\pi$$
$$= -\frac{1}{n} \cos(n\pi) + \frac{1}{n} \cos 0$$

$$= \begin{cases} 0 & : n \text{ は偶数} \\ \frac{2}{n} & : n \text{ は奇数} \end{cases}$$

$$(2) I_n = \int_0^\pi x \sin(nx) dx = \left[ -\frac{x \cos(nx)}{n} \right]_0^\pi - \int_0^\pi 1 \times \left( -\frac{\cos(nx)}{n} \right) dx$$
$$= -\frac{\pi}{n} \cos(n\pi) + \int_0^\pi \frac{\cos(nx)}{n} dx$$

$$= -\frac{\pi}{n} \cos(n\pi) + \left[ \frac{\sin(nx)}{n^2} \right]_0^\pi$$

$$= -\frac{\pi}{n} \cos(n\pi)$$

$$= \begin{cases} \frac{\pi}{n} & : n \text{ は奇数} \\ -\frac{\pi}{n} & : n \text{ は偶数} \end{cases}$$

## &lt; 21 ページ. 三角多項式の係数 1 &gt;

## 問の解答

$$(1) f(x) = 5.2 - 3.1 \cos x + 2.7 \sin(5x)$$

$$\int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^{\pi} 5.2 dx = 5.2 \times 2\pi = 10.4\pi$$

$$(2) f(x) = -3.7 - 4.9 \cos(3x) - 6.8 \sin(7x)$$

$$\int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^{\pi} (-3.7) dx = -3.7 \times 2\pi = -7.4\pi$$

$$(3) f(x) = a_0 + a_n \cos(nx) + b_n \sin(nx)$$

$$\int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^{\pi} a_0 dx = 2\pi a_0$$

## &lt; 22 ページ. 三角多項式の係数 2 &gt;

## 問の解答

$$(1) \int_{-\pi}^{\pi} f(x) \sin x dx = \int_{-\pi}^{\pi} (-4 \sin^2 x) dx = -4\pi$$

$$(2) \int_{-\pi}^{\pi} f(x) \cos(2x) dx = \int_{-\pi}^{\pi} 5 \cos^2(2x) dx = 5\pi$$

$$(3) \int_{-\pi}^{\pi} f(x) \sin(2x) dx = 0$$

$$(4) \int_{-\pi}^{\pi} f(x) \cos(3x) dx = 0$$

$$(5) \int_{-\pi}^{\pi} f(x) \sin(3x) dx = \int_{-\pi}^{\pi} (-8 \sin^2(3x)) dx = -8\pi$$

$$(6) \int_{-\pi}^{\pi} f(x) \cos(4x) dx = 0$$

$$(7) \int_{-\pi}^{\pi} f(x) \sin(4x) dx = 0$$

$$(8) \int_{-\pi}^{\pi} f(x) \cos(5x) dx = 0$$

## &lt; 23 ページ. 三角多項式の係数 3 &gt;

## 問 1 の解答

$$(1) \int_{-\pi}^{\pi} f(x) \cos x dx = a_1 \pi$$

$$(2) \int_{-\pi}^{\pi} f(x) \sin x dx = b_1 \pi$$

$$(3) \int_{-\pi}^{\pi} f(x) \sin(2x) dx = b_2 \pi$$

$$(4) \int_{-\pi}^{\pi} f(x) \cos(3x) dx = a_3 \pi$$

$$(5) \int_{-\pi}^{\pi} f(x) \sin(3x) dx = b_3 \pi$$

$$(6) \int_{-\pi}^{\pi} f(x) \cos(4x) dx = 0$$

$$(7) \int_{-\pi}^{\pi} f(x) \sin(4x) dx = 0$$

$$(8) \int_{-\pi}^{\pi} f(x) \cos(5x) dx = 0$$

## &lt; 24 ページ. 三角多項式の係数 4 &gt;

## 問 1 の解答

$$(1) a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos x dx$$

$$(2) b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin x dx$$

$$(3) b_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(2x) dx$$

$$(4) a_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(3x) dx$$

$$(5) b_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(3x) dx$$

## 問 2 の解答

$$(1) a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$(2) a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx) dx$$

$$(3) b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(kx) dx$$

## &lt; 25 ページ. フーリエ級数 1 &gt;

## 問の解答

$$f(x) \sim \sum_{k=1}^{\infty} b_k \sin(kx)$$

## &lt; 26 ページ. フーリエ級数 2 &gt;

## 問の解答

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{2\pi} \int_{-\pi}^0 1 dx = \frac{1}{2\pi} [x]_{-\pi}^0 = \frac{1}{2}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx = \frac{1}{\pi} \int_{-\pi}^0 \cos(nx) dx = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx = \frac{1}{\pi} \int_{-\pi}^0 \sin(nx) dx$$
$$= \begin{cases} 0 & : n \text{ が偶数} \\ -\frac{2}{n\pi} & : n \text{ が奇数} \end{cases}$$

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}$$
$$= \frac{1}{2} - \frac{2}{\pi} \sin x - \frac{2}{3\pi} \sin(3x) - \frac{2}{5\pi} \sin(5x) - \frac{2}{7\pi} \sin(7x) \cdots$$

## &lt; 27 ページ. フーリエ級数 3 &gt;

## 問の解答

$f(x)$  は奇関数より

$$a_0 = a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (-x \sin(nx)) dx = -\frac{2}{\pi} \int_0^{\pi} x \sin(nx) dx$$

$$= \begin{cases} -\frac{2}{\pi} \times \frac{\pi}{n} = -\frac{2}{n} & : n \text{ が奇数} \\ -\frac{2}{\pi} \times \left(-\frac{\pi}{n}\right) = \frac{2}{n} & : n \text{ が偶数} \end{cases}$$

よって

$$\begin{aligned} f(x) \sim & -\frac{2}{1} \sin x + \frac{2}{2} \sin(2x) - \frac{2}{3} \sin(3x) \\ & + \frac{2}{4} \sin(4x) - \frac{2}{5} \sin(5x) + \frac{2}{6} \sin(6x) \cdots \end{aligned}$$

## &lt; 28 ページ. フーリエ級数 4 &gt;

## 問の解答

$$f(x) = |x| \text{ は偶関数より } b_n = 0$$

$$a_0 = \frac{2}{2\pi} \int_0^\pi |x| dx = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi} \int_0^\pi |x| \cos(nx) dx = \begin{cases} \frac{2}{\pi} \times \left(-\frac{2}{n^2}\right) = -\frac{4}{n^2\pi} & : n \text{ が奇数} \\ 0 & : n \text{ が偶数} \end{cases}$$

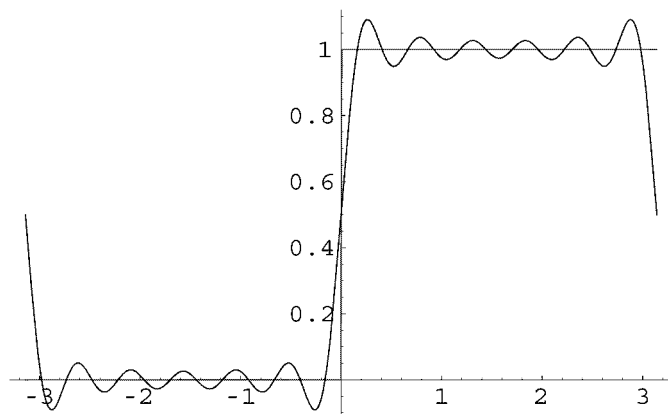
よって

$$f(x) \sim \frac{\pi}{2} - \frac{4}{\pi} \left\{ \cos x + \frac{1}{9} \cos(3x) + \frac{1}{25} \cos(5x) + \frac{1}{49} \cos(7x) + \cdots \right\}$$

