高知工科大学

基礎数学ワークブック

(2001 年度版)

Series A

No. 8

解答

＜１ページ. 高次式の因数分解＞

問の解答

(1) $x^3+8=(x+2)(x^2-2x+4)$

$$=(x+2)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)$$

(2) $x^3-27=(x-3)(x^2+3x+9)$

$$=(x-3)\left(x+\frac{3}{2}-\frac{3\sqrt{3}}{2}i\right)\left(x+\frac{3}{2}+\frac{3\sqrt{3}}{2}i\right)$$

(3) $x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x-i)(x+i)$

＜ 2ページ. 高次方程式 ＞

問の解答

(1) $x^3+8=0 \Longleftrightarrow (x+2)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)=0$

$$\underline{x=-2\ ,\ 1+\sqrt{3}i\ ,\ 1-\sqrt{3}i}$$

(2) $x^3-27=0 \Longleftrightarrow (x-3)\left(x+\frac{3}{2}-\frac{3\sqrt{3}}{2}i\right)\left(x+\frac{3}{2}+\frac{3\sqrt{3}}{2}i\right)=0$

$$\underline{x=3\ ,\ x=-\frac{3}{2}+\frac{3\sqrt{3}}{2}i\ ,\ x=-\frac{3}{2}-\frac{3\sqrt{3}}{2}i}$$

(3) $x^4-1=0 \Longleftrightarrow (x-1)(x+1)(x-i)(x+i)=0$

$$\underline{x=\pm 1\ ,\ \pm i}$$

＜3ページ. 共役複素数＞

問１の解答

(1) $z = 1$, $\overline{z} = 1$ (2) $z = i$, $\overline{z} = -i$

(3) $z = 1 + i$, $\overline{z} = 1 - i$ (4) $z = \dfrac{1+i}{2}$, $\overline{z} = \dfrac{1-i}{2}$

問２の解答

(1) $\dfrac{1}{2}(z + \overline{z})$

$$= \frac{1}{2}(1 - 3i + 1 + 3i)$$

$$= 1$$

(2) $\dfrac{1}{2i}(z - \overline{z})$

$$= \frac{1}{2i}(1 - 3i - (1 + 3i))$$

$$= \frac{1}{2i}(-6i)$$

$$= -3$$

(3) $z\overline{z}$

$$= (1 - 3i)(1 + 3i)$$

$$= 1^2 - (3i)^2$$

$$= 1 + 9 = 10$$

問３の解答

(1) $\dfrac{1}{2}(z + \overline{z})$

$$= \frac{1}{2}(a + bi + a - bi)$$

$$= a$$

(2) $\dfrac{1}{2i}(z - \overline{z})$

$$= \frac{1}{2i}(a + bi - (a - bi))$$

$$= \frac{1}{2i}(2bi)$$

$$= b$$

(3) $z\overline{z}$

$$= (a + bi)(a - bi)$$

$$= a^2 - (bi)^2$$

$$= a^2 + b^2$$

＜ 4 ページ. 絶対値 ＞

問 1 の解答

(1) $|z| = 1$　(2) $|z| = 7$

(3) $|z| = \sqrt{\left(\dfrac{\sqrt{5}}{3}\right)^2 + \left(\dfrac{2}{3}\right)^2} = \sqrt{\dfrac{5+4}{9}} = 1$

問 2 の解答

(1) $z = 2 - 3i$

$$|z|^2 = 2^2 + (-3)^2 = 4 + 9 = 13$$

$$\begin{aligned} z^2 &= (2-3i)^2 \\ &= 2^2 - 12i + (3i)^2 \\ &= 4 - 12i - 9 \\ &= -5 - 12i \end{aligned}$$

$$\begin{aligned} |z^2| &= \sqrt{(-5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} = \sqrt{169} \\ &= 13 \end{aligned}$$

