高知工科大学

基礎数学ワークブック

(2001年度版)

Series A

No. 12

解答

< ベクトル三重積 1 > (1ページ)

問 1 の解答

$$(*)\text{の左辺} = \left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \overrightarrow{c} = \begin{vmatrix} \overrightarrow{\boldsymbol{i}} & \overrightarrow{\boldsymbol{j}} & \overrightarrow{\boldsymbol{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \times \overrightarrow{c}$$

$$= \left\{(a_2b_3 - a_3b_2)\overrightarrow{\boldsymbol{i}} + (a_3b_1 - a_1b_3)\overrightarrow{\boldsymbol{j}} + (a_1b_2 - a_2b_1)\overrightarrow{\boldsymbol{k}}\right\} \times \left\{c_1\overrightarrow{\boldsymbol{i}} + c_2\overrightarrow{\boldsymbol{j}} + c_3\overrightarrow{\boldsymbol{k}}\right\}$$

$$= \begin{vmatrix} \overrightarrow{\boldsymbol{i}} & \overrightarrow{\boldsymbol{j}} & \overrightarrow{\boldsymbol{k}} \\ a_2b_3 - a_3b_2 & a_3b_1 - a_1b_3 & a_1b_2 - a_2b_1 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (a_3b_1c_3 - a_1b_3c_3 - a_1b_2c_2 + a_2b_1c_2)\overrightarrow{\boldsymbol{i}}$$

$$+ (a_1b_2c_1 - a_2b_1c_1 - a_2b_3c_3 + a_3b_2c_3)\overrightarrow{\boldsymbol{j}}$$

$$+ (a_2b_3c_2 - a_3b_2c_2 - a_3b_1c_1 + a_1b_3c_1)\overrightarrow{\boldsymbol{k}}$$

問 2 の解答

$$(*)\text{の右辺} = \left(\overrightarrow{c} \cdot \overrightarrow{a}\right)\overrightarrow{b} - \left(\overrightarrow{b} \cdot \overrightarrow{c}\right)\overrightarrow{a}$$

$$= (c_1a_1 + c_2a_2 + c_3a_3)(b_1\overrightarrow{\boldsymbol{i}} + b_2\overrightarrow{\boldsymbol{j}} + b_3\overrightarrow{\boldsymbol{k}}) - (b_1c_1 + b_2c_2 + b_3c_3)(a_1\overrightarrow{\boldsymbol{i}} + a_2\overrightarrow{\boldsymbol{j}} + a_3\overrightarrow{\boldsymbol{k}})$$

$$= \{a_1b_1c_1 + a_2b_1c_2 + a_3b_1c_3 - a_1b_1c_1 - a_1b_2c_2 - a_1b_3c_3\}\overrightarrow{\boldsymbol{i}}$$

$$+ \{a_1b_2c_1 + a_2b_2c_2 + a_3b_2c_3 - a_2b_1c_1 - a_2b_2c_2 - a_2b_3c_3\}\overrightarrow{\boldsymbol{j}}$$

$$+ \{a_1b_3c_1 + a_2b_3c_2 + a_3b_3c_3 - a_3b_1c_1 - a_3b_2c_2 - a_3b_3c_3\}\overrightarrow{\boldsymbol{k}}$$

$$= \{a_2b_1c_2 + a_3b_1c_3 - a_1b_2c_2 - a_1b_3c_3\}\overrightarrow{\boldsymbol{i}}$$

$$+ \{a_1b_2c_1 + a_3b_2c_3 - a_2b_1c_1 - a_2b_3c_3\}\overrightarrow{\boldsymbol{j}}$$

$$+ \{a_1b_3c_1 + a_2b_3c_2 - a_3b_1c_1 - a_3b_2c_2\}\overrightarrow{\boldsymbol{k}}$$

< ベクトル三重積 2 > (2ページ)

問 1 の解答

$$\text{左辺} = \overrightarrow{a} \times \left(\overrightarrow{b} \times \overrightarrow{c}\right) = -\left(\overrightarrow{b} \times \overrightarrow{c}\right) \times \overrightarrow{a}$$

$$= -\left\{\left(\overrightarrow{a} \cdot \overrightarrow{b}\right)\overrightarrow{c} - \left(\overrightarrow{c} \cdot \overrightarrow{a}\right)\overrightarrow{b}\right\} = \left(\overrightarrow{c} \cdot \overrightarrow{a}\right)\overrightarrow{b} - \left(\overrightarrow{a} \cdot \overrightarrow{b}\right)\overrightarrow{c}$$

問 2 の解答

(1) $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \overrightarrow{c} = \left(\overrightarrow{c} \cdot \overrightarrow{a}\right)\overrightarrow{b} - \left(\overrightarrow{b} \cdot \overrightarrow{c}\right)\overrightarrow{a}$

$$= (-1 + 2 + 15)\,\overrightarrow{b} - (-4 - 1 + 5)\,\overrightarrow{a}$$

$$= 16\left(4\overrightarrow{\boldsymbol{i}} - \overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{k}}\right) = 64\overrightarrow{\boldsymbol{i}} - 16\overrightarrow{\boldsymbol{j}} + 16\overrightarrow{\boldsymbol{k}}$$

