高知工科大学

基礎数学ワークブック

(2001 年度版)

秋期入学者用

VI

解答

< 偏微分係数の幾何学的意味 > (1ページ)

問の解答

$f_x(x,y) = 2x - 3 + y$

$f_y(x,y) = x - 2y + 2$

$f(3,2) = 0$　　$f_x(3,2) = 5$　　$f_y(3,2) = 1$

接線 L の方程式

$y = 2$

$z = 5x - 15$

接線 ℓ の方程式

$x = 3$

$z = y - 2$

< 接平面 > (2ページ)

問の解答

$f(1,2) = 7$

$f_x(x,y) = 2x - y$, $f_y(x,y) = -x + 4y$

$f_x(1,2) = 0$, $f_y(1,2) = 7$

だから接平面の方程式は

$z = 0(x-1) + 7(y-2) + 7$ より (答) $z = 7y - 7$

< 2変数関数の一次近似 > (3ページ)

問の解答

(1) $f_x(x,y) = 3x^2y^{-2}$, $f_y(x,y) = -2x^3y^{-3}$

$$(a+\Delta x)^3(b+\Delta y)^{-2} \fallingdotseq \left(\frac{3a^2}{b^2}\right)\Delta x - \left(\frac{2a^3}{b^3}\right)\Delta y + \frac{a^3}{b^2}$$

(2) $f_x(x,y) = -(\sin x)\sqrt{y}$, $f_y(x,y) = \dfrac{\cos x}{2\sqrt{y}}$

$$\cos(a+\Delta x)\sqrt{b+\Delta y} \fallingdotseq -(\sin a)\sqrt{b}\Delta x + \frac{\cos a}{2\sqrt{b}}\Delta y + (\cos a)\sqrt{b}$$

(3) $f_x(x,y) = \dfrac{1}{y}$, $f_y(x,y) = -\dfrac{x}{y^2}$

$$\frac{a+\Delta x}{b+\Delta y} \fallingdotseq \left(\frac{1}{b}\right)\Delta x - \left(\frac{a}{b^2}\right)\Delta y + \frac{a}{b}$$

< 2変数合成関数の微分 1 > (4ページ)

問の解答

$$\frac{d}{dt}f\left(x(t), y(t)\right) = f_x(x, y)\frac{dx}{dt} + f_y(x, y)\frac{dy}{dt}$$

< 2変数合成関数の微分 2 > (5ページ)

問の解答

(1) $2f_x(1+2t,\ 3-t) - f_y(1+2t,\ 3-t)$

(2) $(\cos\theta)f_x(r\cos\theta,\ r\sin\theta) + (\sin\theta)f_y(r\cos\theta,\ r\sin\theta)$

< 全微分 > (6ページ)

問 1 の解答

(1) $dz = 9\left(3x + 2y^2\right)^2 dx + 12y\left(3x + 2y^2\right)^2 dy$

(2) $dz = \sin(2y)dx + 2x\cos(2y)dy$

問 2 の解答

(1) $dx = du - dv$

(2) $dy = e^u du + \dfrac{1}{v}dv$

< ヤコビアン > (7ページ)

問の解答

(1) $J = \begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix} = 5 - (-3) = 8$

(2) $J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$

< ラプラシアン 1 > (8ページ)

問の解答

(1) $\Delta f(x, y) = 6y - 4$

(2) $\Delta f(x, y) = -5\sin(2x - y)$

< ラプラシアン 3 > (10ページ)

問の解答

(1) $z_r = 2\cos(2r),\quad z_{rr} = -4\sin(2r),\quad z_\theta = 0,\quad z_{\theta\theta} = 0$

$$\Delta z = z_{rr} + \frac{1}{r}z_r = -4\sin(2r) + \frac{2}{r}\cos(2r) = -4\sin\left(2\sqrt{x^2+y^2}\right) + \frac{2\cos\left(2\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}$$

(2) $z_r = 0,\quad z_{rr} = 0,\quad z_\theta = -\sin\theta + \cos\theta,\quad z_{\theta\theta} = -\cos\theta - \sin\theta$

$$\Delta z = \frac{1}{r^2}z_{\theta\theta} = \frac{-\cos\theta - \sin\theta}{r^2} = -\frac{x+y}{(x^2+y^2)\sqrt{x^2+y^2}}$$

