

高知工科大学

基礎数学ワークブック

(2001年度版)

秋期入学者用

V

解答

## < 1対1関係 > (1ページ)

### 問の解答

- (1) 1対1ではない
- (2) 1対1ではない
- (3) 1対1である

## < 逆関数 1 > (2 ページ)

### 問の解答

$$\begin{array}{lll} (1) & b = f(a) = 1 - 3a & (2) & b = f(a) = \frac{2}{1-a} & (3) & b = f(a) = \sqrt{a} \\ & 3a = 1 - b & & 1 - a = \frac{2}{b} & & a = b^2 = f^{-1}(b) \\ & a = \frac{1-b}{3} = f^{-1}(b) & & a = 1 - \frac{2}{b} = f^{-1}(b) & & f^{-1}(b) = b^2 \\ & f^{-1}(b) = \frac{1-b}{3} & & f^{-1}(b) = 1 - \frac{2}{b} & & \end{array}$$

## < 逆関数 2 > (3ページ)

### 問の解答

$$(1) \quad y = \frac{x}{2} - \frac{1}{4}$$

$$4y = 2x - 1$$

$$2x = 4y + 1$$

$$x = 2y + \frac{1}{2}$$

$$f^{-1}(x) = 2x + \frac{1}{2}$$

$$(2) \quad y = 1 - \frac{1}{1-x}$$

$$y - 1 = -\frac{1}{1-x}$$

$$x - 1 = \frac{1}{y - 1}$$

$$x = \frac{1}{y - 1} + 1$$

$$f^{-1}(x) = \frac{1}{x - 1} + 1$$

$$(3) \quad y = \sqrt{1-x}$$

$$y^2 = 1 - x$$

$$x = 1 - y^2$$

$$f^{-1}(x) = 1 - x^2$$

## < 逆関数 3 > (4 ページ)

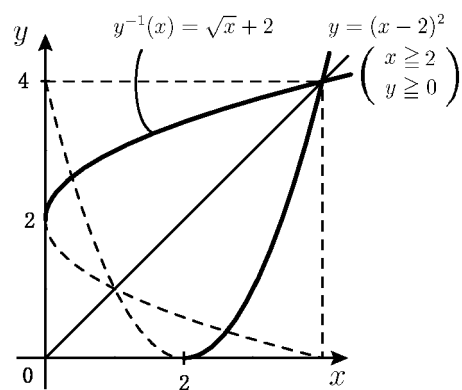
### 問の解答

$$y = (x - 2)^2 \quad (x \geq 2, y \geq 0)$$

$$\sqrt{y} = x - 2$$

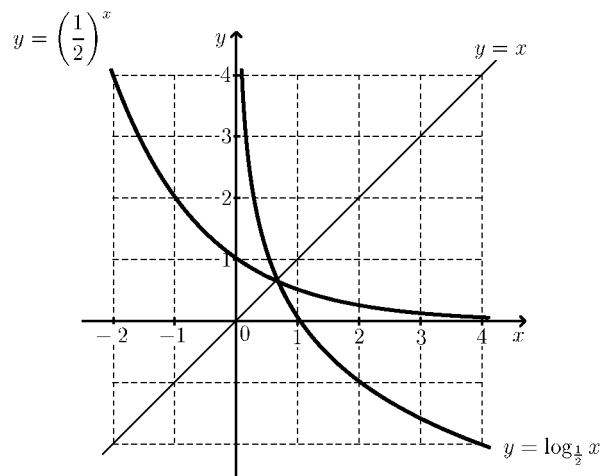
$$x = \sqrt{y} + 2$$

$$f^{-1}(x) = \sqrt{x} + 2 \quad (x \geq 0, y \geq 2)$$



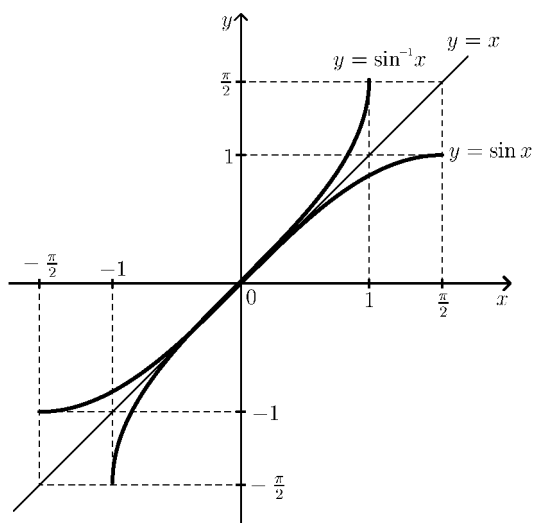
## < 逆関数 4 > (5 ページ)

