高知工科大学

基礎数学ワークブック

(2001年度版)

Series A

No. 7

解答

＜１ページ. 指数の復習＞

問１の解答

(1) $8^{\frac{1}{3}} = 2$

(2) $5^0 = 1$

(3) $2^{-3} = \dfrac{1}{8}$

(4) $9^{-\frac{1}{2}} = \dfrac{1}{3}$

(5) $64^{\frac{4}{3}} = 256$

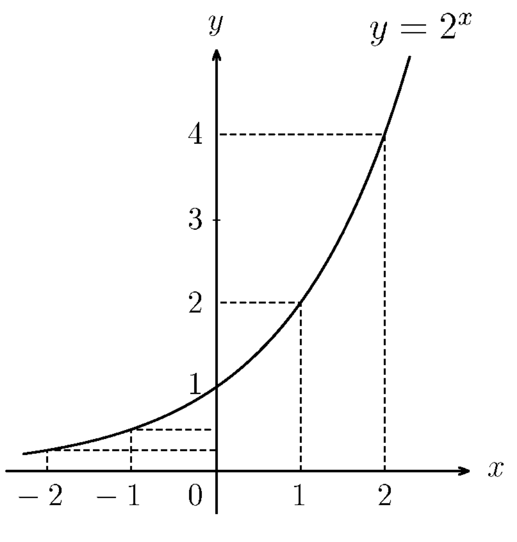
(6) $27^{-\frac{2}{3}} = \dfrac{1}{9}$

(7) $\left(\dfrac{1}{9}\right)^{\frac{1}{2}} = \dfrac{1}{3}$

(8) $(0.0001)^{-0.75} = \left(\dfrac{1}{10000}\right)^{-\frac{3}{4}} = 1000$

問２の解答

x	-2	-1	0	1	2
2^x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



＜ 2ページ. 対数の復習 ＞

問１の解答

(1) $\log_2 16 = 4$

(2) $\log_7 49 = 2$

(3) $\log_2 1 = 0$

(4) $\log_3 \left(\frac{1}{9}\right) = -2$

(5) $\log_{81} 3 = \frac{1}{4}$

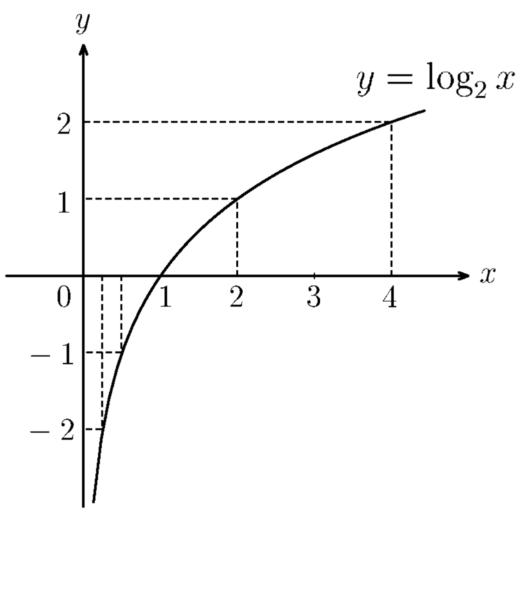
(6) $\log_{32} 4 = \frac{2}{5}$

(7) $\log_{10}(0.01) = -2$

(8) $\log_5(0.2) = -1$

問２の解答

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$\log_2 x$	− 2	− 1	0	1	2

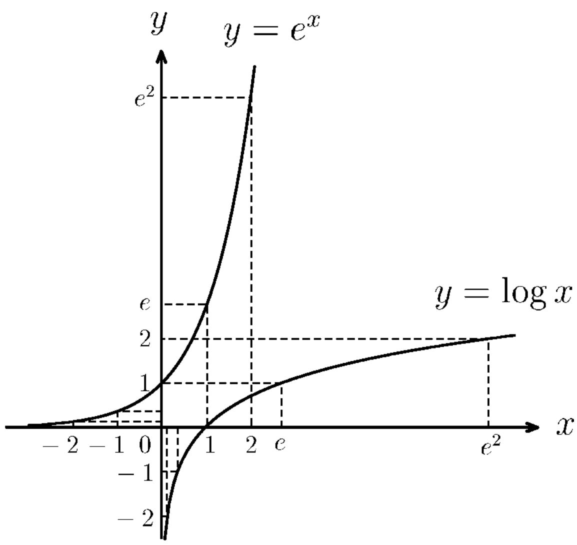


＜ 3ページ.e の復習 ＞

問の解答

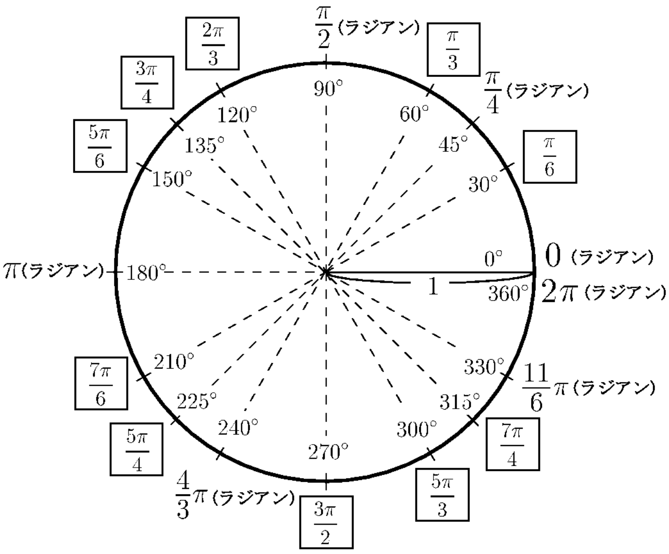
x	-2	-1	0	1	2
e^x	$\frac{1}{e^2}$	$\frac{1}{e}$	1	e	e^2

x	$\frac{1}{e^2}$	$\frac{1}{e}$	1	e	e^2
$\log x$	-2	-1	0	1	2



＜ 4ページ. 三角関数の復習 1 ＞

問 1 の解答



問 2 の解答

角度 θ	度数法	0°	30°	45°	60°	90°	120°	135°	150°	180°