< 面積 1 > (11ページ)

問の解答

$$S = \int_0^2 (-x^2 + 4)dx - \int_0^2 x^2 dx$$

$$= \int_0^2 (-2x^2 + 4)dx = \left[-\frac{2}{3}x^3 + 4x \right]_0^2$$

$$= -\frac{16}{3} + 8 - 0 = \frac{-16 + 24}{3} = \frac{8}{3}$$

＜ 面積 2 ＞ (12ページ)

問 1 の解答

$$S = \int_a^b \{f(x) + C\}\,dx - \int_a^b \{g(x) + C\}\,dx$$

$$= \int_a^b \{f(x) - g(x)\}\,dx$$

問 2 の解答

$$S = \int_0^3 \left\{(-x^2 + 4x) - (x^2 - 2x)\right\} dx$$

$$= \int_0^3 (-2x^2 + 6x)dx = \left[-\frac{2}{3}x^3 + 3x^2\right]_0^3$$

$$= -\frac{2 \times 27}{3} + 3 \times 9 - 0 = 9$$

< 体積 1 > (13ページ)

問の解答

$$V = \lim_{n\to\infty} V_n = \lim_{n\to\infty} \frac{5^2 \times 7}{12} \times \left(1 + \frac{1}{n}\right) \times \left(2 + \frac{1}{n}\right)$$

$$= \frac{5^2 \times 7}{12} \times 1 \times 2 = \frac{175}{6}$$

< 体積 2 > (14 ページ)

問の解答

$$V = \int_0^7 f(x)dx = \int_0^7 \frac{25}{98}x^2 dx = \left[\frac{25}{98} \times \frac{x^3}{3}\right]_0^7$$

$$= \frac{25}{98} \times \frac{7^3}{3} = \frac{175}{6} \quad \left(\fallingdotseq 29.16\right)$$

＜体積 3＞ (15ページ)

問の解答

(1) $f(x) = \dfrac{1}{2}\times\mathrm{DE}\times\mathrm{DF}\times\sin 30^\circ = \dfrac{1}{2}\times\dfrac{x}{3}\times\dfrac{4x}{9}\times\dfrac{1}{2} = \dfrac{x^2}{27}$

(2) $V = \displaystyle\int_0^9 f(x)dx = \int_0^9 \frac{x^2}{27}dx = \left[\frac{x^3}{81}\right]_0^9 = 9$

< 体積 4 > (16 ページ)

問 1 の解答

$$f(x) = \frac{\pi x^2}{4}$$

$$V = \int_0^6 \frac{\pi x^2}{4} dx = \left[\frac{\pi x^3}{12}\right]_0^6 = \frac{\pi \times 6^3}{12} = 18\pi$$

問 2 の解答

$$f(x) = \left(\frac{2x}{3}\right)^2 = \frac{4x^2}{9}$$

$$V = \int_0^6 \frac{4x^2}{9} dx = \left[\frac{4x^3}{27}\right]_0^6 = \frac{4 \times 6^3}{27} = 32$$

< 体積 5 > (17ページ)

問の解答

$$V = \int_{-r}^{r} \pi \left(\sqrt{r^2 - x - 2}\right)^2 dx = \int_{-r}^{r} \pi (r^2 - x^2) dx$$

$$= \left[\pi \left(r^2 x - \frac{1}{3}x^3\right)\right]_{-r}^{r} = \pi \left\{\left(r^3 - \frac{1}{3}r^3\right) - \left(-r^3 + \frac{1}{3}r^3\right)\right\}$$

$$= \pi \left\{2r^3 - \frac{2}{3}r^3\right\} = \frac{4}{3}\pi r^3$$

＜ 体積 6 ＞ (18ページ)