### 問の解答



## < 逆三角関数 1 > (6 ページ)

### 問 1 の解答



### 問 2 の解答

$\theta$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

### 問 3 の解答

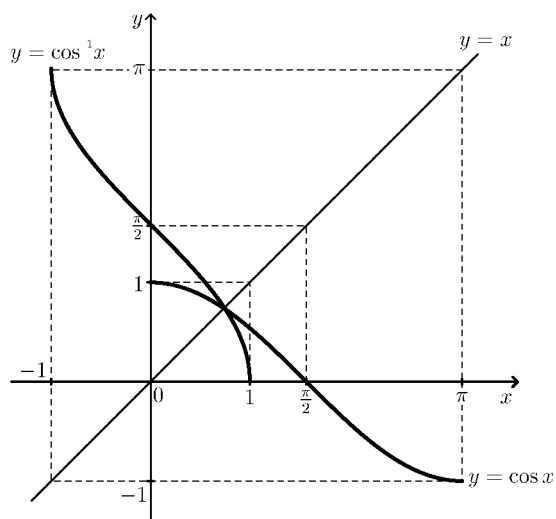
(1)  $\frac{\pi}{6}$

(2)  $-\frac{\pi}{4}$

(3)  $-\frac{\pi}{2}$

## < 逆三角関数 2 > (7ページ)

### 問1の解答



### 問2の解答

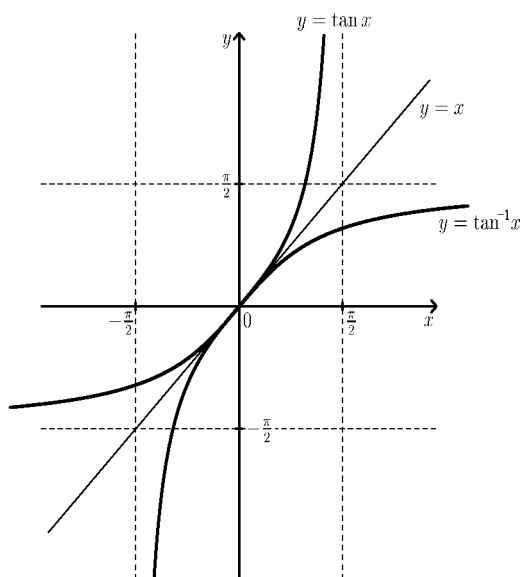
$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

### 問3の解答

- (1)  $\frac{\pi}{4}$
- (2)  $\frac{5\pi}{6}$
- (3)  $\frac{\pi}{2}$

## < 逆三角関数 3 > (8 ページ)

### 問 1 の解答



### 問 2 の解答

$\theta$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan \theta$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

### 問 3 の解答

(1)  $\frac{\pi}{6}$

(2)  $-\frac{\pi}{3}$

(3)  $-\frac{\pi}{4}$

## < 逆関数の微分 > (9 ページ)

### 問の解答

$$(1) \quad y = \cos^{-1} x \quad \Leftrightarrow \quad x = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\cos y)'} = \frac{1}{-\sin y} = -\frac{1}{\sqrt{1 - \cos^2 y}} = -\frac{1}{\sqrt{1 - x^2}}$$

$$(2) \quad y = \tan^{-1} x \quad \Leftrightarrow \quad x = \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\tan y)'} = \frac{1}{\frac{1}{\cos^2 y}} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

## < 逆三角関数の積分 > (10 ページ)

### 問1の解答

$x = au$  とおくと  $dx = adu$  より

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \int \frac{1}{\sqrt{a^2 - a^2u^2}} adu = \int \frac{1}{\sqrt{1 - u^2}} du = \sin^{-1}(u) + C = \sin^{-1}\left(\frac{x}{a}\right) + C$$

### 問2の解答

$x = au$  とおくと  $dx = adu$  より

$$\int \frac{1}{a^2 + x^2} dx = \int \frac{1}{a^2 + a^2u^2} adu = \frac{1}{a} \int \frac{1}{1 + u^2} du = \frac{1}{a} \tan^{-1}(u) + C = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