(2) $z = 1 - i$

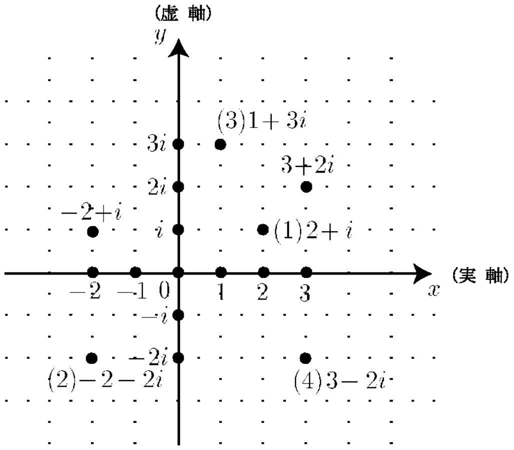
$$|z|^2 = 1^2 + (-1)^2 = 1 + 1 = 2$$

$$\begin{aligned} z^2 &= (1-i)^2 \\ &= 1^2 - 2i + i^2 \\ &= -2i \end{aligned}$$

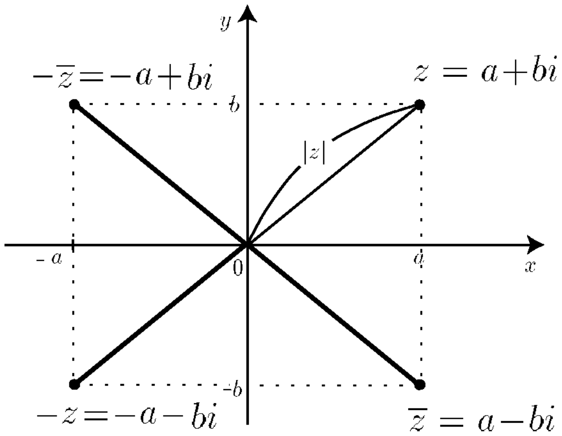
$$|z^2| = \sqrt{(-2)^2} = 2$$

＜５ページ.複素平面１＞

問１の解答



問２の解答



＜ 6 ページ. 複素平面 2 ＞

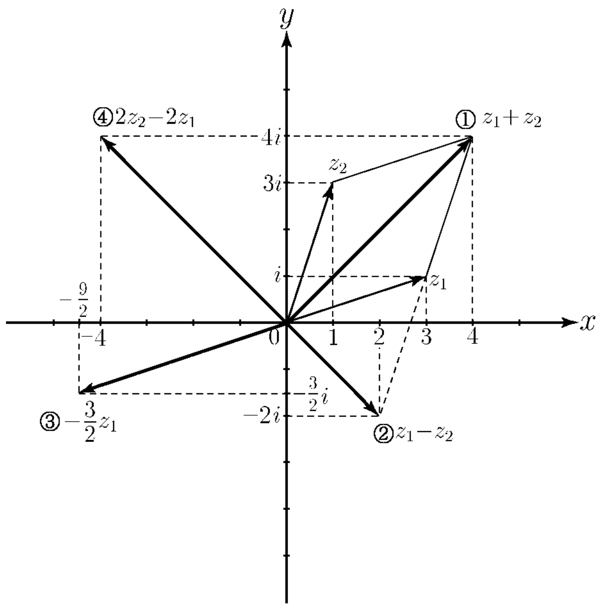
問の解答

① $z_1 + z_2 = (3+i) + (1+3i)$
$= 4 + 4i$

② $z_1 - z_2 = (3+i) - (1+3i)$
$= 2 - 2i$

③ $-\dfrac{3}{2}z_1 = -\dfrac{3}{2}(3+i)$
$= -\dfrac{9}{2} - \dfrac{3}{2}i$

④ $2z_2 - 2z_1 = 2(1+3i) - 2(3+i)$
$= 2 + 6i - (6+2i)$
$= -4 + 4i$



＜ 7ページ. 複素数の i 倍 ＞

問の解答

(1) $z = 1 + i$

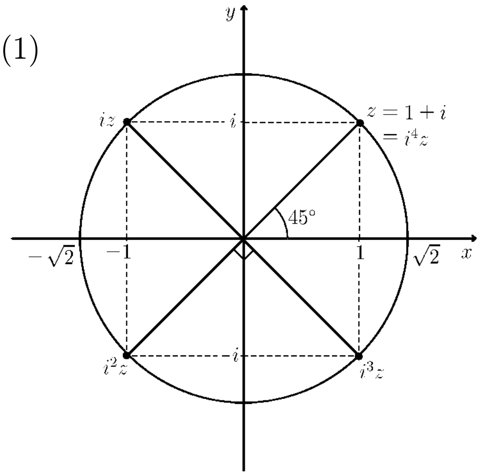
$iz = i - 1$

$i^2 z = -1 - i$

$i^3 z = -i + 1$

$i^4 z = 1 + i$

(1)