問の解答

$$
\begin{aligned}
V &= \int_0^{r\cos\theta} \pi \{(\tan\theta)\, x\}^2\, dx + \int_{r\cos\theta}^{r} \pi \left\{\sqrt{r^2 - x^2}\right\}^2 dx \\
&= \pi \tan^2\theta \left[\frac{x^3}{3}\right]_0^{r\cos\theta} + \pi \left[r^2 x - \frac{1}{3}x^3\right]_{r\cos\theta}^{r} \\
&= \pi \left(\tan^2\theta\right) \frac{r^3\cos^3\theta}{3} + \pi \left\{\frac{2r^3}{3} - \left(r^3\cos\theta - \frac{r^3}{3}\cos^3\theta\right)\right\} \\
&= \frac{\pi r^3}{3} \left\{2 + \left(1 + \tan^2\theta\right)\cos^3\theta - 3\cos\theta\right\} = \frac{2\pi r^3}{3}(1 - \cos\theta)
\end{aligned}
$$

$<$ 体積 7 $>$ (19 ページ)

問 1 の解答

$$a = 1 + \frac{x}{3}$$

問 2 の解答

$$b = 2 + \frac{x}{3}$$

問 3 の解答

$$\begin{aligned} S(x) &= \frac{1}{2} \times (a+b) \times 4 = 2a + 2b = 2\left(1 + \frac{x}{3}\right) + 2\left(2 + \frac{x}{3}\right) \\ &= 6 + \frac{4}{3}x \end{aligned}$$

問 4 の解答

$$\begin{aligned} V &= \int_0^3 S(x)dx = \int_0^3 \left(6 + \frac{4}{3}x\right) dx = \left[6x + \frac{2}{3}x^2\right]_0^3 \\ &= 18 + \frac{2}{3} \times 3^2 = 24 \end{aligned}$$

< 体積 8 > (20 ページ)

問の解答

$$
\begin{aligned}
S(x) &= \int_0^3 (5 - x + 0.2y)dy \\
&= \left[(5-x)y + 0.1y^2\right]_{y=0}^{y=3} \\
&= (5-x) \times 3 + 0.9 = 15.9 - 3x
\end{aligned}
$$

$$
\begin{aligned}
V &= \int_0^2 S(x)dx = \int_0^2 (15.9 - 3x)dx \\
&= \left[15.9x - \frac{3}{2}x^2\right]_0^2 \\
&= 31.8 - 6 = 25.8
\end{aligned}
$$

< 体積 9 > (21ページ)

問の解答

$$S(x) = \int_0^3 \left(3 - \frac{x^2}{2} + \frac{xy}{2} + 2y - y^2\right) dy = \left[3y - \frac{x^2}{2}y + \frac{xy^2}{4} + y^2 - \frac{y^3}{3}\right]_{y=0}^{y=3}$$

$$= 9 - \frac{3}{2}x^2 + \frac{9x}{4} + 9 - 9 = 9 - \frac{3}{2}x^2 + \frac{9}{4}x$$

$$V = \int_0^2 S(x)dx = \int_0^2 \left(9 - \frac{3}{2}x^2 + \frac{9}{4}x\right) dx = \left[9x - \frac{1}{2}x^3 + \frac{9}{8}x^2\right]_0^2$$

$$= 18 - \frac{1}{2} \times 8 + \frac{9}{8} \times 4 = \frac{37}{2}$$

＜体積 10 ＞ (22 ページ)

問の解答

$$S(y) = \int_0^3 \left(\frac{1}{3}x - \frac{1}{4}y + 2\right) dx = \left[\frac{1}{6}x^2 - \frac{1}{4}yx + 2x\right]_{x=0}^{x=3}$$

$$= \frac{9}{6} - \frac{3}{4}y + 6 = \frac{15}{2} - \frac{3}{4}y$$

$$V = \int_0^4 S(y)dy = \int_0^4 \left(\frac{15}{2} - \frac{3}{4}y\right) dy = \left[\frac{15}{2}y - \frac{3}{8}y^2\right]_0^4$$

$$= 30 - 6 = 24$$

< 累次積分 1 > (23 ページ)

問の解答

$$\int_1^2 \left\{ \int_1^3 (x^2 - xy - 1)dy \right\} dx = \int_1^2 \left\{ \left[x^2 y - \frac{x}{2} y^2 - y \right]_{y=1}^{y=3} \right\} dx$$

$$= \int_1^2 \left\{ \left(3x^2 - \frac{9}{2}x - 3 \right) - \left(x^2 - \frac{1}{2}x - 1 \right) \right\} dx = \int_1^2 \left(2x^2 - 4x - 2 \right) dx$$

$$= \left[\frac{2}{3}x^3 - 2x^2 - 2x \right]_{x=1}^{x=2} = \left(\frac{16}{3} - 8 - 4 \right) - \left(\frac{2}{3} - 2 - 2 \right) = -\frac{10}{3}$$