## < 不定積分の特例 1 > (11 ページ)

### 問の解答

$$(1) \int \sin^2 x dx = \int \frac{1 - \cos(2x)}{2} dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

$$(2) \int \cos(5x) \cos(2x) dx = \frac{1}{2} \int \{\cos(7x) + \cos(3x)\} dx = \frac{1}{14}\sin(7x) + \frac{1}{6}\sin(3x) + C$$

$$(3) \int \cos(-x) \sin\left(\frac{x}{2}\right) dx = \int \left\{ \sin\left(-\frac{x}{2}\right) + \sin\left(\frac{3}{2}x\right) \right\} dx$$
$$= \frac{1}{2} \int \left\{ \sin\left(\frac{3}{2}x\right) - \sin\left(\frac{x}{2}\right) \right\} dx = \cos\left(\frac{x}{2}\right) - \frac{1}{3}\cos\left(\frac{3}{2}x\right) + C$$

## < 不定積分の特例 2 > (12 ページ)

### 問の解答

$$x = au \text{ とおくと } dx = a du$$

$$\begin{aligned} \int \sqrt{a^2 - x^2} dx &= \int \sqrt{a^2 - a^2 u^2} a du = a^2 \int \sqrt{1 - u^2} du \\ &= a^2 \left( \frac{1}{2} \sin^{-1}(u) + \frac{1}{2} u \sqrt{1 - u^2} \right) + C \\ &= \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + \frac{ax}{2} \sqrt{1 - \left( \frac{x}{a} \right)^2} + C \\ &= \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + \frac{x}{2} \sqrt{a^2 - x^2} + C \end{aligned}$$

## < 円の面積 > (13ページ)

### 問の解答

$$(1) \int_0^a \sqrt{a^2 - x^2} dx = \left[ \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + \frac{x}{2} \sqrt{a^2 - x^2} \right]_0^a = \frac{a^2}{2} \sin^{-1}(1)$$

$$(2) S = 4 \int_0^a \sqrt{a^2 - x^2} dx = 4 \times \frac{a^2}{2} \times \sin^{-1}(1) = 4 \times \frac{a^2}{2} \times \frac{\pi}{2} = \pi a^2$$

## < 高階導関数 > (14 ページ)

### 問1の解答

$$(1) \quad f'(x) = 9x^2 - 4x + 1 \quad (2) \quad f'(x) = \frac{1}{2} \cos\left(\frac{x}{2}\right) \quad (3) \quad f'(x) = 2e^{2x}$$
$$f''(x) = 18x - 4 \quad f''(x) = -\frac{1}{4} \sin\left(\frac{x}{2}\right) \quad f''(x) = 4e^{2x}$$

### 問2の解答

$$(1) \quad f'(x) = 4x^3 - 10x \quad (2) \quad f'(x) = -\sin x$$
$$f''(x) = 12x^2 - 10 \quad f''(x) = -\cos x$$
$$f'''(x) = 24x \quad f'''(x) = \sin x$$
  
$$(3) \quad f'(x) = -e^{1-x} \quad (4) \quad f'(x) = 3x^2 \log x + x^2$$
$$f''(x) = e^{1-x} \quad f''(x) = 6x \log x + 3x + 2x = 6x \log x + 5x$$
$$f'''(x) = -e^{1-x} \quad f'''(x) = 6 \log x + 6 + 5 = 6 \log x + 11$$