(2) $z = 1 + \sqrt{3}i$

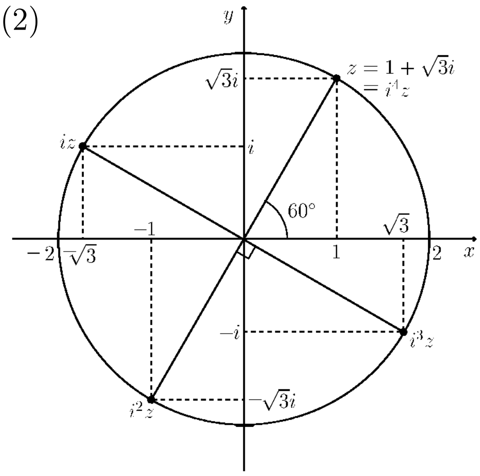
$iz = i - \sqrt{3}$

$i^2 z = -1 - \sqrt{3}i$

$i^3 z = -i + \sqrt{3}$

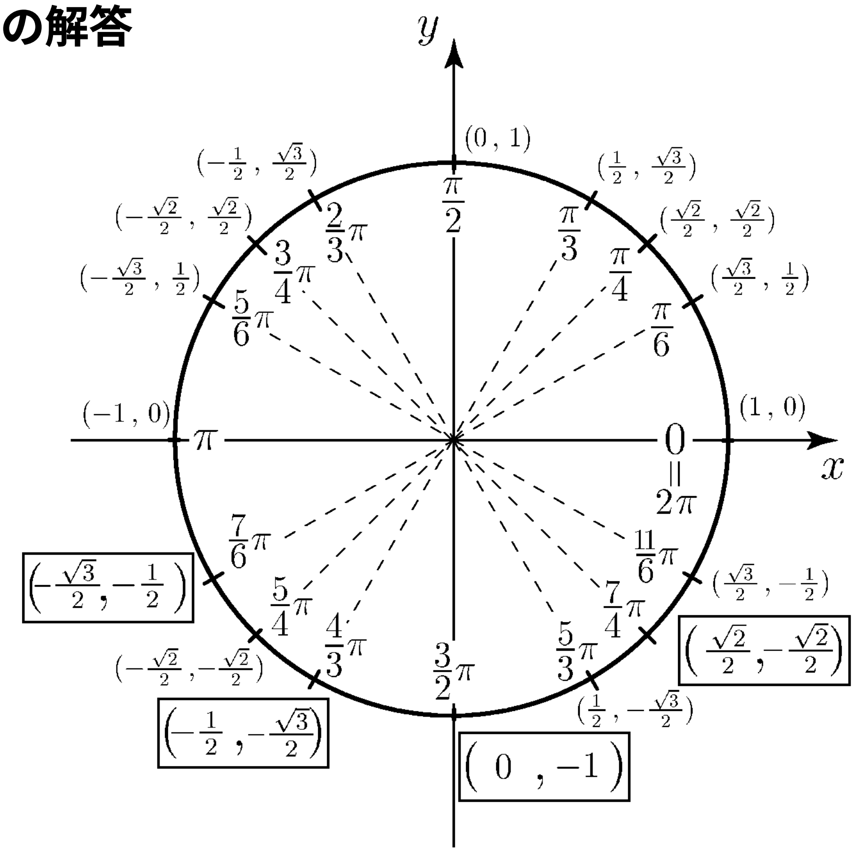
$i^4 z = 1 + \sqrt{3}i$

(2)



＜ 8ページ. 極座標 1 ＞

問 1 の解答



問 2 の解答

(1) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \left(\cos\left(\frac{\pi}{3}\right)\ ,\ \sin\left(\frac{\pi}{3}\right)\right)$

(2) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(\cos\left(\frac{3}{4}\pi\right)\ ,\ \sin\left(\frac{3}{4}\pi\right)\right)$

(3) $\left(-1,\ 0\right) = (\cos\pi\ ,\ \sin\pi)$

(4) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = \left(\cos\left(\frac{5}{6}\pi\right)\ ,\ \sin\left(\frac{5}{6}\pi\right)\right)$

(5) $\left(0,\ -1\right) = \left(\cos\left(\frac{3}{2}\pi\right)\ ,\ \sin\left(\frac{3}{2}\pi\right)\right)$

(6) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = \left(\cos\left(\frac{7}{4}\pi\right)\ ,\ \sin\left(\frac{7}{4}\pi\right)\right)$

＜ 9ページ. 極座標 2 ＞

問の解答

(1) $\left(\ 2\ ,\ 2\ \right) = \left(\ 2\sqrt{2}\cos\left(\frac{\pi}{4}\right)\ ,\ 2\sqrt{2}\sin\left(\frac{\pi}{4}\right)\ \right)$

(2) $\left(-\sqrt{3},\ 1\right) = \left(\ 2\cos\left(\frac{5\pi}{6}\right)\ ,\ 2\sin\left(\frac{5\pi}{6}\right)\ \right)$

(3) $\left(\sqrt{3},\ -1\right) = \left(\ 2\cos\left(-\frac{\pi}{6}\right)\ ,\ 2\sin\left(-\frac{\pi}{6}\right)\ \right)$

$= \left(\ 2\cos\left(\frac{11}{6}\pi\right)\ ,\ 2\sin\left(\frac{11}{6}\pi\right)\ \right)$

(4) $\left(\ -3\ ,\ -3\ \right) = \left(\ 3\sqrt{2}\cos\left(\frac{5}{4}\pi\right)\ ,\ 3\sqrt{2}\sin\left(\frac{5}{4}\pi\right)\ \right)$

$= \left(\ 3\sqrt{2}\cos\left(-\frac{3}{4}\pi\right)\ ,\ 3\sin\left(-\frac{3}{4}\pi\right)\ \right)$

< 10ページ. 絶対値 1 の複素数 >

問の解答

(1) $\cos\left(\dfrac{\pi}{4}\right)+i\sin\left(\dfrac{\pi}{4}\right)=\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$, (2) $\cos\left(\dfrac{\pi}{3}\right)+i\sin\left(\dfrac{\pi}{3}\right)=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$, (3) $\cos\left(\dfrac{2}{3}\pi\right)+i\sin\left(\dfrac{2}{3}\pi\right)=-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

(4) $\cos\left(\dfrac{3}{4}\pi\right)+i\sin\left(\dfrac{3}{4}\pi\right)=-\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$, (5) $\cos\left(\dfrac{5}{6}\pi\right)+i\sin\left(\dfrac{5}{6}\pi\right)=-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$, (6) $\cos(\pi)+i\sin(\pi)=-1$

(7) $\cos\left(\dfrac{7}{6}\pi\right)+i\sin\left(\dfrac{7}{6}\pi\right)=-\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i$, (8) $\cos\left(\dfrac{5}{4}\pi\right)+i\sin\left(\dfrac{5}{4}\pi\right)=-\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$, (9) $\cos\left(\dfrac{4}{3}\pi\right)+i\sin\left(\dfrac{4}{3}\pi\right)=-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