＜ 累次積分 2 ＞ (24 ページ)

問の解答

$$\int_1^3 \left\{ \int_1^2 (x^2 - xy - 1) dx \right\} dy = \int_1^3 \left\{ \left[\frac{x^3}{3} - \frac{x^2}{2} y - x \right]_{x=1}^{x=2} \right\} dy$$

$$= \int_1^3 \left\{ \left(\frac{8}{3} - \frac{4}{2} y - 2 \right) - \left(\frac{1}{3} - \frac{1}{2} y - 1 \right) \right\} dy = \int_1^3 \left(\frac{7}{3} - \frac{3}{2} y - 1 \right) dy$$

$$= \int_1^3 \left(\frac{4}{3} - \frac{3}{2} y \right) dy = \left[\frac{4}{3} y - \frac{3}{4} y^2 \right]_{y=1}^{y=3} = \left(4 - \frac{27}{4} \right) - \left(\frac{4}{3} - \frac{3}{4} \right) = -\frac{10}{3}$$

< 長方形領域の 2 重積分 1 > (25 ページ)

問の解答

$$\iint_D (2x-3y^2)dxdy = \int_0^2 \left\{ \int_{-1}^1 (2x-3y^2)dx \right\} dy$$

$$= \int_0^2 \left\{ \left[x^2 - 3y^2 x \right]_{x=-1}^{x=1} \right\} dy = \int_0^2 \left\{ -6y^2 \right\} dy = \left[-2y^3 \right]_{y=0}^{y=2} = -16$$

(別解)

$$\iint_D (2x-3y^2)dxdy = \int_{-1}^1 \left\{ \int_0^2 (2x-3y^2)dy \right\} dx$$

$$= \int_{-1}^1 \left\{ \left[2xy - y^3 \right]_{y=0}^{y=2} \right\} dx = \int_{-1}^1 (4x-8)dy = \left[2x^2 - 8x \right]_{x=-1}^{x=1} = -16$$

< 長方形領域の 2 重積分 2 > (26 ページ)

問の解答

(1) $\displaystyle\iint_D x^3\sin(2y)dxdy = \left(\int_0^2 x^3dx\right)\times\left(\int_0^{\frac{\pi}{2}}\sin(2y)dy\right)$

$\displaystyle= \left\{\left[\frac{1}{4}x^4\right]_{x=0}^{x=2}\right\}\times\left\{\left[-\frac{1}{2}\cos(2y)\right]_{y=0}^{y=\frac{\pi}{2}}\right\} = \left\{\frac{2^4}{4}\right\}\times\left\{-\frac{1}{2}\cos\pi+\frac{1}{2}\cos 0\right\} = 4\times 1 = 4$

(2) $\displaystyle\iint_D e^{2x-y}dxdy = \iint_D e^{2x}\times e^{-y}dxdy = \left(\int_0^1 e^{2x}dx\right)\times\left(\int_0^1 e^{-y}dy\right)$

$\displaystyle= \left\{\left[\frac{1}{2}e^{2x}\right]_{x=0}^{x=1}\right\}\times\left\{\left[-e^{-y}\right]_{y=0}^{y=1}\right\} = \left\{\frac{1}{2}e^2-\frac{1}{2}\right\}\times\left\{-e^{-1}-(-1)\right\}$

$\displaystyle= \frac{1}{2}(e^2-1)\times\left(1-\frac{1}{e}\right) = \frac{1}{2}\left\{e^2-e-1+\frac{1}{e}\right\}$

< 一般領域の 2 重積分 1 > (27ページ)

問の解答

$$\iint_D (x+y)dxdy = \int_1^2 \left\{ \int_0^1 (x+y)dy \right\} dx + \int_2^3 \left\{ \int_1^2 (x+y)dy \right\} dx$$

$$= \int_1^2 \left(x + \frac{1}{2} \right) dx + \int_2^3 \left(x + \frac{3}{2} \right) dx = 2 + 4 = 6$$