(2) $\left(\overrightarrow{b} \times \overrightarrow{c}\right) \times \overrightarrow{a} = \left(\overrightarrow{a} \cdot \overrightarrow{b}\right)\overrightarrow{c} - \left(\overrightarrow{c} \cdot \overrightarrow{a}\right)\overrightarrow{b}$

$$= 5\overrightarrow{c} - 16\overrightarrow{b}$$

$$= -69\overrightarrow{\boldsymbol{i}} + 21\overrightarrow{\boldsymbol{j}} + 9\overrightarrow{\boldsymbol{k}}$$

(3) $\overrightarrow{a} \times \left(\overrightarrow{b} \times \overrightarrow{c}\right) = \left(\overrightarrow{c} \cdot \overrightarrow{a}\right)\overrightarrow{b} - \left(\overrightarrow{a} \cdot \overrightarrow{b}\right)\overrightarrow{c}$

$$= 16\overrightarrow{b} - 5\overrightarrow{c}$$

$$= 69\overrightarrow{\boldsymbol{i}} - 21\overrightarrow{\boldsymbol{j}} - 9\overrightarrow{\boldsymbol{k}}$$

(4) $\overrightarrow{c} \times \left(\overrightarrow{a} \times \overrightarrow{b}\right) = -\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \overrightarrow{c}$

$$= -64\overrightarrow{\boldsymbol{i}} + 16\overrightarrow{\boldsymbol{j}} - 16\overrightarrow{\boldsymbol{k}}$$

< 直交系 > (3ページ)

問の解答

(1) $\overrightarrow{b} = \overrightarrow{r} - \left(\overrightarrow{a} \cdot \overrightarrow{r}\right)\overrightarrow{a}$

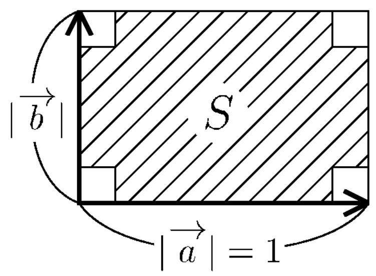
(2) $\left(\overrightarrow{a} \times \overrightarrow{r}\right) \times \overrightarrow{a} = \left(\overrightarrow{a} \cdot \overrightarrow{a}\right)\overrightarrow{r} - \left(\overrightarrow{r} \cdot \overrightarrow{a}\right)\overrightarrow{a} = \overrightarrow{r} - \left(\overrightarrow{r} \cdot \overrightarrow{a}\right)\overrightarrow{a}$

(3) $\overrightarrow{c} = \overrightarrow{a} \times \overrightarrow{b}$ より $|\overrightarrow{c}|$ は $\overrightarrow{a}$ と $\overrightarrow{b}$ の作る平行四辺形の面積 S に等しい。

一方、右図より $S = |\overrightarrow{a}| \times |\overrightarrow{b}| = |\overrightarrow{b}|$ より

$|\overrightarrow{c}| = S = |\overrightarrow{b}|$

がわかる。



< 空間の回転 1 > (4ページ)

問 1 の解答

$$\overrightarrow{\mathrm{OH}} = \frac{|\overrightarrow{\mathrm{OH}}|}{|\overrightarrow{a}|}\overrightarrow{a} = \left(\frac{\overrightarrow{a}\cdot\overrightarrow{r}}{|\overrightarrow{a}|^2}\right)\overrightarrow{a} = \left(\overrightarrow{a}\cdot\overrightarrow{r}\right)\overrightarrow{a}$$

問 2 の解答

$$\overrightarrow{b} = \overrightarrow{\mathrm{HR}} = \overrightarrow{\mathrm{OR}} - \overrightarrow{\mathrm{OH}} = \overrightarrow{r} - \left(\overrightarrow{a}\cdot\overrightarrow{r}\right)\overrightarrow{a}$$

問 3 の解答

$$\overrightarrow{c} = \overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{a} \times \overrightarrow{r}$$

問 4 の解答

$$|\overrightarrow{c}| = |\overrightarrow{b}|$$

問 5 の解答

(1) $\overrightarrow{\mathrm{HP}} = \dfrac{\mathrm{HP}}{|\overrightarrow{b}|}\overrightarrow{b} = \dfrac{|\overrightarrow{b}|\cos\psi}{|\overrightarrow{b}|}\overrightarrow{b} = (\cos\psi)\,\overrightarrow{b}$

(2) $\overrightarrow{\mathrm{HQ}} = \dfrac{\mathrm{HQ}}{|\overrightarrow{c}|}\overrightarrow{c} = \dfrac{|\overrightarrow{c}|\sin\psi}{|\overrightarrow{c}|}\overrightarrow{c} = (\sin\psi)\,\overrightarrow{c}$

(3) $\overrightarrow{\mathrm{HR'}} = \overrightarrow{\mathrm{HP}} + \overrightarrow{\mathrm{PR'}} = \overrightarrow{\mathrm{HP}} + \overrightarrow{\mathrm{HQ}} = (\cos\psi)\,\overrightarrow{b} + (\sin\psi)\,\overrightarrow{c}$

< 空間の回転 2 > (5ページ)