(10) $\cos\left(\dfrac{3}{2}\pi\right)+i\sin\left(\dfrac{3}{2}\pi\right)=-i$, (11) $\cos\left(\dfrac{5}{3}\pi\right)+i\sin\left(\dfrac{5}{3}\pi\right)=\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$, (12) $\cos\left(\dfrac{7}{4}\pi\right)+i\sin\left(\dfrac{7}{4}\pi\right)=\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$

< 11ページ. 極形式 1 >

問の解答

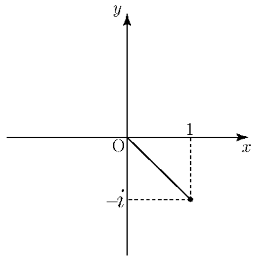
(1) $-i = \cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right) = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)$

(2) $-1 = \cos(\pi) + i\sin(\pi)$

(3) $\sqrt{2} = \sqrt{2}\,(\cos 0 + i\sin 0)$

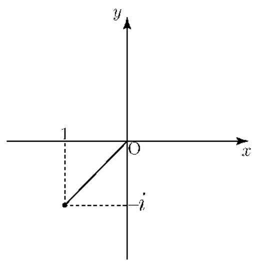
＜ 12 ページ. 極形式 2 ＞

問の解答



(1) $z = 1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

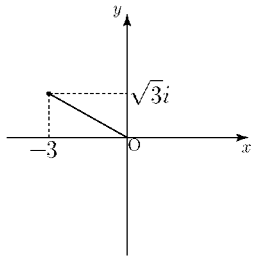
$$= \sqrt{2}\left(\cos\left(\frac{7}{4}\pi\right) + i\sin\left(\frac{7}{4}\pi\right)\right)$$



(2) $z = -1 - i = \sqrt{2}\left(\cos\left(-\frac{3}{4}\pi\right) + i\sin\left(-\frac{3}{4}\pi\right)\right)$

$$= \sqrt{2}\left(\cos\left(\frac{5}{4}\pi\right) + i\sin\left(\frac{5}{4}\pi\right)\right)$$

(3) $z = \sqrt{2} + \sqrt{2}i = 2\left(\cos\left(\frac{\pi}{4}\pi\right) + i\sin\left(\frac{\pi}{4}\right)\right)$



(4) $z = -3 + \sqrt{3}i = 2\sqrt{3}\left(\cos\left(\frac{5}{6}\pi\right) + i\sin\left(\frac{5}{6}\pi\right)\right)$

(5) $z = \sqrt{6} + \sqrt{2}i = 2\sqrt{2}\left(\cos\left(\frac{\pi}{6}\pi\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

< 13 ページ. 複素数の積 >

問の解答

(1) $\left(\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)z=\left(\cos\left(-\dfrac{\pi}{3}\right)+i\sin\left(-\dfrac{\pi}{3}\right)\right)z=r\left\{\cos\left(\theta-\dfrac{\pi}{3}\right)+i\sin\left(\theta-\dfrac{\pi}{3}\right)\right\}$

$-\dfrac{\pi}{3}$ 回転 $\quad$ $\left(\text{または } \dfrac{5}{3}\pi \text{ 回転}\right)$

(2) $\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}i\right)z=\left(\cos\left(\dfrac{\pi}{4}\right)+i\sin\left(\dfrac{\pi}{4}\right)\right)z=r\left\{\cos\left(\theta+\dfrac{\pi}{4}\right)+i\sin\left(\theta+\dfrac{\pi}{4}\right)\right\}$

$\dfrac{\pi}{4}$ 回転

(3) $-z=(\cos\pi+i\sin\pi)\,z=r\left\{\cos(\theta+\pi)+i\sin(\theta+\pi)\right\}$

π 回転

＜ 14 ページ. 複素数の商 ＞

問の解答

(1) $$\frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{2\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)}{2\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)}$$

$$= \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$$

(2) $$\frac{-1-i}{1+i} = \frac{\sqrt{2}\left(\cos\left(\frac{5}{4}\pi\right)+i\sin\left(\frac{5}{4}\pi\right)\right)}{\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)}$$

$$= \cos(\pi) + i\sin(\pi)$$

(3) $$\frac{-1-i}{-\sqrt{3}+i} = \frac{\sqrt{2}\left(\cos\left(\frac{5}{4}\pi\right)+i\sin\left(\frac{5}{4}\pi\right)\right)}{2\left(\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right)}$$

$$= \frac{\sqrt{2}}{2}\left(\cos\left(\frac{5}{12}\pi\right) + i\sin\left(\frac{5}{12}\pi\right)\right)$$

＜ 15 ページ. ド・モアブルの定理 ＞

問の解答

(1) $$\left(\frac{\sqrt{3}+i}{2}\right)^3 = \left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)^3 = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$$

(2) $$\left(\frac{-1-\sqrt{3}i}{2}\right)^6 = \left(\cos\left(-\frac{2}{3}\pi\right) + i\sin\left(-\frac{2}{3}\pi\right)\right)^6$$
$$= \cos(-4\pi) + i\sin(-4\pi) = 1$$

(3) $$(1-i)^4 = \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right)^4$$
$$= \left(\sqrt{2}\right)^4 \times (\cos(-\pi) + i\sin(-\pi)) = -4$$