## &lt; 29 ページ. フーリエ級数 5 &gt;

## 問の解答

```
f[x_] := Sign[1 + Sign[x]]; n = 6;  
S[x_] := 0.5 +  $\frac{2}{\text{Pi}}$  *  $\left\{ \sum_{k=1}^n \frac{\text{Sin}[(2 * k - 1) * x]}{2 * k - 1} \right\};$   
Plot[{f[x], S[x]}, {x, -Pi, Pi},  
PlotStyle -> {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}}
```



- Graphics -

## &lt; 30 ページ. フーリエ級数 6 &gt;

## 問の解答

$$f(x) = \begin{cases} 0 & : 1 < x \leq \pi \\ 1 & : -1 \leq x \leq 1 \\ 0 & : -\pi \leq x < -1 \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{2\pi} \int_{-1}^1 1 dx = \frac{1}{\pi}$$

$$\begin{aligned} a_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx) dx = \frac{1}{\pi} \int_{-1}^1 \cos(kx) dx = \left[ \frac{1}{\pi k} \sin(kx) \right]_{-1}^1 \\ &= \frac{1}{\pi k} \{ \sin(k) - \sin(-k) \} = \frac{2 \sin k}{\pi k} \end{aligned}$$

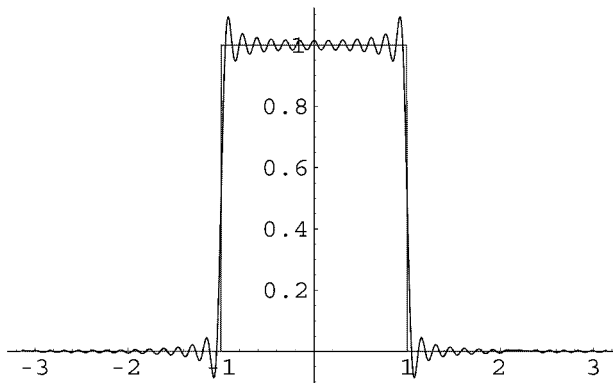
$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(kx) dx = \frac{1}{\pi} \int_{-1}^1 \sin(kx) dx = 0 \quad (\text{奇関数の積分})$$

より

$$\begin{aligned} S_n(x) &= a_0 + \sum_{k=1}^n \{ a_k \cos(kx) + b_k \sin(kx) \} \\ &= \frac{1}{\pi} + \sum_{k=1}^n \left\{ \frac{2 \sin k}{\pi k} \cos(kx) \right\} \end{aligned}$$

## 参考

```
p[x_] := Sign[1 + Sign[x]]; m[x_] := Sign[1 + Sign[-x]];
f[x_] := p[x+1] * m[x-1]; n = 40; S[x_] := 1/π + Sum[{2 * Sin[k]/(π * k) * Cos[k * x]}, {k, 1, n}];
Plot[{f[x], S[x]}, {x, -Pi, Pi},
PlotStyle -> {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}}
```



- Graphics -

## &lt; 31 ページ. フーリエ級数 7 &gt;

## 問の解答

$$f(0) = 1 \quad , \quad S_{\infty}(0) = \frac{1}{2}$$

$$f(\pi) = 0 \quad , \quad S_{\infty}(\pi) = \frac{1}{2}$$

## &lt; 32 ページ. 左極限・右極限 &gt;

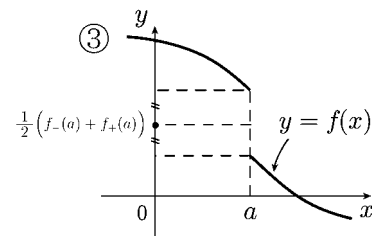
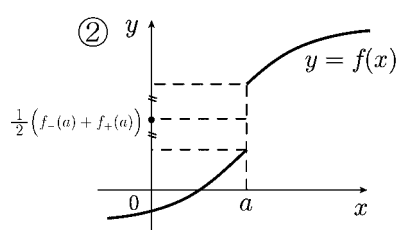
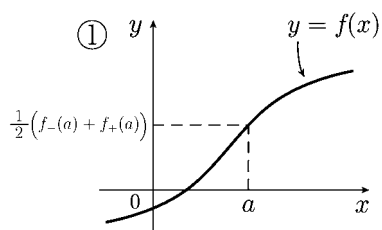
## 問 1 の解答

$$\textcircled{1} f_-(0) = -1 \quad \textcircled{2} f_+(0) = 0 \quad \textcircled{3} f_-(2) = 1 \quad \textcircled{4} f_+(2) = 2$$

## 問 2 の解答

$$\textcircled{1} m = \frac{a+b}{2} \quad \textcircled{2} m = \frac{a+b}{2} \quad \textcircled{3} m = \frac{a+b}{2} \quad \textcircled{4} m = \frac{a+b}{2}$$

## 問 3 の解答



## &lt; 33 ページ. フーリエ級数の収束 &gt;

## 問の解答

$$(1) S_{\infty}\left(\frac{3\pi}{2}\right) = \frac{\pi}{2} \quad (2) S_{\infty}(0) = 0 \quad (3) S_{\infty}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$$

## &lt; 34 ページ. フーリエ級数の部分和 &gt;

$$\begin{aligned}
S_n(x) &= a_0 + \sum_{k=1}^n \{a_k \cos(kx) + b_k \sin(kx)\} \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt + \sum_{k=1}^n \left\{ \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(kt) dt \cos(kx) + \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(kt) dt \sin(kx) \right\} \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt + \sum_{k=1}^n \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \left\{ \boxed{\cos(kt) \cos(kx) + \sin(kt) \sin(kx)} \right\} dt \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt + \sum_{k=1}^n \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(\boxed{kt - kx}) dt \\
&= \int_{-\pi}^{\pi} f(t) \left\{ \frac{1}{2\pi} + \frac{1}{\pi} \sum_{k=1}^n \cos(\boxed{k(x-t)}) \right\} dt = \int_{-\pi}^{\pi} f(t) D_n(x-t) dt
\end{aligned}$$

## &lt; 35 ページ. オイラーの公式 1 &gt;

## 問 1 の解答

(1)  $e^{2\pi i} = 1$

(2)  $e^{-\pi i} = -1$

(3)  $e^{\frac{\pi}{2}i} = i$

(4)  $e^{\frac{\pi}{3}i} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

(5)  $e^{-\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{i}{2}$

(6)  $e^{\frac{3}{4}\pi i} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

## 問 2 の解答

$|e^{i\theta}| = 1$

## 問 3 の解答

$e^{-i\theta} = \cos \theta - i \sin \theta$

## 問 4 の解答

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} = \left( \frac{e^{-i\theta} - e^{i\theta}}{2} \right) i$$