問 1 の解答

$$\overrightarrow{\mathrm{OR}'} = (1-\cos\psi)(a_1x+a_2y+a_3z)\left(a_1\overrightarrow{\boldsymbol{i}}+a_2\overrightarrow{\boldsymbol{j}}+a_3\overrightarrow{\boldsymbol{k}}\right)$$
$$+(\cos\psi)\left(x\overrightarrow{\boldsymbol{i}}+y\overrightarrow{\boldsymbol{j}}+z\overrightarrow{\boldsymbol{k}}\right)$$
$$+(\sin\psi)\left\{(a_2z-a_3y)\overrightarrow{\boldsymbol{i}}+(a_3x-a_1z)\overrightarrow{\boldsymbol{j}}+(a_1y-a_2x)\overrightarrow{\boldsymbol{k}}\right\}$$
$$=\{(1-\cos\psi)(a_1x+a_2y+a_3z)a_1+x(\cos\psi)+(\sin\psi)(a_2z-a_3y)\}\overrightarrow{\boldsymbol{i}}$$
$$+\{(1-\cos\psi)(a_1x+a_2y+a_3z)a_2+y(\cos\psi)+(\sin\psi)(a_3x-a_1z)\}\overrightarrow{\boldsymbol{j}}$$
$$+\{(1-\cos\psi)(a_1x+a_2y+a_3z)a_3+z(\cos\psi)+(\sin\psi)(a_1y-a_2x)\}\overrightarrow{\boldsymbol{k}}$$

問 2 の解答

$$x' = \left\{a_1^2(1-\cos\psi)+\cos\psi\right\}x+\{a_1a_2(1-\cos\psi)-a_3\sin\psi\}y+\{a_1a_3(1-\cos\psi)+a_2\sin\psi\}z$$

$$y' = \{a_1a_2(1-\cos\psi)+a_3\sin\psi\}x+\left\{a_2^2(1-\cos\psi)+\cos\psi\right\}y+\{a_2a_3(1-\cos\psi)-a_1\sin\psi\}z$$

$$z' = \{a_1a_3(1-\cos\psi)-a_2\sin\psi\}x+\{a_2a_3(1-\cos\psi)+a_1\sin\psi\}y+\left\{a_3^2(1-\cos\psi)+\cos\psi\right\}z$$

問 3 の解答

(1) $x' = (\cos\psi)x-(\sin\psi)y$

$y' = (\sin\psi)x+(\cos\psi)y$

$z' = \{(1-\cos\psi)+\cos\psi\}z = z$

(2) $x' = \left\{\frac{1}{3}\times\frac{3}{2}-\frac{1}{2}\right\}x+\left\{\frac{1}{3}\times\frac{3}{2}-\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right\}+\left\{\frac{1}{3}\times\frac{3}{2}+\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right\}z = z$

$y' = \left\{\frac{1}{3}\times\frac{3}{2}+\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right\}x+\left\{\frac{1}{3}\times\frac{3}{2}-\frac{1}{2}\right\}y+\left\{\frac{1}{3}\times\frac{3}{2}-\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right\}z = x$

$z' = \left\{\frac{1}{3}\times\frac{3}{2}-\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right\}x+\left\{\frac{1}{3}\times\frac{3}{2}+\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right\}y+\left\{\frac{1}{3}\times\frac{3}{2}-\frac{1}{2}\right\}z = y$

< 微分の復習 > (6ページ)

問の解答

(1) $\dfrac{d}{dx}\left(3x^2 - x + 6\right) = 6x - 1$

(2) $\dfrac{d}{dy}\left(\dfrac{1}{y^2 + 2y - 1}\right) = -\dfrac{2y + 2}{(y^2 + 2y - 1)^2}$

(3) $\dfrac{d}{dx}\left((x^2 - 1)^6\right) = 12x(x^2 - 1)^5$

(4) $\dfrac{d}{dt}\left(e^{-t}\right) = -e^{-t}$

(5) $\dfrac{d}{dx}\left(\sin(1 - x^2)\right) = -2x\cos(1 - x^2)$

(6) $\dfrac{d}{dy}(\log(2y - 1)) = \dfrac{2}{2y - 1}$

$<$ 2変数関数 $>$ (7ページ)

問の解答

(1) $f(1,2)=1-1\times 2+3\times 4=11$

(2) $f(3,11)=\dfrac{9-2}{22-1}=\dfrac{1}{3}$

(3) $f(\pi,e)=\cos\pi\log e=-1$

(4) $f(5,-1)=5^{-1}=\dfrac{1}{5}$

< 偏導関数 1 > (8ページ)

問の解答

(1) $f(x,0) = x^2 - 3x + 1$, $f_x(x,0) = 2x - 3$

$f(x,1) = x^2 + x + 2$, $f_x(x,1) = 2x + 1$

$f(x,2) = x^2 + 5x + 7$, $f_x(x,2) = 2x + 5$

$f(x,b) = x^2 - 3x + 4bx + 2b^2 - b + 1$, $f_x(x,b) = 2x - 3 + 4b$

(2) $f(x,0) = x^3$, $f_x(x,0) = 3x^2$

$f(x,1) = x^3 - 2x^2 + 3x - 1$, $f_x(x,1) = 3x^2 - 4x + 3$

$f(x,2) = x^3 - 16x^2 + 48x - 32$, $f_x(x,2) = 3x^2 - 32x + 48$

$f(x,b) = x^3 - 2b^3x^2 + 3b^4x - b^5$, $f_x(x,b) = 3x^2 - 4b^3x + 3b^4$

< 偏導関数 2 > (9ページ)

問の解答

(1) $f(x,b) = 1 - 3x + 5bx - b^3$

$f_x(x,b) = -3 + 5b$

$f_x(x,y) = -3 + 5y$

(2) $f(x,b) = 3x^4 - 2bx^2 + b^3x - 7b^5$

$f_x(x,b) = 12x^3 - 4bx + b^3$

$f_x(x,y) = 12x^3 - 4xy + y^3$

< 偏導関数 3 > (10 ページ)