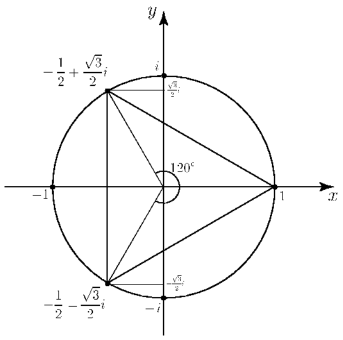
(4) $$\left(\frac{-1-i}{-\sqrt{3}+i}\right)^{12} = \left(\frac{\sqrt{2}\left(\cos\left(\frac{5}{4}\pi\right) + i\sin\left(\frac{5}{4}\pi\right)\right)}{2\left(\cos\left(\frac{5}{6}\pi\right) + i\sin\left(\frac{5}{6}\pi\right)\right)}\right)^{12}$$
$$= \left(\frac{1}{\sqrt{2}}\right)^{12} \times \frac{\cos(15\pi) + i\sin(15\pi)}{\cos(10\pi) + i\sin(10\pi)}$$
$$= \left(\frac{1}{2}\right)^6 \times \frac{-1}{1} = -\frac{1}{64}$$

＜ 16ページ.1の累乗根 ＞

問の解答

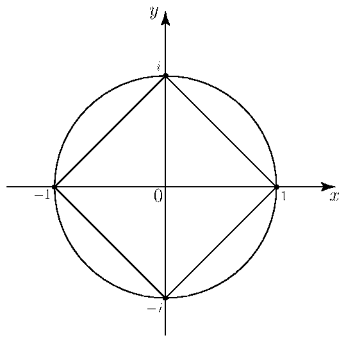
(1) $z^3 = 1$

$$\underline{z \ = \ 1\ ,\ -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i}$$



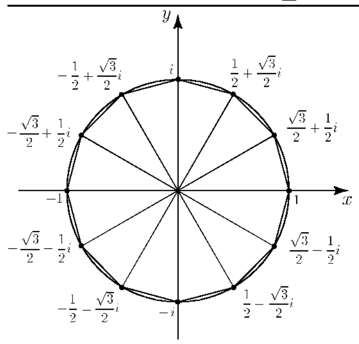
(2) $z^4 = 1$

$$\underline{z \ = \ \pm 1\ ,\ \pm i}$$



(3) $z^{12} = 1$

$$\underline{z \ = \ \pm 1\ ,\ \pm i\ ,\ \frac{1}{2} \pm \frac{\sqrt{3}}{2} i\ ,\ \frac{\sqrt{3}}{2} \pm \frac{1}{2} i\ ,\ -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i\ ,\ -\frac{\sqrt{3}}{2} \pm \frac{1}{2} i}$$



< 17 ページ. 平面上の回転移動 >

問の解答

(1) 角度 $\dfrac{\pi}{2}$

$$\begin{cases} x' = x\cos\left(\frac{\pi}{2}\right) - y\sin\left(\frac{\pi}{2}\right) = -y \\ y' = x\sin\left(\frac{\pi}{2}\right) + y\cos\left(\frac{\pi}{2}\right) = x \end{cases}$$

(2) 角度 $\dfrac{\pi}{4}$

$$\begin{cases} x' = x\cos\left(\frac{\pi}{4}\right) - y\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \\ y' = x\sin\left(\frac{\pi}{4}\right) + y\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \end{cases}$$

(3) 角度 $\dfrac{\pi}{3}$

$$\begin{cases} x' = x\cos\left(\frac{\pi}{3}\right) - y\sin\left(\frac{\pi}{3}\right) = \frac{1}{2}x - \frac{\sqrt{3}}{2}y \\ y' = x\sin\left(\frac{\pi}{3}\right) + y\cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}x + \frac{1}{2}y \end{cases}$$

(4) 角度 θ

$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases}$$

＜ 18 ページ. オイラーの公式 1 ＞

問の解答

(1) $e^{0i} = \cos 0 + i \sin 0 = 1$

(2) $e^{\pi i} = \cos(\pi) + i \sin(\pi) = -1$

(3) $e^{\frac{\pi}{4}i} = \cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

(4) $e^{\frac{5}{6}\pi i} = \cos\left(\frac{5}{6}\pi\right) + i \sin(\frac{5}{6}\pi) = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

(5) $e^{-\frac{2}{3}\pi i} = \cos\left(-\frac{2}{3}\pi\right) + i \sin(-\frac{2}{3}\pi) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

(6) $e^{-\frac{\pi}{2}i} = \cos\left(-\frac{\pi}{2}\right) + i \sin(-\frac{\pi}{2}) = -i$

< 19 ページ. オイラーの公式 2 >

問の解答

(1) $e^{1-\pi i} = e\left(\cos(-\pi) + i\sin(-\pi)\right) = -e$

(2) $e^{0+\frac{\pi}{2}i} = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$

(3) $e^{2+\frac{\pi}{4}i} = e^2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) = e^2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \frac{\sqrt{2}}{2}e^2 + \frac{\sqrt{2}}{2}e^2 i$

(4) $e^{\frac{1}{2}-\frac{\pi}{2}i} = e^{\frac{1}{2}}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right) = \sqrt{e}i$