	弧度法	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$		0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$		1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$		0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	╳	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

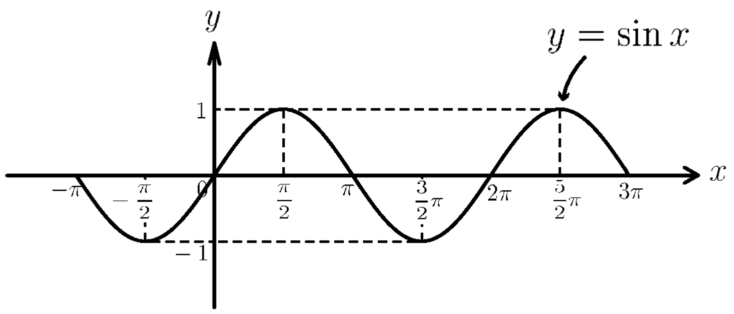
角度 θ	度数法	210°	225°	240°	270°	300°	315°	330°	360°
	弧度法	$\frac{7}{6}\pi$	$\frac{5\pi}{4}$	$\frac{4}{3}\pi$	$\frac{3\pi}{2}$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11\pi}{6}$	2π
$\sin\theta$		$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\theta$		$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$		$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	╳	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

＜ 5ページ. 三角関数の復習 2 ＞

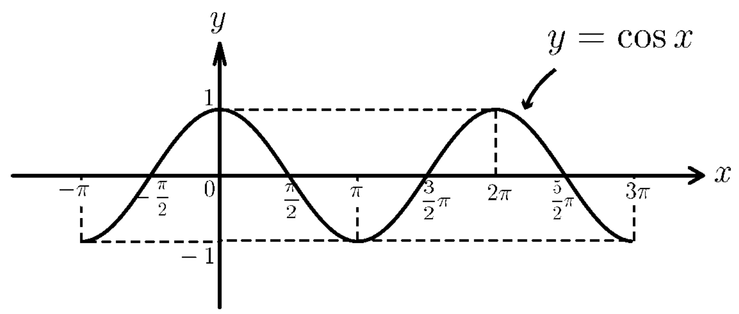
問の解答

x																		
x	度数法	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	225°	270°	315°	360°	405°	450°	495°	540°
	弧度法	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3}{4}\pi$	π	$\frac{5\pi}{4}$	$\frac{3}{2}\pi$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5}{2}\pi$	$\frac{11\pi}{4}$	3π
$\sin x$		0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\cos x$		-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1