問の解答

(1) $f(0,y) = 2y^2 - y + 1$, $f_y(0,y) = 4y - 1$

$f(1,y) = 2y^2 + 3y - 1$, $f_y(1,y) = 4y + 3$

$f(2,y) = 2y^2 + 7y - 1$, $f_y(2,y) = 4y + 7$

$f(a,y) = 2y^2 + (4a-1)y + a^2 - 3a + 1$, $f_y(a,y) = 4y + 4a - 1$

(2) $f(0,y) = -y^5$, $f_y(0,y) = -5y^4$

$f(1,y) = -y^5 + 3y^4 - 2y^3 + 1$, $f_y(1,y) = -5y^4 + 12y^3 - 6y^2$

$f(2,y) = -y^5 + 6y^4 - 8y^3 + 8$, $f_y(2,y) = -5y^4 + 24y^3 - 24y^2$

$f(a,y) = -y^5 + 3ay^4 - 2a^2y^3 + a^3$, $f_y(a,y) = -5y^4 + 12ay^3 - 6a^2y^2$

< 偏導関数 4 > (11ページ)

問の解答

(1) $f(a,y) = 1 - 3a + 5ay - y^3$

$f_y(a,y) = 5a - 3y^2$

$f_y(x,y) = 5x - 3y^2$

(2) $f(a,y) = 3a^4 - 2a^2y + ay^3 - 7y^5$

$f_y(a,y) = -2a^2 + 3ay^2 - 35y^4$

$f_y(x,y) = -2x^2 + 3xy^2 - 35y^4$

< 偏微分 1 > (12ページ)

問の解答

(1)　$f_x(x, y) = 2x - 2y + 1$

(2)　$f_x(x, y) = 5x^4 - 8x^3y + 9x^2y^2 - 2xy^3 - 2y^4$

(3)　$f_x(x, y) = -e^{-x} - \sin x \sin(2y) - \log\left(\frac{1}{y}\right) + \frac{y}{2x^2}$

< 偏微分 2 > (13ページ)

問の解答

(1) $f_y(x, y) = -2x + 6y - 2$

(2) $f_y(x, y) = -2x^4 + 6x^3y - 3x^2y^2 - 8xy^3 + 15y^4$

(3) $f_y(x, y) = 2\cos x \cos(2y) + \dfrac{x}{y} - \dfrac{1}{2x}$

< 偏微分 3 > (14ページ)

問の解答

(1) $\dfrac{\partial}{\partial x}\left(x^2-3xy^2+2y^3\right)=2x-3y^2$, $\dfrac{\partial}{\partial y}\left(x^2-3xy^2+2y^3\right)=-6xy+6y^2$

(2) $\dfrac{\partial}{\partial x}\left(\dfrac{2x}{y^2}\right)=\dfrac{2}{y^2}$, $\dfrac{\partial}{\partial y}\left(\dfrac{2x}{y^2}\right)=-\dfrac{4x}{y^3}$

(3) $\dfrac{\partial}{\partial x}\left(x^{-y}\right)=-yx^{-y-1}$, $\dfrac{\partial}{\partial y}\left(x^{-y}\right)=-x^{-y}\log x$

< 偏微分 4 > (15ページ)

問の解答

(1) $\dfrac{\partial z}{\partial x} = 5(x-y^2)^4$, $\dfrac{\partial z}{\partial y} = -10y(x-y^2)^4$

(2) $\dfrac{\partial z}{\partial x} = \dfrac{-1}{\sqrt{1-2x+y^2}}$, $\dfrac{\partial z}{\partial y} = \dfrac{y}{\sqrt{1-2x+y^2}}$

(3) $\dfrac{\partial z}{\partial x} = -e^{1-x+3y^2}$, $\dfrac{\partial z}{\partial y} = 6ye^{1-x+3y^2}$

(3) $\dfrac{\partial z}{\partial x} = (2x-\log y)\cos\left(x^2 - x\log y\right)$, $\dfrac{\partial z}{\partial y} = -\dfrac{x}{y}\cos\left(x^2 - x\log y\right)$

< 偏微分 5 > (16 ページ)

問の解答

(1) $f_x(x,y) = 2x - y^2$, $f_y(x,y) = -2xy + 12y^3$

(2) $f_x(x,y) = y^2 \sin(1 - xy^2)$, $f_y(x,y) = 2xy \sin(1 - xy^2)$

(3) $\dfrac{\partial z}{\partial x} = -\dfrac{1}{x^2(2-y)}$, $\dfrac{\partial z}{\partial y} = \dfrac{x}{(2x - xy)^2} = \dfrac{1}{x(2-y)^2}$

(4) $z_x = -\dfrac{x}{(x^2 - y^4)\sqrt{x^2 - y^4}}$, $z_y = \dfrac{2y^3}{(x^2 - y^4)\sqrt{x^2 - y^4}}$

(5) $z_x = 2e^{2x - y^3}$, $z_y = -3y^2 e^{2x - y^3}$

＜ 2 階偏導関数 1 ＞ (17ページ)