(別解)

$$\iint_D (x+y)dxdy = \int_0^1 \left\{ \int_1^2 (x+y)dx \right\} dy + \int_1^2 \left\{ \int_2^3 (x+y)dx \right\} dy$$

$$= \int_0^1 \left(\frac{3}{2} + y \right) dy + \int_1^2 \left(\frac{5}{2} + y \right) dy = 2 + 4 = 6$$

< 一般領域の 2 重積分 2 > (28 ページ)

問の解答

$$\iint_D (6xy^2 - 1)dxdy = \int_0^1 \left\{ \int_0^{-x+1} (6xy^2 - 1)dy \right\} dx$$

$$= \int_0^1 \left\{ \left[2xy^3 - y \right]_{y=0}^{y=-x+1} \right\} dx = \int_0^1 \left\{ 2x(-x+1)^3 - (-x+1) \right\} dx$$

$$= \int_0^1 \left\{ -2x^4 + 6x^3 - 6x^2 + 3x - 1 \right\} dx = \left[-\frac{2}{5}x^5 + \frac{3}{2}x^4 - 2x^3 + \frac{3}{2}x^2 - x \right]_{x=0}^{x=1}$$

$$= -\frac{2}{5} + \frac{3}{2} - 2 + \frac{3}{2} - 1 = -\frac{2}{5}$$

< 一般領域の 2 重積分 3 > (29 ページ)

問の解答

$$\iint_D (xy-y)dxdy = \int_0^1 \left\{ \int_0^y (xy-y)dx \right\} dy$$

$$= \int_0^1 \left\{ \left[\frac{1}{2}x^2y - xy \right]_{x=0}^{x=y} \right\} dy = \int_0^1 \left\{ \frac{1}{2}y^3 - y^2 \right\} dy$$

$$= \left[\frac{1}{8}y^4 - \frac{1}{3}y^3 \right]_{y=0}^{y=1} = \frac{1}{8} - \frac{1}{3} = \frac{3-8}{24} = -\frac{5}{24}$$

＜ 面積比 ＞ (30ページ)

問の解答

(1) $\varDelta(u,v) = 2 \times 2 = 4$, $\varDelta(x,y) = \begin{vmatrix} 6 & 4 \\ 2 & 8 \end{vmatrix} = 48 - 8 = 40$, $\dfrac{\varDelta(x,y)}{\varDelta(u,v)} = \dfrac{40}{4} = 10$

(2) $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10$

＜ 重積分の変数変換 1 ＞ (31 ページ)

問の解答

$$\iint_D (y-x)dxdy = \iint_\Omega \{(u+4v)-(3u+2v)\}\,10dudv$$

$$= 10\int_0^2 \left\{\int_0^2 (-2u+2v)du\right\} dv = 10\int_0^2 \left\{\left[-u^2+2uv\right]_{u=0}^{u=2}\right\} dv$$

$$= 10\int_0^2 \{-4+4v\}\,dv = 10\times\left[-4v+2v^2\right]_0^2 = 10\times(-8+8) = 0$$

＜ 重積分の変数変換 2 ＞ (32ページ)

問の解答

$\Omega = \left\{(r,\ \theta) : 0 \leqq r \leqq R,\ 0 \leqq \theta \leqq \frac{\pi}{2}\right\}$ とすると

$$\iint_D e^{-x^2-y^2} dxdy = \iint_\Omega e^{-r^2} r dr d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left\{\int_0^R e^{-r^2} r dr\right\} d\theta = \int_0^{\frac{\pi}{2}} \left\{\left[-\frac{1}{2}e^{-r^2}\right]_{r=0}^{r=R}\right\} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(-\frac{1}{2}e^{-R^2} + \frac{1}{2}\right) d\theta = \frac{\pi}{4}\left\{1 - e^{-R^2}\right\}$$

< 重量と重心 1 > (33ページ)

問の解答

$$g = \frac{1}{M}\left\{m_1x_1 + m_2x_2 + \cdots + m_nx_n\right\}$$

< 重量と重心 3 > (35ページ)