(1) $y = \sin x$

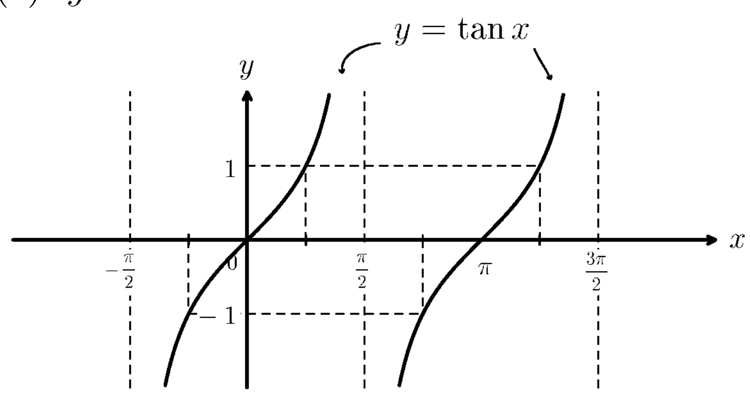


(2) $y = \cos x$



x																		
x	度数法	-90°	-60°	-45°	-30°	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°
	弧度法	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3}{4}\pi$	$\frac{5\pi}{6}$	π	$\frac{7}{6}\pi$	$\frac{5\pi}{4}$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$
$\tan x$		×	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	×	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	×

(3) $y = \tan x$



＜ 6 ページ. 微分の復習 1 ＞

問の解答

(1) $(x^3 + x^2)' = 3x^2 + 2x$

(2) $(3x^4 - 2x + 1)' = 12x^3 - 2$

(3) $(\sqrt[5]{x})' = \dfrac{1}{5\sqrt[5]{x^4}}$

(4) $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$

(5) $\left(\dfrac{1}{\sqrt{x}}\right)' = -\dfrac{1}{2x\sqrt{x}}$

(6) $(x^2\sqrt{x})' = \dfrac{5}{2}x\sqrt{x}$

(7) $\left(\dfrac{2}{x^3}\right)' = -\dfrac{6}{x^4}$

(8) $\left(\dfrac{x^3 - 2x^2 - 1}{x^2}\right)' = \left(x - 2 - \dfrac{1}{x^2}\right) = 1 + \dfrac{2}{x^3} \quad \left(= \dfrac{x^3 + 2}{x^3}\right)$

(9) $\left(\dfrac{x^2 - x}{\sqrt{x}}\right)' = (x\sqrt{x} - \sqrt{x})' = \dfrac{3}{2}\sqrt{x} - \dfrac{1}{2\sqrt{x}} \quad \left(= \dfrac{3x - 1}{2\sqrt{x}}\right)$

＜ 7ページ. 微分の復習 2 ＞

問の解答

(1) $(x\cos x)' = \cos x - x\sin x$

(2) $(e^x \sin x)' = e^x \sin x + e^x \cos x$

(3) $(xe^x)' = e^x + xe^x$

(4) $(x - x\log x)' = 1 - \left(1\log x + x \times \dfrac{1}{x}\right) = -\log x$

(5) $\left(\dfrac{1}{1-x}\right)' = -\dfrac{-1}{(1-x)^2} = \dfrac{1}{(1-x)^2}$

(6) $\left(\dfrac{x}{1-x}\right)' = \left(\dfrac{1}{1-x} - 1\right)' = \dfrac{1}{(1-x)^2}$

(7) $\left(\dfrac{2}{x^2+x}\right)' = 2 \times \left(-\dfrac{(x^2+x)'}{(x^2+x)^2}\right) = -\dfrac{2(2x+1)}{(x^2+x)^2} = -\dfrac{4x+2}{(x^2+x)^2}$

(8) $\left(\dfrac{1}{e^x}\right)' = \left(e^{-x}\right)' = -e^{-x} = -\dfrac{1}{e^x}$

(9) $\left(\dfrac{\cos x}{\sin x}\right)' = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \dfrac{-1}{\sin^2 x}$