問の解答

(1) $f_{xx}(x,y) = 12x - 2y$, $f_{yy}(x,y) = 6x - 6y$

$\left(f_x = 6x^2 - 2xy + 3y^2\right)$ 　　$\left(f_y = -x^2 + 6xy - 3y^2\right)$

(2) $\dfrac{\partial^2 z}{\partial x^2} = -\dfrac{2y^2}{x^3}\sin\left(\dfrac{y^2}{x}\right) - \dfrac{y^4}{x^4}\cos\left(\dfrac{y^2}{x}\right)$, $\dfrac{\partial^2 z}{\partial y^2} = -\dfrac{2}{x}\sin\left(\dfrac{y^2}{x}\right) - \dfrac{4y^2}{x^2}\cos\left(\dfrac{y^2}{x}\right)$

$\left(z_x = \dfrac{y^2}{x^2}\sin\left(\dfrac{y^2}{x}\right)\right)$ 　　$\left(z_y = -\dfrac{2y}{x}\sin\left(\dfrac{y^2}{x}\right)\right)$

< 2階偏導関数 2 > (18ページ)

問の解答

(1) $f_{xy}(x,y) = 2x - 9y^2$, $f_{yx}(x,y) = 2x - 9y^2$

$\left(f_x = 4x^3 + 2xy - 3y^3\right)$　　$\left(f_y = x^2 - 9xy^2 + 10y^4\right)$

(2) $\dfrac{\partial^2 z}{\partial y \partial x} = -8y\sin(4x)\cos(y^2)$, $\dfrac{\partial^2 z}{\partial x \partial y} = -8y\sin(4x)\cos(y^2)$

$\left(z_x = -4\sin(4x)\sin(y^2)\right)$　　$\left(z_y = 2y\cos(4x)\cos(y^2)\right)$

＜偏微分係数＞(19ページ)

問の解答

(1) $f_x(2,1)=19$, $f_y(2,1)=2$

$\left(f_x(x,y)=3x^2+4xy-y^3\right)$ $\left(f_y(x,y)=2x^2-3xy^2\right)$

(2) $f_x\left(\frac{\pi}{2},\frac{\pi}{3}\right)=0$, $f_y\left(\frac{\pi}{2},\frac{\pi}{3}\right)=2$

$\left(f_x(x,y)=-2\sin(2x)\sin(4y)\right)$ $\left(f_y(x,y)=4\cos(2x)\cos(4y)\right)$

(3) $f_x(1,e)=-\frac{2}{e}$, $f_y(1,e)=\frac{2}{e^2}$

$\left(f_x(x,y)=-e^{-x}\log(y^2)\right)$ $\left(f_y(x,y)=\frac{2}{ye^x}\right)$

＜ 2変数関数のグラフ ＞ (20ページ)

問 1 の解答

(1) x 軸方向の傾き $= 1$

y 軸方向の傾き $= -1$

z 切片 $= -2$

(2) x 軸方向の傾き $= m$

y 軸方向の傾き $= n$

z 切片 $= k$

問 2 の解答

(1) ℓ_1 の方程式

$y = 1$

$z = 3$

(2) ℓ_2 の方程式

$y = 2$

$z = x$

(3) L_2 の方程式

$x = 2$

$z = -y^2 + 2y + 2$

< 偏微分係数の幾何学的意味 > (21 ページ)

問の解答

$f_x(3,2)=4$

接線 L の方程式

$y=2$

$z=4x-6$

$$\left(\begin{array}{l} f_x(x,y)=2x-2 \\ z=4(x-3)+6=4x-12+6 \\ f(a,b)=f(3,2)=6 \end{array}\right)$$

$f_y(3,2)=-3$

接線 ℓ の方程式

$x=3$

$z=-3y+12$

$$\left(\begin{array}{l} f_y(x,y)=-2y+1 \\ z=-3y+12 \end{array}\right)$$

< 接平面 > (22ページ)

問の解答

$f_x(x,y)=6x-y$, $f_y(x,y)=-x+6y^2$

$f_x(1,-1)=7$, $f_y(1,-1)=5$

$f(1,-1)=2$

(答) $z=7x+5y$

＜2変数関数の一次近似＞(23ページ)

問の解答

(1) $f_x = 5x^4y^{-2}$, $f_y = -2x^5y^{-3}$

$$(a+\Delta x)^5(b+\Delta y)^{-2} \fallingdotseq 5a^4b^{-2}\Delta x - 2a^5b^{-3}\Delta y + a^5b^{-2} = \frac{5a^4}{b^2}\Delta x - \frac{2a^5}{b^3}\Delta y + \frac{a^5}{b^2}$$

(2) $f_x = (\cos x)y\sqrt{y}$, $f_y = \frac{3}{2}(\sin x)\sqrt{y}$

$$\sin(a+\Delta x)(b+\Delta y)\sqrt{b+\Delta y} \fallingdotseq \left\{(\cos a)b\sqrt{b}\right\}\Delta x + \left\{\frac{3}{2}(\sin a)\sqrt{b}\right\}\Delta y + (\sin a)b\sqrt{b}$$

(3) $f_x = \frac{1}{y^2}$, $f_y = -\frac{2x}{y^3}$

$$\frac{a+\Delta x}{(b+\Delta y)^2} \fallingdotseq \left(\frac{1}{b^2}\right)\Delta x - \left(\frac{2a}{b^3}\right)\Delta y + \frac{a}{b^2}$$

< 2変数合成関数の微分 1 > (24ページ)