## &lt; 36 ページ. オイラーの公式 2 &gt;

## 問 1 の解答

$$\begin{aligned} e^{i\alpha} e^{i\beta} &= (\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta) \\ &= \cos \alpha \cos \beta - \sin \alpha \sin \beta + i(\sin \alpha \cos \beta + \cos \alpha \sin \beta) \\ &= \cos(\alpha + \beta) + i \sin(\alpha + \beta) \\ &= e^{i(\alpha + \beta)} \quad (\text{証明終}) \end{aligned}$$

## 問 2 の解答

$$\begin{aligned} (1) \quad \frac{1 + e^{10ti}}{e^{5ti}} &= e^{-5ti} + e^{5ti} \\ &= 2 \cos(5t) \\ (2) \quad (e^{6ti} - 1) \times e^{-3ti} &= e^{3ti} - e^{-3ti} \\ &= 2i \sin(3t) \end{aligned}$$

## 問 3 の解答

$$e^{-8ti} \times \frac{1 - e^{18ti}}{1 - e^{2ti}} = \frac{e^{-8ti} - e^{10ti}}{1 - e^{2ti}} = \frac{e^{-9ti} - e^{9ti}}{e^{-ti} - e^{ti}} = \frac{e^{9ti} - e^{-9ti}}{e^{ti} - e^{-ti}} = \frac{2i \sin(9t)}{2i \sin t} = \frac{\sin(9t)}{\sin t}$$

## &lt; 37 ページ. 三角数列の和 &gt;

## 問 1 の解答

$$(1) \quad a + ar + ar^2 + \cdots + ar^n = \frac{a - ar^{n+1}}{1 - r}$$

$$(2) \quad r + r^2 + r^3 + \cdots + r^n = \frac{r - r^{n+1}}{1 - r}$$

$$(3) \quad \sum_{k=0}^n ar^k = \frac{a - ar^{n+1}}{1 - r}$$

$$(4) \quad \sum_{k=0}^{2n} r^k = \frac{1 - r^{2n+1}}{1 - r}$$

## 問 2 の解答

$$(1) \quad \sum_{k=1}^n (e^{it})^k = \frac{e^{it} - e^{it(n+1)}}{1 - e^{it}}$$

$$(2) \quad \sum_{k=0}^{2n} (e^{it})^k = \frac{1 - e^{it(2n+1)}}{1 - e^{it}}$$

## 問 3 の解答

$$\begin{aligned} 1 + 2 \sum_{k=1}^n \cos(kt) &= 1 + 2 \sum_{k=1}^n \frac{e^{ikt} + e^{-ikt}}{2} = e^0 + \sum_{k=1}^n e^{ikt} + \sum_{k=1}^n e^{-ikt} \\ &= e^{-int} \left\{ e^{int} + \sum_{k=1}^n e^{i(n+k)t} + \sum_{k=1}^n e^{i(n-k)t} \right\} = e^{-int} \sum_{k=0}^{2n} e^{ikt} \end{aligned}$$

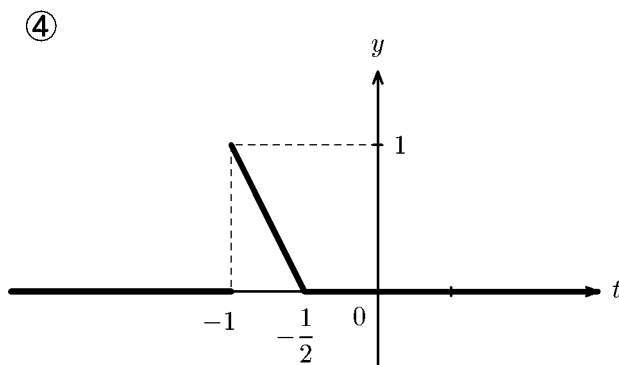
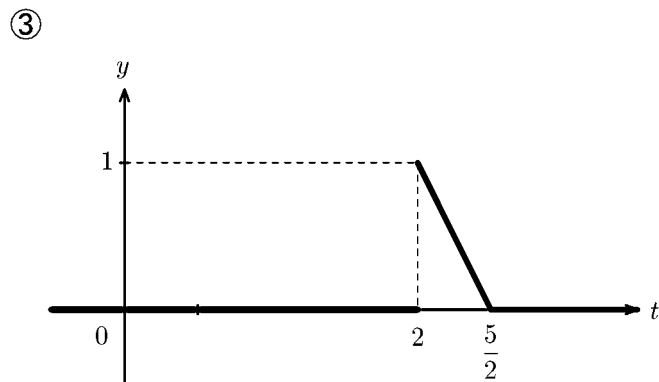
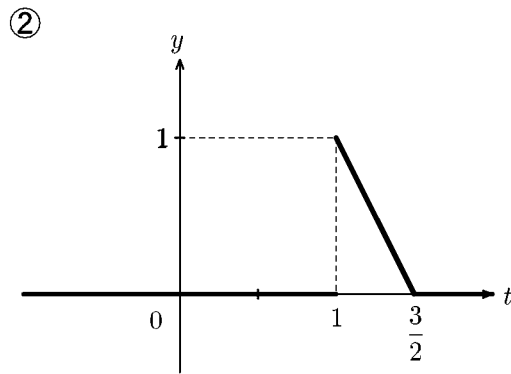
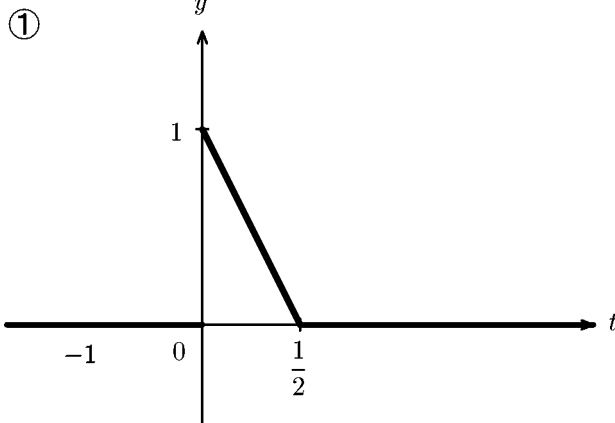
## &lt; 38 ページ. ディリクレ核 &gt;