## < グラフの凹凸 1 > (15 ページ)

### 問 1 の解答

$$(1) \quad y'' = 6x - 18 = 6(x - 3)$$

$$\left( y' = 3x^2 - 18x + 20 \right)$$

$x$	...	3	...
$y''$	-	0	+
$y$	凸	6	凹

$$(2) \quad y'' = 12x^2 - 12 = 12(x + 1)(x - 1)$$

$$\left( y' = 4x^3 - 12x \right)$$

$x$	...	-1	...	1	...
$y''$	+	0	-	0	+
$y$	凹	10	凸	10	凹

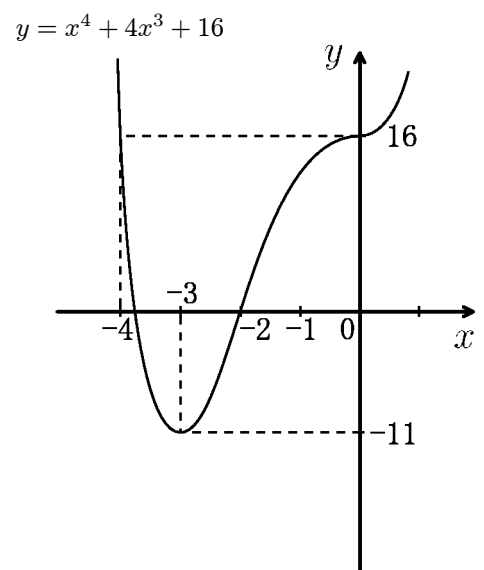
## < グラフの凹凸 2 > (16 ページ)

### 問の解答

$$y'(x) = 4x^3 + 12x^2 = 4x^2(x + 3)$$

$$y''(x) = 12x^2 + 24x = 12x(x + 2)$$

$x$	...	-3	...	-2	...	0	...
$y'$	-	0	+	+	+	0	+
$y''$	+	+	+	0	-	0	+
$y$	↘	-11	↗	0	↖	16	↗



## < 関数の一次近似 > (17 ページ)

### 問の解答

$$f(x) = \sqrt{x} \text{ とおくと}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

より、一次近似式は

$$x \doteq a \text{ のとき、} \sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a)$$

となる。ここで  $a = 1$ ,  $x = 1.1$  とおけば

$$\sqrt{1.1} \doteq \sqrt{1} + \frac{1}{2\sqrt{1}}(1.1 - 1) = 1 + \frac{1}{2} \times 0.1 = 1 + \frac{1}{20} = \frac{21}{20}$$

## < ロピタルの定理 1 > (18 ページ)

### 問の解答

$$(1) \lim_{x \rightarrow 3} \frac{4x^3}{27} = 4$$

$$(2) \lim_{x \rightarrow 5} \frac{\frac{1}{x-4}}{1} = 1$$

$$(3) \lim_{x \rightarrow 2} \frac{3x^2 - 12}{2(x - 2)} = \lim_{x \rightarrow 2} \frac{6x}{2} = 6$$

$$(4) \lim_{x \rightarrow a} \frac{4x^3 - 4a^3 - 12a^2(x - a)}{3(x - a)^2} = \lim_{x \rightarrow a} \frac{12x^2 - 12a^2}{6(x - a)} = \lim_{x \rightarrow a} \frac{24x}{6} = 4a$$

## < ロピタルの定理 2 > (19 ページ)

### 問の解答

$$(1) \lim_{x \rightarrow a} \frac{f'(x) - f'(a) - f''(a)(x-a)}{3(x-a)^2} = \lim_{x \rightarrow a} \frac{f''(x) - f''(a)}{6(x-a)}$$

$$= \lim_{x \rightarrow a} \frac{f'''(x)}{6} = \frac{1}{6} f'''(a)$$

$$(2) \lim_{x \rightarrow a} \frac{f'(x) - f'(a) - f''(a)(x-a) - \frac{1}{2} f'''(a)(x-a)^2}{4(x-a)^3}$$

$$= \lim_{x \rightarrow a} \frac{f''(x) - f''(a) - f'''(a)(x-a)}{12(x-a)^2} = \lim_{x \rightarrow a} \frac{f'''(x) - f'''(a)}{24(x-a)}$$

$$= \lim_{x \rightarrow a} \frac{f''''(x)}{24} = \frac{1}{24} f''''(a)$$

## < 関数の高次近似 > (20 ページ)

### 問の解答

$x \doteq a$  のとき

$$f(x) \doteq f(a) + f'(a)(x - a) + \frac{1}{2}f''(a)(x - a)^2 + \frac{1}{6}f'''(a)(x - a)^3 + \frac{1}{24}f^{(4)}(a)(x - a)^4$$