$<$ 8ページ.微分の復習 3 $>$

問の解答

(1) $\bigl(\cos(1-x)\bigr)' = -\sin(1-x)\times(-1) = \sin(1-x)$

(2) $\bigl(\sin(x^2+2)\bigr)' = \cos(x^2+2)\times(2x) = 2x\cos(x^2+2)$

(3) $\left(e^{x^3}\right)' = 3x^2e^{x^3}$

(4) $\bigl((2x+1)^5\bigr)' = 5\times(2x+1)^4\times 2 = 10(2x+1)^4$

(5) $\bigl(\sqrt{2x-5}\bigr)' = \dfrac{1}{\sqrt{2x-5}}$

(6) $\bigl(\sin(2x)+e^{x-1}\bigr)' = 2\cos(2x)+e^{x-1}$

(7) $\bigl(e^{2x}\cos(2x)\bigr)' = 2e^{2x}\cos(2x) - 2e^{2x}\sin(2x)$

(8) $\bigl(e^{1-x}\sin(3x)\bigr)' = -e^{1-x}\sin(3x) + 3e^{1-x}\cos(3x)$

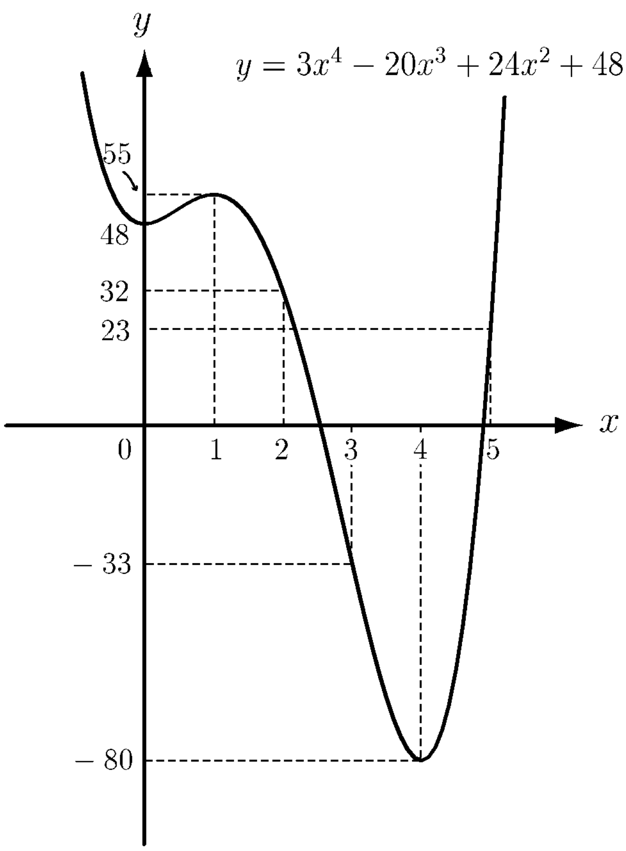
(9) $\bigl(\cos(2x)\sin(-3x)\bigr)' = -2\sin(2x)\sin(-3x) - 3\cos(2x)\cos(-3x)$

＜ 9ページ. 接線の傾き 1 ＞

問の解答

$$
\begin{aligned}
f'(x) &= 12x^3 - 60x^2 + 48x \\
&= 12x(x^2 - 5x + 4) \\
&= 12x(x-1)(x-4)
\end{aligned}
$$

$f(5) = 23$	$f'(5) = 240$
$f(4) = -80$	$f'(4) = 0$
$f(3) = -33$	$f'(3) = -72$
$f(2) = 32$	$f'(2) = -48$
$f(1) = 55$	$f'(1) = 0$
$f(0) = 48$	$f'(0) = 0$
$f(-1) = 95$	$f'(-1) = -120$



＜ 10 ページ. 接線の傾き 2 ＞

問の解答

(1) $y = e^x$ の $x = 1$ における接線の傾き $= e^1 = e$

$$(y' = e^x)$$

(2) $y = e^x$ の $x = -1$ における接線の傾き $= e^{-1} = \dfrac{1}{e}$

$$(y' = e^x)$$

(3) $f(x) = \sin x$ の $x = 0$ における接線の傾き $= \cos 0 = 1$

$$(f'(x) = \cos x)$$

(4) $f(x) = \sin x$ の $x = \pi$ における接線の傾き $= \cos \pi = -1$

$$(f'(x) = \cos x)$$

(5) $f(x) = \cos x$ の $x = 0$ における接線の傾き $= -\sin 0 = 0$

$$(f'(x) = -\sin x)$$

(6) $f(x) = \cos x$ の $x = \dfrac{\pi}{2}$ における接線の傾き $= -\sin\left(\dfrac{\pi}{2}\right) = -1$