問の解答

$$M = \int_0^1 f(x)dx = \int_0^1 (-x^2 + x)dx = \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_0^1 = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$$

$$g = \frac{1}{M}\int_0^1 xf(x)dx = \frac{1}{\frac{1}{6}}\int_0^1 (-x^3 + x^2)dx = 6\left[-\frac{1}{4}x^4 + \frac{1}{3}x^3\right]_0^1$$

$$= 6 \times \left(-\frac{1}{4} + \frac{1}{3}\right) = 6 \times \frac{1}{12} = \frac{1}{2}$$

< 重量と重心 4 > (36ページ)

問の解答

$$
\begin{aligned}
g_y &= \frac{1}{M}\iint_D yf(x,y)dxdy = \frac{1}{3}\iint_{D_1} ydxdy + \frac{1}{3}\iint_{D_2} ydxdy \\
&= \frac{1}{3}\int_0^1 \left\{\int_0^{2x} ydy\right\} dx + \frac{1}{3}\int_1^3 \left\{\int_0^{-x+3} ydy\right\} dx \\
&= \frac{1}{3}\int_0^1 \left\{\left[\frac{y^2}{2}\right]_{y=0}^{y=2x}\right\} dx + \frac{1}{3}\int_1^3 \left\{\left[\frac{1}{2}y^2\right]_{y=0}^{y=-x+3}\right\} dx \\
&= \frac{1}{3}\int_0^1 2x^2dx + \frac{1}{3}\int_1^3 \frac{1}{2}(-x+3)^2dx = \frac{1}{3}\left[\frac{2}{3}x^3\right]_{x=0}^{x=1} + \frac{1}{3}\left[-\frac{1}{6}(-x+3)^3\right]_{x=1}^{x=3} \\
&= \frac{1}{3}\times\frac{2}{3} + \frac{1}{3}\left(0 + \frac{1}{6}\times 2^3\right) = \frac{2}{3}
\end{aligned}
$$

< 広義積分 1 > (37ページ)

問の解答

(1) $$\int_0^\infty e^{-\lambda x}dx = \lim_{b\to\infty}\int_0^b e^{-\lambda x}dx = \lim_{b\to\infty}\left[-\frac{1}{\lambda}e^{-\lambda x}\right]_0^b = \lim_{b\to\infty}\left(-\frac{1}{\lambda}e^{-\lambda b}+\frac{1}{\lambda}e^0\right) = \frac{1}{\lambda}$$

(2) $$\int_1^\infty \frac{1}{x^3}dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^3}dx = \lim_{b\to\infty}\left[-\frac{1}{2}x^{-2}\right]_1^b = \lim_{b\to\infty}\left(-\frac{1}{2b^2}+\frac{1}{2}\right) = \frac{1}{2}$$

(3) $$\int_1^\infty \frac{1}{x^r}dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^r}dx = \lim_{b\to\infty}\left[\frac{1}{-r+1}x^{-r+1}\right]_1^b$$

$$= \lim_{b\to\infty}\left(-\frac{1}{(r-1)b^{r-1}}+\frac{1}{r-1}\right) = \frac{1}{r-1}$$

< 広義積分 2 > (38 ページ)

問の解答

$$\int_0^\infty \int_0^\infty e^{-x^2-y^2} dxdy = \lim_{R\to\infty} \iint_{D_R} e^{-x^2-y^2} dxdy = \lim_{R\to\infty} \frac{\pi}{4} \left\{ 1 - \frac{1}{e^{R^2}} \right\} = \frac{\pi}{4}$$

< 広義積分 4 > (40 ページ)

問の解答

(1) $\frac{x}{2} = t$ とおく

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{4}} dx = \int_{-\infty}^{\infty} e^{-t^2} 2dt = 2\int_{-\infty}^{\infty} e^{-t^2} dt = 2\sqrt{\pi}$$

(2) $x = \sqrt{2\lambda}t$ とおく

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\lambda}} dx = \int_{-\infty}^{\infty} e^{-t^2} \sqrt{2\lambda} dt = \sqrt{2\lambda}\int_{-\infty}^{\infty} e^{-t^2} = \sqrt{2\lambda\pi}$$