## < 高階微分係数 > (21 ページ)

### 問の解答

$$(1) \quad f^{(4)}(x) = 2^4 e^{2x} \quad , \quad f^{(4)}(0) = 2^4 e^0 = 2^4 = 16$$

$$(2) \quad f^{(n)}(x) = 2^n e^{2x} \quad , \quad f^{(n)}(0) = 2^n e^0 = 2^n$$

$$(3) \quad f^{(1)}(x) = -\sin x \quad , \quad f^{(1)}(0) = 0$$

$$f^{(2)}(x) = -\cos x \quad , \quad f^{(2)}(0) = -1$$

$$f^{(3)}(x) = \sin x \quad , \quad f^{(3)}(0) = 0$$

$$f^{(4)}(x) = \cos x \quad , \quad f^{(4)}(0) = 1$$

$$f^{(5)}(x) = -\sin x \quad , \quad f^{(5)}(0) = 0$$

$$f^{(6)}(x) = -\cos x \quad , \quad f^{(6)}(0) = -1$$

$$f^{(7)}(x) = \sin x \quad , \quad f^{(7)}(0) = 0$$

$$f^{(8)}(x) = \cos x \quad , \quad f^{(8)}(0) = 1$$

## < 関数の $n$ 次近似 > (22 ページ)

### 問の解答

$$f^{(n)}(x) = e^x \text{ より } f^{(n)}(a) = e^a$$

$x = a$  のとき

$$e^x = e^a + e^a(x - a) + \frac{e^a}{2!}(x - a)^2 + \cdots + \frac{e^a}{n!}(x - a)^n$$

## < テーラー展開 > (23 ページ)

### 問の解答

$$(1) e^x = e^3 + e^3(x-3) + \frac{1}{2!}e^3(x-3)^2 + \frac{1}{3!}e^3(x-3)^3 + \cdots + \frac{1}{n!}e^3(x-3)^n + \cdots$$

$$(2) e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$$

## < マクローリン展開 > (24 ページ)

### 問の解答

$$f(0) = \cos 0 = 1$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10} + \dots$$

## < 近似の練習 1 > (25 ページ)

### 問 1 の解答

$$(1) \sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a)$$

$$\sqrt{9.1} \doteq \sqrt{9} + \frac{1}{2\sqrt{9}}(9.1 - 9) = 3 + \frac{1}{6}(0.1) = \frac{181}{60}$$

$$(2) \log x \doteq \log a + \frac{1}{a}(x - a)$$

$$\log 1.1 \doteq \log 1 + \frac{1}{1}(1.1 - 1) = 0 + 0.1 = 0.1$$

### 問 2 の解答

$$(1) \sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a) - \frac{1}{8a\sqrt{a}}(x - a)^2$$

$$(2) \log x \doteq \log a + \frac{1}{a}(x - a) - \frac{1}{2a^2}(x - a)^2$$

$$(3) \sin x \doteq \sin a + (\cos a)(x - a) - \frac{1}{2}(\sin a)(x - a)^2$$

## < 近似の練習 2 > (26 ページ)

### 問 1 の解答

$$e \doteq 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} = \frac{326}{120} \doteq 2.7167$$

### 問 2 の解答

$$\sin 1 \doteq 1 - \frac{1}{6} = \frac{5}{6} \doteq 0.8333$$

## < 2変数関数 > (27ページ)

### 問の解答

$$(1) 16 + 24 + 27 = 67$$

$$(2) \frac{\frac{1}{2} + 2}{\frac{3}{2} - 1} = \frac{\frac{5}{2}}{\frac{1}{2}} = 5$$

$$(3) \sin \frac{\pi}{2} \log e = 1$$

$$(4) 2^{-3} = \frac{1}{8}$$

## < 偏導関数 1 > (28 ページ)

### 問の解答

$$\begin{aligned} (1) \quad f(x, 0) &= x + 1 & , \quad f_x(x, 0) &= 1 \\ f(x, 1) &= 2x & , \quad f_x(x, 1) &= 2 \\ f(x, 2) &= 3x - 1 & , \quad f_x(x, 2) &= 3 \\ f(x, b) &= (1 + b)x - b + 1 & , \quad f_x(x, b) &= 1 + b \end{aligned}$$

$$\begin{aligned} (2) \quad f(x, 0) &= x^3 & , \quad f_x(x, 0) &= 3x^2 \\ f(x, 1) &= x^3 - 2x^2 + x - 3 & , \quad f_x(x, 1) &= 3x^2 - 4x + 1 \\ f(x, 2) &= x^3 - 4x^2 + 8x - 48 & , \quad f_x(x, 2) &= 3x^2 - 8x + 8 \\ f(x, b) &= x^3 - 2bx^2 + b^3x - 3b^4 & , \quad f_x(x, b) &= 3x^2 - 4bx + b^3 \end{aligned}$$