問の解答

$$\frac{d}{dt}f(x(t),y(t)) = f_x(x,y)\frac{dx}{dt} + f_y(x,y)\frac{dy}{dt}$$

< 2変数合成関数の微分 2 > (25ページ)

問の解答

(1) $5f_x(3+5t, 1-t) - f_y(3+5t, 1-t)$

(2) $(\cos\theta)f_x(r\cos\theta, r\sin\theta) + (\sin\theta)f_y(r\cos\theta, r\sin\theta)$

< 全微分 > (26 ページ)

問 1 の解答

(1) $dz = 6x^2dx - 5y^4dy$

(2) $dz = -\dfrac{\sin y}{x^2}dx + \dfrac{\cos y}{x}dy$

問 2 の解答

(1) $dx = du - dv$

(2) $dy = (4u - 2v^2)du - 4uvdv$

< ヤコビアン > (27ページ)

問の解答

(1) $J = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$

(2) $J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$

< 積分の練習 > (28 ページ)

問 1 の解答

(1) $x + C$ (2) $\dfrac{1}{n+1}x^{n+1} + C$ (3) $\log|x| + C$

(4) $-\cos x + C$ (5) $\sin x + C$ (6) $e^x + C$

(7) $\dfrac{1}{4a}(ax+b)^4 + C$ (8) $-\dfrac{1}{a}\cos(ax+b) + C$ (9) $\dfrac{1}{a}e^{ax+b} + C$

問 2 の解答

(1) $\left[\dfrac{1}{3}x^3 - x^2\right]_{-1}^{2} = \left(\dfrac{8}{3} - 4\right) - \left(-\dfrac{1}{3} - 1\right) = 0$

(2) $\left[y^2 + \dfrac{1}{4}y^4\right]_{-1}^{1} = \left(1 + \dfrac{1}{4}\right) - \left(1 + \dfrac{1}{4}\right) = 0$

(3) $\left[\dfrac{a}{3}x^3 + \dfrac{b}{2}x\right]_{0}^{2} = \dfrac{8}{3}a + 2b$

(4) $\left[\dfrac{a}{2}y^2 + \dfrac{b}{3}y^3 + \dfrac{c}{4}y^4\right]_{-1}^{1} = \left(\dfrac{a}{2} + \dfrac{b}{3} + \dfrac{c}{4}\right) - \left(\dfrac{a}{2} - \dfrac{b}{3} + \dfrac{c}{4}\right) = \dfrac{2}{3}b$

問 3 の解答

(1) $S = \displaystyle\int_0^2 (-x^2 + 2x)dx = \left[-\dfrac{1}{3}x^3 + x^2\right]_0^2 = -\dfrac{8}{3} + 4 = \dfrac{4}{3}$

(2) $S = 2 + \displaystyle\int_0^1 (x^3 - x + 2)dx = 2 + \left[\dfrac{x^4}{4} - \dfrac{x^2}{2} + 2x\right]_0^1 = 2 + \dfrac{1}{4} - \dfrac{1}{2} + 2 = \dfrac{15}{4}$

(3) $S = \displaystyle\int_{-1}^2 (x - x^2 + 2)dx = \left[\dfrac{x^2}{2} - \dfrac{x^3}{3} + 2x\right]_{-1}^2 = \left(2 - \dfrac{8}{3} + 4\right) - \left(\dfrac{1}{2} + \dfrac{1}{3} - 2\right) = \dfrac{9}{2}$

< 体積 1 > (29 ページ)

問の解答

$$V = \int_0^{r\cos\theta} \pi \left\{(\tan\theta)x\right\}^2 dx + \int_{r\cos\theta}^{r} \pi \left\{\sqrt{r^2 - x^2}\right\}^2 dx$$

$$= \pi \tan^2\theta \left[\frac{x^3}{3}\right]_0^{r\cos\theta} + \pi \left[r^2 x - \frac{x^3}{3}\right]_{r\cos\theta}^{r}$$

$$= \pi \tan^2\theta \times \frac{(r\cos\theta)^3}{3} + \pi \left\{\left(r^3 - \frac{r^3}{3}\right) - \left(r^3\cos\theta - \frac{r^3}{3}\cos^3\theta\right)\right\}$$

$$= \frac{\pi r^3}{3}\left\{\tan^2\theta\cos^3\theta + 2 - 3\cos\theta + \cos^3\theta\right\}$$

$$= \frac{\pi r^3}{3}\left\{2 + (1 + \tan^2\theta)\cos^3\theta - 3\cos\theta\right\}$$

$$= \frac{2\pi}{3} r^3 \left\{1 - \cos\theta\right\}$$

< 体積 2 > (30 ページ)

問の解答

$$S(x) = \int_0^5 \left(\frac{1}{4}x - \frac{1}{5}y + 5\right) dy$$

$$= \left[\frac{1}{4}xy - \frac{1}{10}y^2 + 5y\right]_{y=0}^{y=5}$$

$$= \frac{5}{4}x - \frac{5}{2} + 25 = \frac{5}{4}x + \frac{45}{2}$$

$$V = \int_0^4 S(x)dx = \int_0^4 \left(\frac{5}{4}x + \frac{45}{2}\right) dx = \left[\frac{5}{8}x^2 + \frac{45}{2}x\right]_0^4$$

$$= 10 + 90 = 100$$

< 体積 3 > (31ページ)