## 問の解答

$$\begin{aligned}
1 + 2 \sum_{k=1}^n \cos(kt) &= 1 + 2 \sum_{k=1}^n \frac{e^{ikt} + e^{-ikt}}{2} = 1 + \sum_{k=1}^n (e^{it})^k + \sum_{k=1}^n (e^{it})^{-k} \\
&= 1 + (e^{it}) + (e^{it})^2 + \cdots + (e^{it})^n + (e^{it})^{-1} + (e^{it})^{-2} + \cdots + (e^{it})^{-n} \\
&= \sum_{k=-n}^n (e^{it})^k = (e^{it})^{-n} \sum_{k=-n}^n (e^{it})^{n+k} = e^{-itn} \sum_{k=0}^{2n} (e^{it})^k \\
&= e^{-itn} \{1 + e^{it} + (e^{it})^2 + (e^{it})^3 + \cdots + (e^{it})^{2n}\} = e^{-itn} \times \frac{1 - e^{\boxed{it(2n+1)}}}{1 - e^{\boxed{it}}} \\
&= \frac{e^{\boxed{-itn}} - e^{\boxed{it(n+1)}}}{1 - e^{\boxed{it}}} = \frac{\left( e^{\boxed{-itn}} - e^{\boxed{it(n+1)}} \right) \times e^{-\frac{it}{2}}}{\left( 1 - e^{\boxed{it}} \right) \times e^{-\frac{it}{2}}} \\
&= \frac{e^{-i(n+\frac{1}{2})t} - e^{i(n+\frac{1}{2})t}}{e^{-\frac{it}{2}} - e^{\frac{it}{2}}} = \frac{-2i \sin\left(n + \frac{1}{2}\right)t}{-2i \sin\left(\frac{t}{2}\right)} = \frac{\sin\left(n + \frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)}
\end{aligned}$$

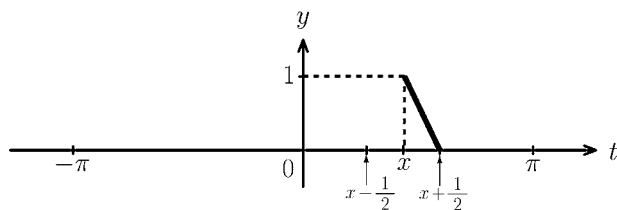
## &lt; 39 ページ. たたみこみ 1 &gt;

## 問の解答



< 40 ページ. たたみこみ 2 >

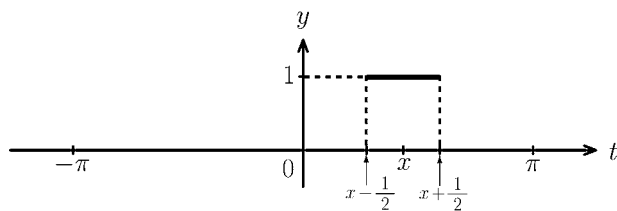
### 問 1 の解答



### 問 2 の解答

「 $y = g(x - t)$  のグラフは  $y = g(t)$  のグラフを  $y$  軸に関し対称に折り返して、  
 $t$  軸 (横軸) に平行に  $x$  だけ移動したグラフである。」

### 問 3 の解答



### 問 4 の解答

「関数  $g(t)$  が偶関数 ( $y$  軸に対称) のとき、 $y = g(x - t)$  のグラフは  
 $y = g(t)$  のグラフを  $t$  軸に平行に  $x$  だけ移動したグラフである。」

< 41 ページ. たたみこみ 3 >

### 問の解答

$$(1) \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_{x-\frac{1}{2}}^2 1dt = \left[ t \right]_{x-\frac{1}{2}}^2 = \frac{5}{2} - x$$

$$(2) \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_0^2 g(x-t)dt = 0$$

< 42 ページ. たたみこみ 4 >

### 問の解答

$$(1) \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_{x-\frac{1}{4}}^{x+\frac{1}{4}} 2dt = 2\left(x + \frac{1}{4} - x + \frac{1}{4}\right) = 1$$

$$(2) \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_0^2 g(x-t)dt = \int_{x-\frac{1}{4}}^2 2dt = 2\left(2 - x + \frac{1}{4}\right) = \frac{9}{2} - 2x$$

$$(3) \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_0^2 g(x-t)dt = 0$$

< 43 ページ. たたみこみ 5 >

### 問の解答

$$(1) (f * g)(x) = \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_0^2 g(x-t)dt = 0$$

$$(2) (f * g)(x) = \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_{x-h}^{x+h} \frac{1}{2h} dt \\ = \frac{1}{2h}(x+h) = \frac{1}{2h}x + \frac{1}{2}$$

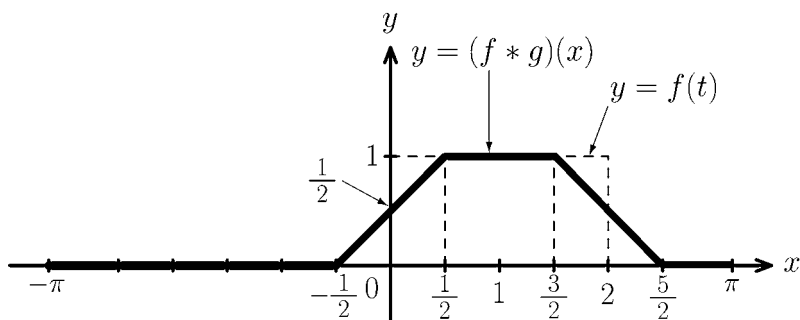
$$(3) (f * g)(x) = \int_{-\pi}^{\pi} f(t)g(x-t)dt = \int_{x-h}^{x+h} \frac{1}{2h} dt \\ = \frac{1}{2h}(x+h-x+h) = 1$$