(5) $e^{\log 4+\frac{2}{3}\pi i} = e^{\log 4}\left(\cos\left(\frac{2}{3}\pi\right) + i\sin\left(\frac{2}{3}\pi\right)\right) = 4\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -2 + 2\sqrt{3}i$

(6) $e^{\frac{1}{2}\log 4-\frac{\pi}{6}i} = e^{\log 2}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$

＜ 21 ページ. 指数法則 ＞

問 1 の解答

(2) $\dfrac{e^{z_1}}{e^{z_2}} = e^{z_1 - z_2}$

(3) $\left(e^{z}\right)^{n} = e^{nz}$

問 2 の解答

(1) $e^{5+\pi i} \times e^{-1-\pi i} = e^{4}$

(2) $e^{-1+\frac{\pi}{4}i} \div e^{-2+\frac{\pi}{4}i} = e$

(3) $\left(e^{\frac{3}{2}-\frac{2}{3}\pi i}\right)^{2} = e^{3-\frac{4}{3}\pi i} = e^{3}\left(\cos(-\frac{4}{3}\pi) + i\sin(-\frac{4}{3}\pi)\right) = -\frac{1}{2}e^{3} + \frac{\sqrt{3}}{2}e^{3}i$

問 3 の解答

$$\frac{\left(-1+i\right)^{4}}{\left(1+\sqrt{3}i\right)^{3}} = \frac{\left(\sqrt{2}e^{\frac{3}{4}\pi i}\right)^{4}}{\left(2e^{\frac{\pi}{3}i}\right)^{3}} = \frac{4e^{3\pi i}}{8e^{\pi i}} = \frac{1}{2}e^{2\pi i} = \frac{1}{2}$$

< 22ページ. 円関数 >

問1の解答

(1) $\cos(i\theta) = \dfrac{e^{i(i\theta)} + e^{-i(i\theta)}}{2} = \dfrac{1}{2}(e^{-\theta} + e^{\theta})$

(2) $(\cos z)^2 = \left(\dfrac{e^{iz} + e^{-iz}}{2}\right)^2 = \dfrac{e^{2iz} + 2 + e^{-2iz}}{4}$

問2の解答

(1) $\cos(-iz) + i\sin(-iz)$

$$= \frac{e^{i(-iz)} + e^{-i(-iz)}}{2} + i\left(\frac{e^{i(-iz)} - e^{-i(-iz)}}{2i}\right)$$

$$= \frac{e^{z} + e^{-z}}{2} + \frac{e^{z} - e^{-z}}{2}$$

$$= e^{z}$$

(2) $(\cos z)^2 + (\sin z)^2$

$$= \left(\frac{e^{iz} + e^{-iz}}{2}\right)^2 + \left(\frac{e^{iz} - e^{-iz}}{2i}\right)^2$$

$$= \frac{e^{2iz} + 2 + e^{-2iz}}{4} + \frac{e^{2iz} - 2 + e^{-2iz}}{-4}$$

$$= 1$$

＜ 23 ページ. 双曲線関数 ＞

問の解答

(1) $\cosh(i\theta) = \dfrac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$

(2) $\cosh(z) + \sinh(z) = \dfrac{e^z + e^{-z}}{2} + \dfrac{e^z - e^{-z}}{2} = e^z$

(3) $\cosh(a + bi) - \cosh(a - bi) + \sinh(a + bi) - \sinh(a - bi)$

$$= \frac{e^{a+bi} + e^{-a-bi}}{2} - \frac{e^{a-bi} + e^{-a+bi}}{2} + \frac{e^{a+bi} - e^{-a-bi}}{2} - \frac{e^{a-bi} - e^{-a+bi}}{2}$$

$$= \frac{e^{a+bi} + e^{-a-bi} - e^{a-bi} - e^{-a+bi} + e^{a+bi} - e^{-a-bi} - e^{a-bi} + e^{-a+bi}}{2}$$

$$= e^{a+bi} - e^{a-bi} = e^a(\cos b + i\sin b) - e^a(\cos b - i\sin b) = 2ie^a \sin b$$

< 24 ページ. 時間変数 t による微分 1 >

問の解答

(1) $\dfrac{d}{dt}\left(10+8t-5t^2\right)=8-10t$

(2) $\dfrac{d}{dt}\left(t^8-10t^4+5e^t\right)=8t^7-40t^3+5e^t$

(3) $\dfrac{d}{dt}\left(4t^5-5\cos t+2\log t\right)=20t^4+5\sin t+\dfrac{2}{t}$

(4) $\dfrac{d}{dt}\left(\dfrac{3}{t}+\dfrac{4}{\sqrt{t}}\right)=-\dfrac{3}{t^2}-\dfrac{2}{t\sqrt{t}}$