問の解答

$$S(x) = \int_0^3 \left(4 - 2x + xy + 2y - y^2\right) dy$$

$$= \left[4y - 2xy + \frac{x}{2}y^2 + y^2 - \frac{1}{3}y^3\right]_{y=0}^{y=3}$$

$$= 12 - 6x + \frac{9}{2}x + 9 - 9$$

$$= -\frac{3}{2}x + 12$$

$$V = \int_0^2 S(x)dx = \int_0^2 \left(-\frac{3}{2}x + 12\right) dx = \left[-\frac{3}{4}x^2 + 12x\right]_0^2$$

$$= -3 + 24 = 21$$

＜ 体積 4 ＞ (32 ページ)

問の解答

$$S(x) = \int_0^3 \left(\frac{1}{3}x - \frac{1}{4}y + 2\right) dx$$

$$= \left[\frac{1}{6}x^2 - \frac{1}{4}xy + 2x\right]_{x=0}^{x=3}$$

$$= \frac{9}{6} - \frac{3}{4}y + 6 = \frac{15}{2} - \frac{3}{4}y$$

$$V = \int_0^4 S(y)dy = \int_0^4 \left(\frac{15}{2} - \frac{3}{4}y\right) dy = \left[\frac{15}{2}y - \frac{3}{8}y^2\right]_0^4$$

$$= 30 - 6 = 24$$

< 累次積分 1 > (33 ページ)

問の解答

$$\int_2^3 \left\{ \int_1^2 (2xy - y^2) dy \right\} dx = \int_2^3 \left\{ \left[xy^2 - \frac{1}{3}y^3 \right]_{y=1}^{y=2} \right\} dx$$

$$= \int_2^3 \left\{ \left(4x - \frac{8}{3} \right) - \left(x - \frac{1}{3} \right) \right\} dx = \int_2^3 \left(3x - \frac{7}{3} \right) dx = \left[\frac{3}{2}x^2 - \frac{7}{3}x \right]_2^3$$

$$= \left(\frac{27}{2} - \frac{21}{3} \right) - \left(\frac{12}{2} - \frac{14}{3} \right) = \frac{31}{6}$$

< 累次積分 2 > (34ページ)

問の解答

$$\int_1^2 \left\{ \int_2^3 \left(2xy - y^2\right) dx \right\} dy = \int_1^2 \left\{ \left[x^2y - xy^2\right]_{x=2}^{x=3} \right\} dy$$

$$= \int_1^2 \left\{ (9y - 3y^2) - (4y - 2y^2) \right\} dy = \int_1^2 \left\{ 5y - y^2 \right\} dy = \left[\frac{5}{2}y^2 - \frac{1}{3}y^3 \right]_{y=1}^{y=2}$$

$$= \left(\frac{20}{2} - \frac{8}{3} \right) - \left(\frac{5}{2} - \frac{1}{3} \right) = \frac{31}{6}$$

< 長方形領域の 2 重積分 > (35 ページ)

問の解答

$$\iint_D \left(2xy - y^2\right) dxdy = \int_0^2 \left\{ \int_{-1}^1 \left(2xy - y^2\right) dy \right\} dx$$

$$= \int_0^2 \left\{ \left[xy^2 - \frac{1}{3}y^3 \right]_{y=-1}^{y=1} \right\} dx = \int_0^2 \left(-\frac{2}{3} \right) dy = -\frac{4}{3}$$

(別解)

$$\iint_D \left(2xy - y^2\right) dxdy = \int_{-1}^1 \left\{ \int_0^2 \left(2xy - y^2\right) dx \right\} dy$$

$$= \int_{-1}^1 \left\{ \left[x^2y - xy^2 \right]_{x=0}^{x=2} \right\} dy = \int_{-1}^1 \left(4y - 2y^2\right) dy = \left[2y^2 - \frac{2}{3}y^3 \right]_{y=-1}^{y=1}$$

$$= \left(2 - \frac{2}{3} \right) - \left(2 + \frac{2}{3} \right) = -\frac{4}{3}$$

< 一般領域の 2 重積分 1 > (36 ページ)

問の解答

$$\iint_D (x+y)\,dxdy = \int_1^2 \left\{\int_0^1 (x+y)\,dy\right\} dx + \int_2^3 \left\{\int_1^2 (x+y)\,dy\right\} dx$$

$$= \int_1^2 \left\{\left[xy + \frac{1}{2}y^2\right]_{y=0}^{y=1}\right\} dx + \int_2^3 \left\{\left[xy + \frac{1}{2}y^2\right]_{y=1}^{y=2}\right\} dx$$

$$= \int_1^2 \left(x + \frac{1}{2}\right) dx + \int_2^3 \left(x + \frac{3}{2}\right) dx = \left[\frac{x^2}{2} + \frac{1}{2}x\right]_1^2 + \left[\frac{x^2}{2} + \frac{3}{2}x\right]_2^3$$

$$= \left(\frac{4}{2} + \frac{2}{2}\right) - \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{9}{2} + \frac{9}{2}\right) - \left(\frac{4}{2} + \frac{6}{2}\right) = 6$$