$$(4) (f * g)(x) = \int_{x-h}^2 \frac{1}{2h} dt = \frac{1}{2h}(2-x+h)$$

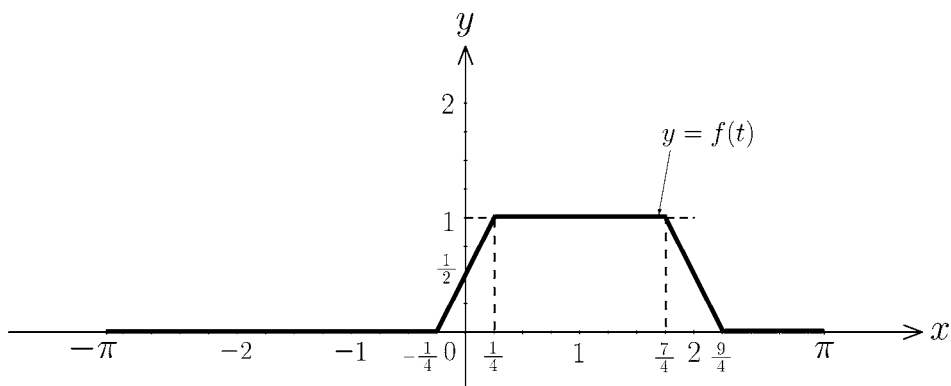
$$(5) (f * g)(x) = 0$$

< 44 ページ. たたみこみ 6 >

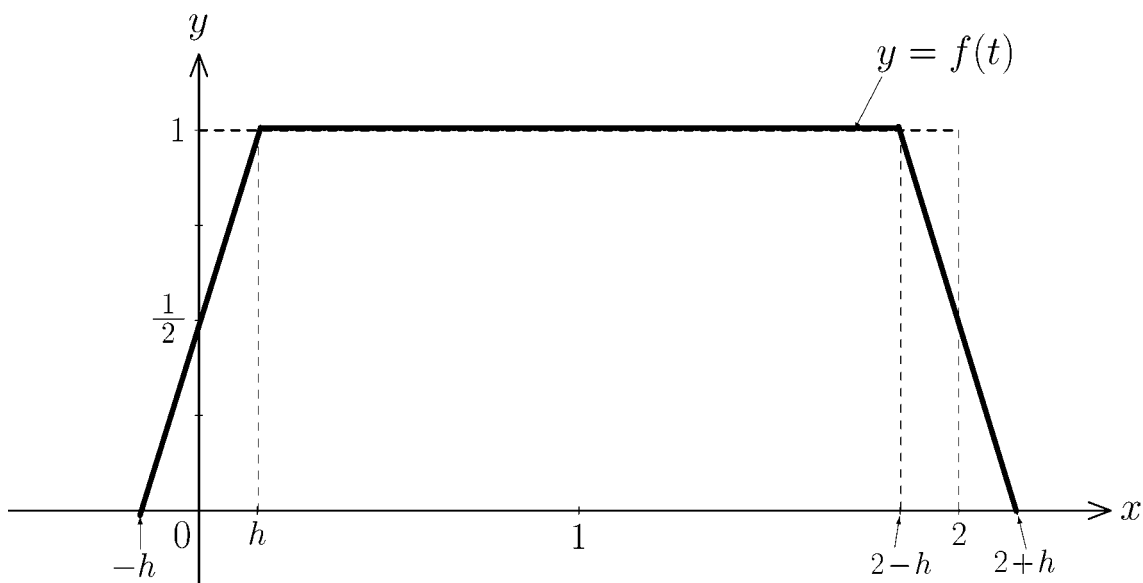
問 1 の解答



問 2 の解答



問 3 の解答



< 47 ページ. たたみこみ 9 >

### 問の解答

(図 2)  $(f * \delta_{0.2})(x)$

(図 3)  $(f * p_1)(x)$

(図 4)  $(f * D_8)(x)$

(図 5)  $(f * \delta_{0.1})(x)$

(図 6)  $(f * p_2)(x)$

(図 7)  $(f * D_{16})(x)$

< 48 ページ. たたみこみ 10 >

### 問の解答

$$(図 2) \quad (f * D_{16})(x)$$

$$(図 3) \quad (f * p_2)(x)$$

$$(図 4) \quad (f * \delta_{0.2})(x)$$

$$(図 5) \quad (f * p_1)(x)$$

$$(図 6) \quad (f * \delta_{0.1})(x)$$

$$(図 7) \quad (f * D_8)(x)$$

## &lt; 49 ページ. たたみこみ近似 &gt;

## 問の解答

$$\begin{aligned}(f * \delta_h)(x) &= \int_{-\pi}^{\pi} f(t) \delta_h(x-t) dt \\ &= \int_{x-h}^{x+h} f(t) \frac{1}{2h} dt \\ &= \frac{1}{2h} \left[ F(t) \right]_{x-h}^{x+h} = \frac{F(x+h) - F(x-h)}{2h}\end{aligned}$$

## &lt; 50 ページ. デルタ関数 &gt;

## 問 1 の解答

(1)  $\frac{3}{2}$

(2) 2

## 問 2 の解答

$$(f * \delta)(x) = \int_{-\pi}^{\pi} f(t)\delta(x-t)dt = f(x)$$

## &lt; 51 ページ. デルタ関数とディリクレ核 &gt;

## 問 1 の解答

$$(1) \quad a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \delta_h(x) dx = \frac{1}{2\pi} \int_{-h}^h \frac{1}{2h} dx = \frac{1}{2\pi}$$

$$\begin{aligned} a_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} \delta_h(x) \cos(kx) dx = \frac{1}{2\pi h} \int_{-h}^h \cos(kx) dx \\ &= \frac{1}{\pi h k} \sin(hk) \end{aligned}$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} \delta_h(x) \sin(kx) dx = \frac{1}{2\pi h} \int_{-h}^h \sin(kx) dx = 0$$

$$\begin{aligned} (2) \quad S_n(x) &= a_0 + \sum_{k=1}^n \left\{ a_k \cos(kx) + b_k \sin(kx) \right\} \\ &= \frac{1}{2\pi} + \sum_{k=1}^n \frac{\sin(hk) \cos(kx)}{\pi h k} \end{aligned}$$

## 問 2 の解答

$$(1) \quad \lim_{h \rightarrow +0} a_k = \lim_{h \rightarrow +0} \frac{1}{\pi h k} \sin(hk) = \frac{1}{\pi}$$

$$(2) \quad \lim_{h \rightarrow +0} S_n(x) = \lim_{h \rightarrow +0} \left\{ \frac{1}{2\pi} + \sum_{k=1}^n \frac{\sin(hk) \cos(kx)}{\pi h k} \right\} = \frac{1}{2\pi} + \sum_{k=1}^n \frac{\cos(kx)}{\pi}$$

## 問 3 の解答

$$D_n(x) = \frac{1}{2\pi} + \sum_{k=1}^n \frac{\cos(kx)}{\pi}$$

## 問 4 の解答

$\delta(x)$ , デルタ

## &lt; 52 ページ. 一般の周期関数 1 &gt;

## 問の解答

- (1) 周期 5      (2) 周期 7      (3) 周期 9      (4) 周期 6      (5) 周期 4  
(6) 周期 2      (7) 周期  $\frac{2}{3}$       (8) 周期  $\frac{2}{n}$       (9) 周期  $L$       (10) 周期  $2l$   
(11) 周期  $\frac{L}{n}$       (12) 周期  $\frac{2l}{n}$

< 53 ページ. 一般の周期関数 2 >

**問の解答**

- (1) 周期 5      (2) 周期  $L$       (3) 周期  $2l$

## &lt; 54 ページ. 一般周期のフーリエ級数 1 &gt;

## 問の解答

$$(1) f(x) \sim a_0 + \sum_{k=1}^{\infty} \left\{ a_k \cos\left(\frac{k\pi}{\ell}x\right) + b_k \sin\left(\frac{k\pi}{\ell}x\right) \right\}$$

$$a_0 = \frac{1}{2\ell} \int_{-\ell}^{\ell} f(x) dx \quad , \quad a_k = \frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \cos\left(\frac{k\pi}{\ell}x\right) dx$$

$$b_k = \frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \sin\left(\frac{k\pi}{\ell}x\right) dx$$

$$(2) f(x) \sim a_0 + \sum_{k=1}^{\infty} \left\{ a_k \cos\left(\frac{k}{\ell}x\right) + b_k \sin\left(\frac{k}{\ell}x\right) \right\}$$

$$a_0 = \frac{1}{2\pi\ell} \int_{-\pi\ell}^{\pi\ell} f(x) dx \quad , \quad a_k = \frac{1}{\pi\ell} \int_{-\pi\ell}^{\pi\ell} f(x) \cos\left(\frac{k}{\ell}x\right) dx$$

$$b_k = \frac{1}{\pi\ell} \int_{-\pi\ell}^{\pi\ell} f(x) \sin\left(\frac{k}{\ell}x\right) dx$$

## &lt; 55 ページ. 一般周期のフーリエ級数 2 &gt;

## 問 1 の解答

$$a_0 = 0, \quad a_k = 0$$

$$b_k = \frac{4}{L} \int_0^{\frac{L}{2}} f(x) \sin\left(\frac{2k\pi}{L}x\right) dx$$