## < 偏導関数 2 > (29 ページ)

### 問の解答

$$(1) f(x, b) = x^2 - 2x + 3bx + 4b^2 - 5b - 6$$

$$f_x(x, b) = 2x - 2 + 3b$$

$$f_x(x, y) = 2x - 2 + 3y$$

$$(2) f(x, b) = 7x^5 - 6bx^4 + 5b^3x^3 + 4b^5x - 3b^7$$

$$f_x(x, b) = 35x^4 - 24bx^3 + 15b^3x^2 + 4b^5$$

$$f_x(x, y) = 35x^4 - 24x^3y + 15x^2y^3 + 4y^5$$

## < 偏導関数 3 > (30 ページ)

### 問の解答

- (1)  $f(0, y) = -y + 1$  ,  $f_y(0, y) = -1$   
 $f(1, y) = 2$  ,  $f_y(1, y) = 0$   
 $f(2, y) = y + 3$  ,  $f_y(2, y) = 1$   
 $f(a, y) = (a - 1)y + a + 1$  ,  $f_y(a, y) = a - 1$
- (2)  $f(0, y) = -3y^4$  ,  $f_y(0, y) = -12y^3$   
 $f(1, y) = -3y^4 + y^3 - 2y + 1$  ,  $f_y(1, y) = -12y^3 + 3y^2 - 2$   
 $f(2, y) = -3y^4 + 2y^3 - 8y + 8$  ,  $f_y(2, y) = -12y^3 + 6y^2 - 8$   
 $f(a, y) = -3y^4 + ay^3 - 2a^2y + a^3$  ,  $f_y(a, y) = -12y^3 + 3ay^2 - 2a^2$

## < 偏導関数 4 > (31 ページ)

### 問の解答

$$(1) f(a, y) = a^2 - 2a + 3ay + 4y^2 - 5y - 6$$

$$f_y(a, y) = 3a + 8y - 5$$

$$f_y(x, y) = 3x + 8y - 5$$

$$(2) f(a, y) = 7a^5 - 6a^4y + 5a^3y^3 + 4ay^5 - 3y^7$$

$$f_y(a, y) = -6a^4 + 15a^3y^2 + 20ay^4 - 21y^6$$

$$f_y(x, y) = -6x^4 + 15x^3y^2 + 20xy^4 - 21y^6$$

## < 偏微分 1 > (32 ページ)

### 問の解答

(1)  $6x - 1 + 2y$

(2)  $5x^4 + 20x^3y - 3x^2y^2 - 4xy^3 + 6y^4$

(3)  $2e^{2x} + \sin x \sin(2y) + 3 \log y - \frac{2x}{y}$

## < 偏微分 2 > (33 ページ)

### 問の解答

(1)  $2x + 10y - 6$

(2)  $5x^4 - 2x^3y - 6x^2y^2 + 24xy^3 + 42y^5$

(3)  $-2 \cos x \cos (2y) + \frac{3x}{y} + \frac{x^2}{y^2}$

## < 偏微分 3 > (34 ページ)

### 問の解答

$$(1) \quad \frac{\partial}{\partial x}(x^3 - x^2y^2 + 3xy^5) \\ = 3x^2 - 2xy^2 + 3y^5$$

$$, \quad \frac{\partial}{\partial y}(x^3 - x^2y^2 + 3xy^5) \\ = -2x^2y + 15xy^4$$

$$(2) \quad \frac{\partial}{\partial x} \left( \frac{x-1}{2y} \right) = \frac{1}{2y}$$

$$, \quad \frac{\partial}{\partial y} \left( \frac{x-1}{2y} \right) = -\frac{x-1}{2y^2}$$

$$(3) \quad \frac{\partial}{\partial x} \left( \left( \frac{1}{y} \right)^{2x} \right) = 2 \left( \frac{1}{y} \right)^{2x} \log \left( \frac{1}{y} \right) \\ \left( = -2y^{-2x} \log y \right)$$

$$, \quad \frac{\partial}{\partial y} \left( \left( \frac{1}{y} \right)^{2x} \right) = 2x \left( \frac{1}{y} \right)^{2x-1} \times \left( -\frac{1}{y^2} \right) \\ \left( = -2xy^{-2x-1} \right)$$