< 25 ページ. 時間変数 t による微分 2 >

問 1 の解答

(1) $\dfrac{d}{dt}\cos(3t+4) = -3\sin(3t+4)$

(2) $\dfrac{d}{dt}e^{5t+2} = 5e^{5t+2}$

(3) $\dfrac{d}{dt}\sin(-2t+1) = -2\cos(-2t+1)$

(4) $\dfrac{d}{dt}\log(4t+3) = \dfrac{4}{4t+3}$

問 2 の解答

(1) $\dfrac{d}{dt}\sin(t^3-2t) = (3t^2-2)\cos(t^3-2t)$

(2) $\dfrac{d}{dt}\left(e^{-t^2}\right) = -2te^{-t^2}$

(3) $\dfrac{d}{dt}\cos(3t+4t^2) = -(3+8t)\sin(3t+4t^2)$

(4) $\dfrac{d}{dt}\log(t^4+3t^2+1) = \dfrac{4t^3+6t}{t^4+3t^2+1}$

< 26 ページ. 時間変数 t による微分 3 >

問１の解答

(1) $\dfrac{d}{dt}(te^t) = e^t + te^t$

(2) $\dfrac{d}{dt}(t^2 \sin t) = 2t \sin t + t^2 \cos t$

(3) $\dfrac{d}{dt}(e^t \sin t) = e^t \sin t + e^t \cos t$

(4) $\dfrac{d}{dt}(t \log t) = \log t + 1$

問２の解答

(1) $\dfrac{d}{dt}\big(e^t \sin(2t)\big) = e^t \sin(2t) + 2e^t \cos(2t)$

(2) $\dfrac{d}{dt}\big(e^{3t} \cos(4t)\big) = 3e^{3t} \cos(4t) - 4e^{3t} \sin(4t)$

(3) $\dfrac{d}{dt}\big(4e^{3t} \sin(-5t)\big) = 12e^{3t} \sin(-5t) - 20e^{3t} \cos(-5t)$

$\big(= -12e^{3t} \sin(5t) - 20e^{3t} \cos(5t)\big)$

(4) $\dfrac{d}{dt}\big(5e^{-2t} \cos(6t)\big) = -10e^{-2t} \cos(6t) - 30e^{-2t} \sin(6t)$

＜ 27 ページ. 複素数値関数の微分 1 ＞

問の解答

(1) $z(t) = t - it^{-2}$

$$\frac{dz}{dt} = 1 + \frac{2}{t^3} i$$

(2) $z(t) = e^{ibt} = \cos(bt) + i\sin(bt)$

$$\frac{dz}{dt} = -b\sin(bt) + bi\cos(bt)$$

(3) $z(t) = e^{(1+2i)t} = e^t\left(\cos(2t) + i\sin(2t)\right)$

$$\begin{aligned}\frac{dz}{dt} &= e^t(\cos(2t) + i\sin(2t)) + e^t(-2\sin(2t) + 2i\cos(2t)) \\ &= \{e^t\cos(2t) - 2e^t\sin(2t)\} + i\{e^t\sin(2t) + 2e^t\cos(2t)\}\end{aligned}$$

(4) $z(t) = e^{(a+bi)t} = e^{at}\cos(bt) + ie^{at}\sin(bt)$

$$\frac{dz}{dt} = \{ae^{at}\cos(bt) - be^{at}\sin(bt)\} + i\{ae^{at}\sin(bt) + be^{at}\cos(bt)\}$$

< 28 ページ. 複素数値関数の微分 2 >

問の解答

(1) $$\frac{d}{dt}\left(e^{2it}\right) = \frac{d}{dt}\left(\cos(2t) + i\sin(2t)\right) = -2\sin(2t) + 2i\cos(2t)$$

$$= 2i\left(-\frac{1}{i}\sin(2t) + \cos(2t)\right) = 2i(i\sin(2t) + \cos(2t))$$

$$= 2i\left(\cos(2t) + i\sin(2t)\right) = 2ie^{2it}$$

(2) $$\frac{d}{dt}\left(e^{bit}\right) = -b\sin(bt) + bi\cos(bt)$$

$$= bi\left(-\frac{1}{i}\sin(bt) + \cos(bt)\right) = bi\left(i\sin(bt) + \cos(bt)\right)$$

$$= bi\left(\cos(bt) + i\sin(bt)\right) = bie^{2it}$$

(3) $$\frac{d}{dt}\left(e^{(1-i)t}\right) = \frac{d}{dt}\Big\{e^t\cos(-t) + ie^t\sin(-t)\Big\}$$

$$= e^t\cos(-t) + e^t\sin(-t) + i\Big\{e^t\sin(-t) - e^t\cos(-t)\Big\}$$

$$= e^t\Big\{\cos(-t) + i\sin(-t) + \sin(-t) - i\cos(-t)\Big\}$$

$$= e^t\Big\{\cos(-t) + i\sin(-t)\Big\} + (-i)e^t\Big\{\cos(-t) - \frac{1}{i}\sin(-t)\Big\}$$

$$= e^te^{-ti} + (-i)e^te^{-ti} = (1-i)e^{t-ti} = (1-i)e^{(1-i)t}$$

(4) $$\frac{d}{dt}\left(e^{(a+bi)t}\right) = \frac{d}{dt}\Big\{e^{at}\cos bt + ie^{at}\sin bt\Big\}$$

$$= ae^{at}\cos bt - be^{at}\sin bt + i\Big\{ae^{at}\sin bt + be^{at}\cos bt\Big\}$$

$$= e^{at}\Big\{a\cos bt + ia\sin bt - b\sin bt + ib\cos bt\Big\}$$

$$= e^{at}\cdot a\Big\{\cos bt + i\sin bt\Big\} + ie^{at}\cdot b\Big\{\cos bt + i\sin bt\Big\}$$

$$= e^{at}\cdot ae^{ibt} + e^{at}\cdot(ib)e^{ibt} = (a+ib)e^{at}\cdot e^{ibt}$$

$$= (a+ib)e^{(a+ib)t}$$

< 30 ページ. 速度 2 >

問の解答

B 地点と C 地点の間

(2 秒後から 12 秒後までの間)