## 問 2 の解答

$$a_0 + \sum_{k=1}^{\infty} \left\{ a_k \cos\left(\frac{2k\pi}{\ell}x\right) + b_k \sin\left(\frac{2k\pi}{\ell}x\right) \right\} = \frac{f_+(x) + f_-(x)}{2}$$

$$a_0 = \frac{1}{2\ell} \int_{-\ell}^{\ell} f(x) dx, \quad a_k = \frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \cos\left(\frac{k\pi}{\ell}x\right) dx$$

$$b_k = \frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \sin\left(\frac{k\pi}{\ell}x\right) dx$$

## 問 3 の解答

$$(1) \quad a_0 = \frac{1}{\ell} \int_0^{\ell} f(x) dx, \quad a_k = \frac{2}{\ell} \int_0^{\ell} f(x) \cos\left(\frac{k\pi}{\ell}x\right) dx$$

$$b_k = 0$$

$$(2) \quad a_0 = 0, \quad a_k = 0$$

$$b_k = \frac{2}{\ell} \int_0^{\ell} f(x) \sin\left(\frac{k\pi}{\ell}x\right) dx$$

## &lt; 56 ページ. 三角多項式の複素数表示 1 &gt;

## 問 1 の解答

$$D_n(x) = \frac{1}{2\pi} + \sum_{k=1}^n \frac{1}{2\pi} (e^{ikx} + e^{-ikx}) = \sum_{k=-n}^n \frac{1}{2\pi} e^{ikx}$$

## 問 2 の解答

$$S(x) = \sum_{k=1}^n \left\{ \alpha k^2 \cos(kx) + \beta k \sin(kx) \right\} = \sum_{k=-n}^n \left( \frac{\alpha k^2}{2} - \frac{\beta k}{2} i \right) e^{ikx} = \sum_{k=-n}^n C_k e^{ikx}$$

$$C_k = \frac{\alpha k^2}{2} - \frac{\beta k}{2} i$$

## &lt; 57 ページ. 三角多項式の複素数表示 2 &gt;

## 問の解答

$$S(x) = a_0 + \sum_{k=1}^n \{a_k \cos(k\omega x) + b_k \sin(k\omega x)\} = \sum_{k=-n}^n C_k e^{ik\omega x}$$

$$C_0 = a_0$$

$$C_{-k} = \frac{a_k}{2} + \frac{b_k}{2}i$$

$$C_k = \frac{a_k}{2} - \frac{b_k}{2}i$$

## &lt; 58 ページ. フーリエ級数の複素数表示 1 &gt;

## 問の解答

$$\begin{aligned}C_{-k} &= \frac{1}{2}(a_k + b_k i) \\&= \frac{1}{2} \left\{ \left( \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) \cos(k\omega x) dx \right) + \left( \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) \sin(k\omega x) dx \right) i \right\} \\&= \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) \{ \cos(k\omega x) + i \sin(k\omega x) \} dx \\&= \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) e^{ik\omega x} dx\end{aligned}$$

$$C_0 = a_0 = \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) dx$$

## &lt; 59 ページ. フーリエ級数の複素数表示 2 &gt;

## 問 1 の解答

$$C_{-k} = \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) e^{ik\omega x} dx = \frac{1}{2}(a_k + b_k i)$$

$$C_0 = \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) dx = a_0$$

## 問 2 の解答

$$(1) f(x) \sim \sum_{k=-\infty}^{\infty} C_k e^{ikx}, \quad (\omega = 1)$$

$$C_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx} dx$$

$$(2) f(x) \sim \sum_{k=-\infty}^{\infty} C_k e^{ik\omega x}, \quad \left(\omega = \frac{1}{m}\right)$$

$$C_k = \frac{1}{2\pi m} \int_{-\pi m}^{\pi m} f(x) e^{-ik\omega x} dx$$

## &lt; 60 ページ. 広義積分 1 &gt;

## 問の解答

$$\begin{aligned}(1) \int_0^{\infty} e^{-tx} dx &= \lim_{b \rightarrow \infty} \int_0^b e^{-tx} dx = \lim_{b \rightarrow \infty} \left[ -\frac{1}{t} e^{-tx} \right]_{x=0}^{x=b} \\ &= \lim_{b \rightarrow \infty} \left( -\frac{1}{te^{tb}} + \frac{1}{t} \right) = \frac{1}{t}\end{aligned}$$

$$\begin{aligned}(2) \int_1^{\infty} \frac{1}{x^2} dx &= \lim_{b \rightarrow \infty} \int_1^b x^{-2} dx = \lim_{b \rightarrow \infty} \left[ -\frac{1}{x} \right]_1^b \\ &= \lim_{b \rightarrow \infty} \left( -\frac{1}{b} + 1 \right) = 1\end{aligned}$$

$$\begin{aligned}(3) \int_1^{\infty} \frac{1}{x^r} dx &= \lim_{b \rightarrow \infty} \int_1^b x^{-r} dx = \lim_{b \rightarrow \infty} \left[ \frac{1}{1-r} x^{1-r} \right]_1^b \\ &= \lim_{b \rightarrow \infty} \left( \frac{1}{(1-r)b^{r-1}} - \frac{1}{1-r} \right) = \frac{1}{r-1}\end{aligned}$$

## &lt; 62 ページ. 広義積分 3 &gt;

## 問の解答

$$(1) \int_{-\infty}^{\infty} \frac{\sin(2x)}{x} dx = 2 \int_0^{\infty} \frac{\sin(2x)}{x} dx = 2 \times \frac{\pi}{2} = \pi$$

$$(2) \int_{-\infty}^{\infty} \frac{\sin(-3x)}{x} dx = 2 \int_0^{\infty} \frac{\sin(-3x)}{x} dx = -\pi$$

$$(3) \lambda x = t \text{ とおくと } dx = \frac{1}{\lambda} dt \text{ より}$$

$$\int_0^{\infty} \frac{\sin(\lambda x)}{x} dx = \int_0^{\infty} \frac{\sin(t)}{\frac{t}{\lambda}} \times \frac{1}{\lambda} dt = \int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$$

$$(4) \sin(-\lambda x) = -\sin(\lambda x) \text{ より}$$

$$\int_0^{\infty} \frac{\sin(-\lambda x)}{x} dx = - \int_0^{\infty} \frac{\sin(\lambda x)}{x} dx = -\frac{\pi}{2}$$

## &lt; 63 ページ. 広義積分 4 &gt;

## 問の解答

$$(1) \quad \frac{1}{\pi} \left\{ \int_0^{\infty} \frac{\sin((8-t)x)}{x} dx + \int_0^{\infty} \frac{\sin(xt)}{x} dx \right\} = I_{\{0, 8\}}(t)$$

$$(2) \quad \frac{6}{\pi} \left\{ \int_0^{\infty} \frac{\sin((8-t)x)}{x} dx + \int_0^{\infty} \frac{\sin((8+t)x)}{x} dx \right\} = 6I_{\{-8, 8\}}(t)$$

## &lt; 64 ページ. 広義積分の近似 &gt;

## 問の解答

$$(1) \lim_{\Delta x \rightarrow 0} \sum_{k=0}^{\infty} \frac{\cos(\alpha k \Delta x) \Delta x}{1 + (k \Delta x)^2} = \int_0^{\infty} \frac{\cos \alpha x}{1 + x^2} dx$$

$$(2) \lim_{\Delta x \rightarrow 0} \sum_{k=-\infty}^{\infty} F(k \Delta x) e^{ik \Delta x t} \Delta x = \int_{-\infty}^{\infty} F(x) e^{ixt} dx$$