$$(f'(x) = -\sin x)$$

(7) $f(x) = \log x$ の $x = 1$ における接線の傾き $= \dfrac{1}{1} = 1$

$$\left(f'(x) = \frac{1}{x}\right)$$

(8) $f(x) = \log x$ の $x = 2$ における接線の傾き $= \dfrac{1}{2}$

$$\left(f'(x) = \frac{1}{x}\right)$$

＜ 11 ページ. 接線の方程式 1 ＞

問 1 の解答

(答) $y = m(x-a) + b$

問 2 の解答

$y' = 3x^2 - 2x$ $x = 1$ のとき $y' = 3 - 2 = 1$

接線 $y = 1(x-1) + 1$ (答) $y = x$

問 3 の解答

(答) $y = f'(a)\big(x-a\big) + b$

$<$ 12ページ. 接線の方程式 2 $>$

問の解答

(1) $y = e^x$ の $x = 0$ における接線

$y' = e^x$　傾き $e^0 = 1$

$x = 1$ のとき $y = 1$　接線 $y = 1(x - 0) + 1$

(答) $y = x + 1$

(2) $y = \log x$ の $x = 1$ における接線

$y' = \dfrac{1}{x}$　傾き $\dfrac{1}{1} = 1$

$x = 1$ のとき $y = 0$　接線 $y = 1(x - 1) + 0$

(答) $y = x - 1$

(3) $y = \sin x$ の $x = \dfrac{\pi}{6}$ における接線

$y' = \cos x$　傾き $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{6}$ のとき $y = \dfrac{1}{2}$　接線 $y = \dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{6}\right) + \dfrac{1}{2}$

(答) $y = \dfrac{\sqrt{3}}{2}x - \dfrac{\sqrt{3}}{12}\pi + \dfrac{1}{2}$

(4) $y = \sqrt{x}$ の $x = 4$ における接線

$y' = \dfrac{1}{2\sqrt{x}}$　傾き $\dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

$x = 4$ のとき $y = 2$　接線 $y = \dfrac{1}{4}(x - 4) + 2$

(答) $y = \dfrac{1}{4}x + 1$

(5) $y = \sqrt[3]{x}$ の $x = 1$ における接線

$y' = \dfrac{1}{3\sqrt[3]{x^2}}$　傾き $\dfrac{1}{3\sqrt[3]{1^2}} = \dfrac{1}{3}$

$x = 1$ のとき $y = 1$　接線 $y = \dfrac{1}{3}(x - 1) + 1$

(答) $y = \dfrac{1}{3}x + \dfrac{2}{3}$

< 13 ページ. 関数の一次近似 >

問の解答

$f(x) = \sqrt{x}$, $f'(x) = \dfrac{1}{2\sqrt{x}}$

一次近似式 $\boxed{x \fallingdotseq a \text{ のとき } \sqrt{x} \fallingdotseq \sqrt{a} + \dfrac{1}{2\sqrt{a}}(x - a)}$

$x = 1.2$, $a = 1$

$$\underline{\sqrt{1.2} \fallingdotseq \sqrt{1} + \frac{1}{2\sqrt{1}}(1.2 - 1) = 1 + \frac{1}{2} \times 0.2 = 1.1}$$

< 14ページ. 高階導関数 >

問１の解答

(1) $f(x) = x^2 - 3x + 2$

$f'(x) = 2x - 3$

$f''(x) = 2$

(2) $f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

(3) $f(x) = \log x$

$f'(x) = \dfrac{1}{x}$

$f''(x) = -\dfrac{1}{x^2}$

問２の解答

(1) $f(x) = x^5 - x^3 + x$

$f'(x) = 5x^4 - 3x^2 + 1$

$f''(x) = 20x^3 - 6x$

$f'''(x) = 60x^2 - 6$

(2) $f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

(3) $f(x) = x\log x - \dfrac{1}{x}$

$f'(x) = 1\log x + x \times \dfrac{1}{x} + \dfrac{1}{x^2} = \log x + 1 + \dfrac{1}{x^2}$