(別解)

$$\iint_D (x+y)\,dxdy = \int_0^1 \left\{\int_1^2 (x+y)\,dx\right\} dy + \int_1^2 \left\{\int_2^3 (x+y)\,dx\right\} dy$$

$$= \int_0^1 \left\{\left[\frac{x^2}{2} + xy\right]_{x=1}^{x=2}\right\} dy + \int_1^2 \left\{\left[\frac{x^2}{2} + xy\right]_{x=2}^{x=3}\right\} dy$$

$$= \int_0^1 \left(\frac{3}{2} + y\right) dy + \int_1^2 \left(\frac{5}{2} + y\right) dy = \left[\frac{3}{2}y + \frac{1}{2}y^2\right]_0^1 + \left[\frac{5}{2}y + \frac{1}{2}y^2\right]_1^2$$

$$= \left(\frac{3}{2} + \frac{1}{2}\right) + \left(\frac{10}{2} + \frac{4}{2}\right) - \left(\frac{5}{2} + \frac{1}{2}\right) = 6$$

< 一般領域の2重積分 2 > (37ページ)

問の解答

$$\iint_D \left(x^3 - 2xy^2\right) dxdy = \int_0^1 \left\{ \int_0^{-x+1} \left(x^3 - 2xy^2\right) dy \right\} dx$$

$$= \int_0^1 \left\{ \left[x^3 y - \frac{2}{3} x y^3 \right]_{y=0}^{y=-x+1} \right\} dx = \int_0^1 \left\{ x^3 (-x+1) - \frac{2}{3} x (-x+1)^3 \right\} dx$$

$$= \int_0^1 \left\{ -x^4 + x^3 + \frac{2}{3} x^4 - 2x^3 + 2x^2 - \frac{2}{3} x \right\} dx = \int_0^1 \left\{ -\frac{1}{3} x^4 - x^3 + 2x^2 - \frac{2}{3} x \right\} dx$$

$$= \left[-\frac{1}{15} x^5 - \frac{1}{4} x^4 + \frac{2}{3} x^3 - \frac{1}{3} x^2 \right]_0^1 = -\frac{1}{15} - \frac{1}{4} + \frac{2}{3} - \frac{1}{3} = \frac{1}{60}$$

(別解)

$$\iint_D \left(x^3 - 2xy^2\right) dxdy = \int_0^1 \left\{ \int_0^{-y+1} \left(x^3 - 2xy^2\right) dx \right\} dy$$

$$= \int_0^1 \left\{ \left[\frac{x^4}{4} - x^2 y^2 \right]_{x=0}^{x=-y+1} \right\} dy = \int_0^1 \left\{ \frac{1}{4} (-y+1)^4 - (-y+1)^2 y^2 \right\} dy$$

$$= \int_0^1 \left\{ \frac{1}{4} (y-1)^4 - y^4 + 2y^3 - y^2 \right\} dx = \left[\frac{1}{20} (y-1)^5 - \frac{1}{5} y^5 + \frac{1}{2} y^4 - \frac{1}{3} y^3 \right]_0^1$$

$$= \left(0 - \frac{1}{5} + \frac{1}{2} - \frac{1}{3} \right) - \left(-\frac{1}{20} \right) = \frac{1}{60}$$

< 一般領域の 2 重積分 3 > (38 ページ)

問の解答

$$\iint_D \left(y^2-2x\right)dxdy=\int_0^1\left\{\int_0^y\left(y^2-2x\right)dx\right\}dy$$

$$=\int_0^1\left\{\left[xy^2-x^2\right]_{x=0}^{x=y}\right\}dy=\int_0^1\left(y^3-y^2\right)dy$$

$$=\left[\frac{1}{4}y^4-\frac{1}{3}y^3\right]_0^1=\frac{1}{4}-\frac{1}{3}=-\frac{1}{12}$$

(別解)

$$\iint_D \left(y^2-2x\right)dxdy=\int_0^1\left\{\int_x^1\left(y^2-2x\right)dy\right\}dx$$

$$=\int_0^1\left\{\left[\frac{1}{3}y^3-2xy\right]_{y=x}^{y=1}\right\}dx=\int_0^1\left\{\left(\frac{1}{3}-2x\right)-\left(\frac{1}{3}x^3-2x^2\right)\right\}dx$$

$$=\int_0^1\left\{-\frac{1}{3}x^3+2x^2-2x+\frac{1}{3}\right\}dx=\left[-\frac{1}{12}x^4+\frac{2}{3}x^3-x^2+\frac{1}{3}x\right]_{x=0}^{x=1}$$

$$=-\frac{1}{12}+\frac{2}{3}-1+\frac{1}{3}=-\frac{1}{12}$$

< 面積比 > (39 ページ)

問の解答

(1) $\begin{cases} \Delta(u,v) = ab \\ \Delta(x,y) = \begin{vmatrix} 4a & 2b \\ a & 5b \end{vmatrix} = 20ab - 2ab = 18ab \end{cases} \quad \Rightarrow \dfrac{\Delta(x,y)}{\Delta(u,v)} = \dfrac{18ab}{ab} = 18$

(2) $J = \begin{vmatrix} 4 & 2 \\ 1 & 5 \end{vmatrix} = 4 \times 5 - 2 \times 1 = 18$

＜ 重積分の変数変換 ＞ (40 ページ)

問の解答

$$\iint_D e^{-x^2-y^2}dxdy = \iint_\Omega e^{-r^2}rdrd\theta$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{ \int_0^R e^{-r^2}rdr \right\} d\theta = \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{ \left[-\frac{1}{2}e^{-r^2} \right]_{r=0}^{r=R} \right\} d\theta$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{ \frac{1}{2}\left(1-e^{-R^2}\right) \right\} d\theta = \frac{\pi}{4}\left(1-e^{-R^2}\right)$$