## &lt; 65 ページ. フーリエ変換 1 &gt;

## 問の解答

$$(1) \lim_{L \rightarrow \infty} F_L(x) = \lim_{L \rightarrow \infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(t) e^{-ixt} dt = \int_{-\infty}^{\infty} f(t) e^{-ixt} dt$$

$$(2) F_L(k\omega) = \int_{-\frac{L}{2}}^{\frac{L}{2}} f(t) e^{-ik\omega t} dt$$

$$(3) C_k = \frac{\omega}{2\pi} F_L(k\omega)$$

$$(4) \sum_{k=-\infty}^{\infty} C_k e^{ik\omega t} = \sum_{k=-\infty}^{\infty} \frac{\omega}{2\pi} F_L(k\omega) e^{ik\omega t}$$

$$(5) \sum_{k=-\infty}^{\infty} C_k e^{ik\omega t} = \sum_{k=-\infty}^{\infty} \frac{1}{2\pi} \times \Delta x F_L(k\Delta x) e^{ik\Delta x t} = \sum_{k=-\infty}^{\infty} \frac{1}{2\pi} F_L(k\Delta x) e^{ik\Delta x t} \Delta x$$

$$(6) \lim_{\Delta x \rightarrow 0} \sum_{k=-\infty}^{\infty} \frac{1}{2\pi} F_L(k\Delta x) e^{ik\Delta x t} \Delta x = \int_{-\infty}^{\infty} \frac{1}{2\pi} F_L(x) e^{ixt} dx$$

$$(7) \lim_{L \rightarrow \infty} \sum_{k=-\infty}^{\infty} \frac{1}{2\pi} F_L(k\Delta x) e^{ik\Delta x t} \Delta x = \int_{-\infty}^{\infty} \frac{1}{2\pi} F_L(x) e^{ixt} dx$$

$$(8) \int_{-\infty}^{\infty} \frac{1}{2\pi} F_L(x) e^{ixt} dx$$

## &lt; 67 ページ. フーリエ変換 3 &gt;

## 問の解答

$$\begin{aligned}(1) \quad F(x) &= \int_{-\infty}^{\infty} f(x)e^{-ixt} dt = \int_{-1}^1 \cos(xt) dt - i \int_{-1}^1 \sin(xt) dt \\ &= 2 \int_0^1 \cos(xt) dt = 2 \left[ \frac{1}{x} \sin(xt) \right]_0^1 \\ &= \frac{2 \sin x}{x}\end{aligned}$$

$$\begin{aligned}(2) \quad F(x) &= \int_{-\infty}^{\infty} f(t)e^{-ixt} dt = \int_{-T}^T k \cos(xt) dt - i \int_{-T}^T k \sin(xt) dt \\ &= k \int_{-T}^T \cos(xt) dt = k \left[ \frac{1}{x} \sin(xt) \right]_{-T}^T \\ &= \frac{2k \sin Tx}{x}\end{aligned}$$

## &lt; 68 ページ. フーリエ変換 4 &gt;

## 問の解答

$$\begin{aligned}
(1) \quad F(x) &= \int_{-\infty}^{\infty} f(x)e^{-ixt} dt = \int_{-\infty}^{\infty} f(x) \cos(xt) dt - i \int_{-\infty}^{\infty} f(x) \sin(xt) dt \\
&= \int_{-1}^0 (-1) \cos(xt) dt + \int_0^1 \cos(xt) dt - i \left\{ \int_{-1}^0 (-1) \sin(xt) dt + \int_0^1 \sin(xt) dt \right\} \\
&= -2i \left[ -\frac{1}{x} \cos(xt) \right]_0^1 \\
&= 2i \left( \frac{\cos x - 1}{x} \right)
\end{aligned}$$

$$\begin{aligned}
(2) \quad F(x) &= \int_{-\infty}^{\infty} f(x)e^{-ixt} dt = \int_{-\infty}^{\infty} f(x) \cos(xt) dt - i \int_{-\infty}^{\infty} f(x) \sin(xt) dt \\
&= \int_{-T}^0 (-k) \cos(xt) dt + \int_0^T k \cos(xt) dt - i \left\{ \int_{-T}^0 (-k) \sin(xt) dt + \int_0^T k \sin(xt) dt \right\} \\
&= -2ki \left[ -\frac{1}{x} \cos(xt) \right]_0^T \\
&= 2ki \left( \frac{\cos Tx - 1}{x} \right)
\end{aligned}$$

## &lt; 69 ページ. フーリエ変換 5 &gt;

## 問の解答

$$\begin{aligned}
(1) \quad F(x) &= \int_{-\infty}^{\infty} f(t)e^{-ixt} dt = \int_{-\infty}^{\infty} f(t) \cos(xt) dt - i \int_{-\infty}^{\infty} f(t) \sin(xt) dt \\
&= \int_0^1 2 \cos(xt) dt - i \int_0^1 2 \sin(xt) dt \\
&= \left[ \frac{2 \sin(xt)}{x} \right]_0^1 - i \left[ -\frac{2 \cos(xt)}{x} \right]_0^1 \\
&= \frac{2 \sin x}{x} + 2i \left( \frac{\cos x - 1}{x} \right)
\end{aligned}$$

$$\begin{aligned}
(2) \quad F(x) &= \int_{-\infty}^{\infty} f(t)e^{-ixt} dt = \int_{-\infty}^{\infty} f(t) \cos(xt) dt - i \int_{-\infty}^{\infty} f(t) \sin(xt) dt \\
&= \int_0^T 2k \cos(xt) dt - i \int_0^T 2k \sin(xt) dt \\
&= \left[ \frac{2k \sin(xt)}{x} \right]_0^T - i \left[ -\frac{2k \cos(xt)}{x} \right]_0^T \\
&= \frac{2k \sin xT}{x} + 2ki \left( \frac{\cos xT - 1}{x} \right)
\end{aligned}$$

## &lt; 70 ページ. フーリエ変換 6 &gt;

## 問 1 の解答

$$\begin{aligned}
\frac{2 \sin x}{x} e^{-3ix} &= \frac{e^{-3ix}}{-ix} (-2i \sin x) = \frac{e^{-3ix}}{-ix} (e^{-ix} - e^{ix}) \\
&= \frac{e^{-4ix} - e^{-2ix}}{-ix} = -\frac{1}{ix} \{ (\cos(4x) - i \sin(4x)) - (\cos(2x) - i \sin(2x)) \} \\
&= \frac{\sin(4x) - \sin(2x)}{x} + i \left( \frac{\cos(4x) - \cos(2x)}{x} \right)
\end{aligned}$$

## 問 2 の解答

$$\begin{aligned}
F(x) &= \int_{-\infty}^{\infty} f(t) e^{-ixt} dt = \int_a^b k \cos(xt) dt - i \int_a^b k \sin(xt) dt \\
&= k \left( \frac{\sin(bx) - \sin(ax)}{x} \right) + ik \left( \frac{\cos(bx) - \cos(ax)}{x} \right)
\end{aligned}$$