$f''(x) = \dfrac{1}{x} - \dfrac{2}{x^3}$

$f'''(x) = -\dfrac{1}{x^2} + \dfrac{6}{x^4}$

(4) $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

$f''(x) = 4e^{2x}$

$f'''(x) = 8e^{2x}$

< 15ページ.グラフの凹凸 1 >

問の解答

(1) $y = x^4 - 2x^3 - 12x^2 + 24$

$y' = 4x^3 - 6x^2 - 24x$

$y'' = 12x^2 - 12x - 24$

$\quad = 12(x^2 - x - 2) = 12(x-2)(x+1)$

x	$\cdots$	-1	$\cdots$	2	$\cdots$
y''	$+$	0	$-$	0	$+$
y	凹	15	凸	-24	凹

(2) $y = 3x^5 - 10x^3 + 6x$

$y' = 15x^4 - 30x^2 + 6$

$y'' = 60x^3 - 60x$

$\quad = 60x(x^2 - 1)$

$\quad = 60x(x-1)(x+1)$

x	$\cdots$	-1	$\cdots$	0	$\cdots$	1	$\cdots$
y''	$-$	0	$+$	0	$-$	0	$+$
y	凸	1	凹	0	凸	-1	凹

< 17ページ. グラフの凹凸 3 >

問の解答

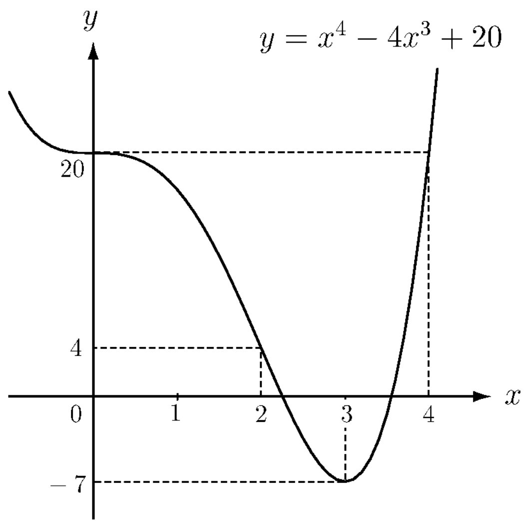
(解) $y' = 4x^3 - 12x^2$

$= 4x^2(x-3)$

$y'' = 12x^2 - 24x$

$= 12x(x-2)$

x	$\cdots$	0	$\cdots$	2	$\cdots$	3	$\cdots$
y'	$-$	0	$-$	$-$	$-$	0	$+$
y''	$+$	0	$-$	0	$+$	$+$	$+$
y	↘	20	↘	4	↘	-7	↗



< 18 ページ. 微分係数と極限値 >

問の解答

(1) $\displaystyle \lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = f'(0) = e^0 = 1$

$(f(x) = e^x \ , \ f'(x) = e^x \ , \ f'(0) = e^0)$

(2) $\displaystyle \lim_{x \to 2} \frac{x^6 - 64}{x - 2} = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = f'(2) = 6 \times 2^5 = 192$

$(f(x) = x^6 \ , \ f'(x) = 6x^5)$

(3) $\displaystyle \lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}} = \lim_{x \to \frac{\pi}{2}} \frac{f(x) - f\left(\frac{\pi}{2}\right)}{x - \frac{\pi}{2}} = f'\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$(f(x) = \sin x \ , \ f'(x) = \cos x)$

< 19 ページ. ロピタルの定理 1 >

問の解答

(1) $\displaystyle \lim_{x\to 2}\frac{x^4-16}{x-2}=\lim_{x\to 2}\frac{4x^3-0}{1-0}=4\times 2^3=32$

(2) $\displaystyle \lim_{x\to 1}\frac{e^x-e}{x-1}=\lim_{x\to 1}\frac{e^x-0}{1-0}=e^1=e$

(3) $\displaystyle \lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\cos x}{1}=\cos 0=1$

$<$ 20 ページ. ロピタルの定理 2 $>$

問の解答

(1) $\displaystyle \lim_{x\to1}\frac{x^5-1-5(x-1)}{(x-1)^2}=\lim_{x\to1}\frac{5x^4-5}{2(x-1)}=\lim_{x\to1}\frac{20x^3}{2}=\frac{20}{2}=10$