＜ 31ページ. 速度 3 ＞

問1の解答

(1) $x(t) = 12 + 6t - 3t^2$, $v(t) = 6 - 6t$

(2) $x(t) = \sin(2t)$, $v(t) = 2\cos(2t)$

(3) $x(t) = e^{2t}\cos(3t)$, $v(t) = 2e^{2t}\cos(3t) - 3e^{2t}\sin(3t)$

問2の解答

(1) $v(t) = -9.8t + 9.8$

(2) 1秒後

(3) $t = 1$のとき　$x(1) = -4.9 + 9.8 + 14.7 = 19.6(m)$

＜ 32 ページ. 速度の応用 1 ＞

問の解答

(1) $v(t) = \dfrac{dy(t)}{dt} = -9.8t + 19.6$

(2) $v(0) = 19.6$

(答) 初速度 19.6 (m/s)

(3) $v(t) = 0 \quad \Rightarrow \quad -9.8t + 19.6 = 0 \quad \Rightarrow \quad t = 2$

(答) 2 秒後

(4) $t = 2$ のとき $y(2) = -4.9 \times 4 + 19.6 \times 2 + 24.5 = 44.1$

(答) 44.1 (m)

(5) $y(t) = -4.9t^2 + 19.6t + 24.5 = 0$

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0 \quad \Rightarrow \quad t = 5\ ,\ -1$

(答) 5 秒後

< 33ページ. 速度の応用 2 >

問の解答

(1) $v_x(t) = \dfrac{dx(t)}{dt} = 14.7$

(2) $v_y(t) = \dfrac{dy(t)}{dt} = -9.8t + 19.6$

(3) $v_y(t) = 0 \quad \Rightarrow \quad -9.8t + 19.6 = 0 \quad \Rightarrow \quad t = 2$

(答) 2 秒後

(4) $t = 2$ のとき　$y(2) = -4.9 \times 4 + 19.6 \times 2 = 19.6$

(答) 19.6 (m)

(5) $y(t) = 0 \quad \Rightarrow \quad -4.9t^2 + 19.6t = 0$

$-4.9t(t-4) = 0$

(答) 4 秒後

(6) $t = 4$ のとき　$x(4) = 14.7 \times 4 = 58.8$

(答) 58.8 (m)

＜ 36 ページ. 加速度 2 ＞

問 1

(1) $x(t) = 1 - t + 2t^2 - 3t^3$

$v(t) = -1 + 4t - 9t^2$, $a(t) = 4 - 18t$

(2) $x(t) = 2\cos(3t)$

$v(t) = -6\sin(3t)$, $a(t) = -18\cos(3t)$

(3) $x(t) = e^{2t}\sin(2t)$

$$v(t) = 2e^{2t}\sin(2t) + 2e^{2t}\cos(2t)$$

$$\begin{aligned} a(t) &= 4e^{2t}\sin(2t) + 4e^{2t}\cos(2t) + 4e^{2t}\cos(2t) - 4e^{2t}\sin(2t) \\ &= 8e^{2t}\cos(2t) \end{aligned}$$

問 2 の解答

$v(t) = 6\sin(-2t)$, $a(t) = -12\cos(-2t)$ $\big(= -4 \times 3\cos(-2t)\big)$

$a(t) = -4x(t)$

< 37 ページ. 加速度 3 >

問の解答

$$a_x = \frac{dv_x}{dt} = 0 \quad , \quad a_y = \frac{dv_y}{dt} = -9.8$$

< 38ページ.等速円運動 1 >

問の解答

(1) $\dfrac{\omega t}{t} = \omega(\mathrm{rad/s})$

(2) $(x(t), y(t)) = (r\cos(\omega t + \alpha))$, $(r\sin(\omega t + \alpha))$

$\Big($ または $x(t) = r\cos(\omega t + \alpha)$, $y(t) = r\sin(\omega t + \alpha)\Big)$

< 39ページ. 等速円運動 2 >

問の解答

$v_x = -r\sin t$, $a_x = -r\cos t$

$v_y = r\cos t$, $a_y = -r\sin t$

＜ 40 ページ. 等速円運動 3 ＞

問１の解答

$$v_x = \frac{dx}{dt} = -6\sin(3t) \quad , \quad v_y = \frac{dy}{dt} = 6\cos(3t)$$

$$a_x = \frac{dv_x}{dt} = -18\cos(3t) \quad , \quad a_y = \frac{dv_y}{dt} = -18\sin(3t)$$

問２の解答

$$v_x = \frac{dx}{dt} = -\omega r\sin(\omega t) \quad , \quad v_y = \frac{dy}{dt} = \omega r\cos(\omega t)$$

$$a_x = \frac{dv_x}{dt} = -\omega^2 r\cos(\omega t) \quad , \quad a_y = \frac{dv_y}{dt} = -\omega^2 r\sin(\omega t)$$