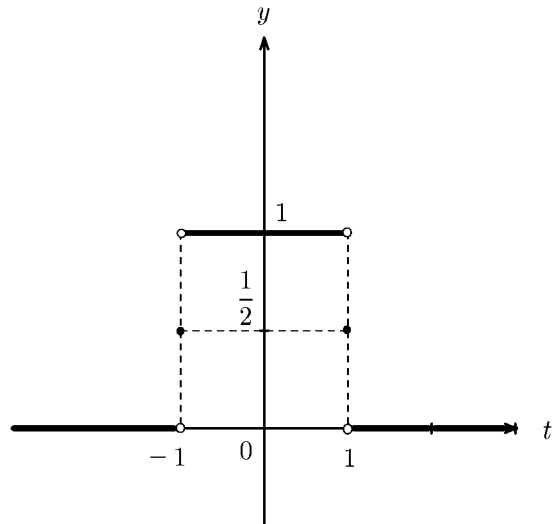
## 問 3 の解答

$$\begin{aligned}
F(x) &= \int_{-\infty}^{\infty} f(t) e^{-ixt} dt = \int_{c-T}^{c+T} k e^{-ixt} dt = -\frac{k}{ix} \{ e^{-ix(c+T)} - e^{-ix(c-T)} \} \\
&= \frac{2k \sin(xT)}{x} e^{-ixc}
\end{aligned}$$

## &lt; 71 ページ. フーリエ逆変換 1 &gt;

## 問の解答

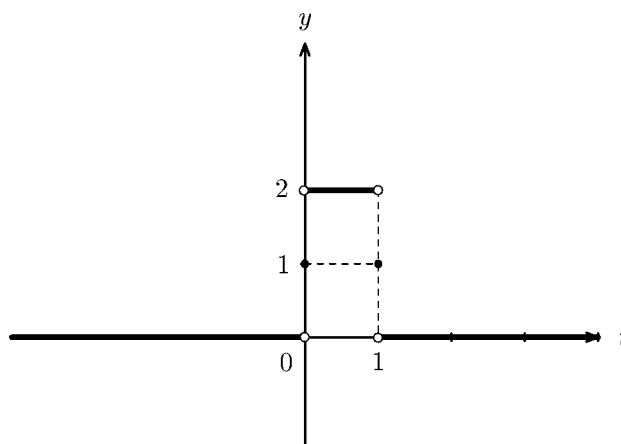
$$\begin{aligned}
& \frac{1}{2\pi} \int_{-\infty}^{\infty} F(x) e^{ixt} dx \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2 \sin x}{x} \left\{ \cos(xt) + i \sin(xt) \right\} dx \\
&= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin x \cos(xt)}{x} dx + \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{\sin x \sin(xt)}{x} dx \\
&= \frac{2}{\pi} \int_0^{\infty} \frac{\sin x \cos(xt)}{x} dx \\
&= \frac{2}{\pi} \int_0^{\infty} \frac{\sin(x+xt) + \sin(x-xt)}{2x} dx \\
&= \frac{1}{\pi} \left\{ \int_0^{\infty} \frac{\sin((1-t)x)}{x} dx + \int_0^{\infty} \frac{\sin((t+1)x)}{x} dx \right\} \\
&= I_{\{-1, 1\}}(t)
\end{aligned}$$



## &lt; 72 ページ. フーリエ逆変換 2 &gt;

## 問の解答

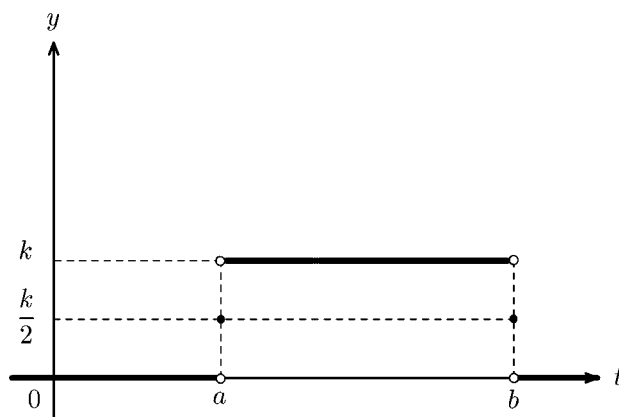
$$\begin{aligned}
& \frac{1}{2\pi} \int_{-\infty}^{\infty} F(x)e^{ixt} dx \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ \frac{2 \sin x}{x} + 2i \left( \frac{\cos x - 1}{x} \right) \right\} \left\{ \cos(xt) + i \sin(xt) \right\} dx \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} 2 \left\{ \frac{\sin x \cos(xt) - (\cos x - 1) \sin(xt)}{x} \right\} dx \\
&= \frac{2}{\pi} \int_0^{\infty} \frac{\sin x \cos(xt) - \cos x \sin(xt) + \sin(xt)}{x} dx \\
&= \frac{2}{\pi} \int_0^{\infty} \frac{\sin((1-t)x) + \sin(xt)}{x} dx \\
&= 2I_{\{0, 1\}}(t)
\end{aligned}$$



## &lt; 73 ページ. フーリエ逆変換 3 &gt;

## 問の解答

$$\begin{aligned}
& \frac{1}{2\pi} \int_{-\infty}^{\infty} F(x)e^{ixt} dx \\
&= \frac{k}{2\pi} \int_{-\infty}^{\infty} \left\{ \frac{\sin(bx) - \sin(ax)}{x} + i \left( \frac{\cos(bx) - \cos(ax)}{x} \right) \right\} \left\{ \cos(xt) + i \sin(xt) \right\} dx \\
&= \frac{k}{\pi} \int_0^{\infty} \frac{1}{x} \{ \sin(bx) \cos(xt) - \sin(ax) \cos(xt) - \cos(bx) \sin(xt) + \cos(ax) \sin(xt) \} dx \\
&= \frac{k}{\pi} \int_0^{\infty} \frac{1}{x} \{ \sin((b-t)x) + \sin((t-a)x) \} dx \\
&= kI_{\{a, b\}}(t)
\end{aligned}$$



## &lt; 74 ページ. 逆変換の収束 1 &gt;

## 問の解答

$$\begin{aligned}\int_{-n}^n \frac{1}{2\pi} e^{i(t-s)x} dx &= \frac{1}{2\pi} \int_{-n}^n \{ \cos(t-s)x + i \sin(t-s)x \} dx \\ &= \frac{1}{\pi} \int_0^n \cos(t-s)x dx = \frac{1}{\pi} \left[ \frac{\sin(t-s)x}{t-s} \right]_0^n \\ &= \frac{\sin\{n(t-s)\}}{\pi(t-s)}\end{aligned}$$

## &lt; 76 ページ. 反転公式 &gt;

## 問 1 の解答

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} F(x)e^{ixt} dx = f(x)$$

## 問 2 の解答

$$F(x) = c \left\{ \frac{\sin(bx) - \sin(ax)}{x} \right\} + ic \left\{ \frac{\cos(bx) - \sin(ax)}{x} \right\}$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} F(x)e^{ixt} dx = cI_{\{a, b\}}(t)$$