## < 偏微分 4 > (35 ページ)

### 問の解答

$$(1) \quad \frac{\partial z}{\partial x} = 10(2x + y^2)^4, \quad \frac{\partial z}{\partial y} = 10y(2x + y^2)^4$$

$$(2) \quad \frac{\partial z}{\partial x} = -\frac{1}{\sqrt{1 - 2x + 3y^2}}, \quad \frac{\partial z}{\partial y} = \frac{3y}{\sqrt{1 - 2x + 3y^2}}$$

$$(3) \quad \frac{\partial z}{\partial x} = 3e^{3x-y^2}, \quad \frac{\partial z}{\partial y} = -2ye^{3x-y^2}$$

$$(4) \quad \frac{\partial z}{\partial x} = -\frac{\cos x \cos(2y)}{1 - \sin x \cos(2y)}, \quad \frac{\partial z}{\partial y} = \frac{2 \sin x \sin(2y)}{1 - \sin x \cos(2y)}$$

## < 偏微分 5 > (36 ページ)

### 問の解答

$$(1) f_x(x, y) = 5x^4 - 4x^3y + 4xy^3, \quad f_y(x, y) = -x^4 + 6x^2y^2 - 28y^3$$

$$(2) f_x(x, y) = -5 \sin(5x - y^2), \quad f_y(x, y) = 2y \sin(5x - y^2)$$

$$(3) \frac{\partial z}{\partial x} = -\frac{y}{(xy - 2y^3)^2}, \quad \frac{\partial z}{\partial y} = -\frac{x - 6y^2}{(xy - 2y^3)^2}$$

$$(4) z_x = -\frac{y}{(2xy - y^2)\sqrt{2xy - y^2}}, \quad z_y = -\frac{x - y}{(2xy - y^2)\sqrt{2xy - y^2}}$$

$$(5) z_x = -y^2 e^{3y - xy^2 + y^3}, \quad z_y = (3 - 2xy + 3y^2) e^{3y - xy^2 + y^3}$$

## < 2階偏導関数 1 > (37 ページ)

### 問の解答

$$(1) \quad f_{xx}(x, y) = 20x^3 - 24x^2y + 6y^2 \quad , \quad f_{yy}(x, y) = 6x^2 + 48y^2$$

$$(2) \quad \frac{\partial^2 z}{\partial y^2} = -4 \cos(2x) \sin(y^2) \quad , \quad \frac{\partial^2 z}{\partial y^2} = 2 \cos(2x) \cos(y^2) - 4y^2 \cos(2x) \sin(y^2)$$

## < 2階偏導関数 2 > (38 ページ)

### 問の解答

$$(1) \quad f_{xy}(x, y) = -8x^3 + 12xy \qquad , \quad f_{yx}(x, y) = -8x^3 + 12xy$$

$$(2) \quad \frac{\partial^2 z}{\partial y \partial x} = -4y \sin(2x) \cos(y^2) \qquad , \quad \frac{\partial^2 z}{\partial x \partial y} = -4y \sin(2x) \cos(y^2)$$

## < 偏微分係数 > (39 ページ)

### 問の解答

$$(1) \quad f_x(x, y) = 3x^2 - 4xy + 5y^2 \qquad , \quad f_y(x, y) = -2x^2 + 10xy - 12y^3$$

$$f_x(2, 1) = 12 - 8 + 5 = 9 \qquad , \quad f_y(2, 1) = -8 + 20 - 12 = 0$$

$$(2) \quad f_x(x, y) = -\sin x \sin(2y) \qquad , \quad f_y(x, y) = 2 \cos x \cos(2y)$$

$$f_x\left(\frac{\pi}{3}, \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2} \qquad , \quad f_y\left(\frac{\pi}{3}, \frac{\pi}{4}\right) = 0$$

$$(3) \quad f_x(x, y) = 2 \log(y^3) \qquad , \quad f_y(x, y) = \frac{2x \times 3y^2}{y^3} = \frac{6x}{y}$$

$$f_x(1, 1) = 0 \qquad , \quad f_y(1, 1) = 6$$

## < 2変数関数のグラフ > (40 ページ)

### 問1の解答

(1)  $x$  軸方向の傾き = 2

$y$  軸方向の傾き = -1

$z$  切片 = 3

(2)  $x$  軸方向の傾き =  $m$

$y$  軸方向の傾き =  $n$

$z$  切片 =  $k$

### 問2の解答

(1)  $y = 1$

$z = 3$

(2)  $y = 2$

$z = x$

(3)  $x = 2$

$z = 2 + 2y - y^2$