(2) $\displaystyle \lim_{x\to2}\frac{x^5-2^5-5\times2^4(x-2)}{(x-2)^2}=\lim_{x\to2}\frac{5x^4-80}{2(x-2)}=\lim_{x\to2}\frac{20x^3}{2}=\frac{20\times8}{2}=80$

(3) $\displaystyle \lim_{x\to1}\frac{x^4-1-4(x-1)-6(x-1)^2}{(x-1)^3}=\lim_{x\to1}\frac{4x^3-4-12(x-1)}{3(x-1)^2}$

$\displaystyle =\lim_{x\to1}\frac{12x^2-12}{6(x-1)}=\lim_{x\to1}\frac{24x}{6}=\frac{24}{6}=4$

(4) $\displaystyle \lim_{x\to1}\frac{x^5-1-5(x-1)-10(x-1)^2-10(x-1)^3}{(x-1)^4}$

$\displaystyle =\lim_{x\to1}\frac{5x^4-5-20(x-1)-30(x-1)^2}{4(x-1)^3}$

$\displaystyle =\lim_{x\to1}\frac{20x^3-20-60(x-1)}{12(x-1)^2}=\lim_{x\to1}\frac{60x^2-60}{24(x-1)}$

$\displaystyle =\lim_{x\to1}\frac{120x}{24}=\frac{120}{24}=5$

＜ 21 ページ. ロピタルの定理 3 ＞

問の解答

(1) $\displaystyle \lim_{x\to a}\frac{f(x)-f(a)-f'(a)(x-a)-\frac{1}{2}f''(a)(x-a)^2}{(x-a)^3}$

$\displaystyle =\lim_{x\to a}\frac{f'(x)-f'(a)-f''(a)(x-a)}{3(x-a)^2}$

$\displaystyle =\lim_{x\to a}\frac{f''(x)-f''(a)}{6(x-a)}$

$\displaystyle =\lim_{x\to a}\frac{f'''(x)}{6}$

$\displaystyle =\frac{1}{6}f'''(a)$

(2) $\displaystyle \lim_{x\to a}\frac{f(x)-f(a)-f'(a)(x-a)-\frac{1}{2}f''(a)(x-a)^2-\frac{1}{6}f'''(a)(x-a)^3}{(x-a)^4}$

$\displaystyle =\lim_{x\to a}\frac{f'(x)-f'(a)-f''(a)(x-a)-\frac{1}{2}f'''(a)(x-a)^2}{4(x-a)^3}$

$\displaystyle =\lim_{x\to a}\frac{f''(x)-f''(a)-f'''(a)(x-a)}{12(x-a)^2}$

$\displaystyle =\lim_{x\to a}\frac{f'''(x)-f'''(a)}{24(x-a)}$

$\displaystyle =\lim_{x\to a}\frac{f''''(x)}{24}$

$\displaystyle =\frac{1}{24}f''''(a)$

< 22ページ. 関数の高次近似 >

問の解答

(解) 4次近似式

$$x \fallingdotseq a \text{ のとき } f(x) \fallingdotseq f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3 + \frac{1}{24}f''''(a)(x-a)^4$$

< 23 ページ. 高階微分係数 >

問の解答

(1) $f^{(4)}(x) = e^x$, $f^{(4)}(0) = e^0 = 1$

(2) $f^{(n)}(x) = e^x$, $f^{(n)}(0) = e^0 = 1$

(3)

$f^{(1)}(x) = \cos x$	$f^{(1)}(0) = \cos 0 = 1$
$f^{(2)}(x) = -\sin x$	$f^{(2)}(0) = -\sin 0 = 0$
$f^{(3)}(x) = -\cos x$	$f^{(3)}(0) = -\cos 0 = -1$
$f^{(4)}(x) = \sin x$	$f^{(4)}(0) = \sin 0 = 0$
$f^{(5)}(x) = \cos x = f^{(1)}(x)$	$f^{(5)}(0) = f^{(1)}(0) = 1$
$f^{(6)}(x) = -\sin x = f^{(2)}(x)$	$f^{(6)}(0) = f^{(2)}(0) = 0$
$f^{(7)}(x) = -\cos x = f^{(3)}(x)$	$f^{(7)}(0) = f^{(3)}(0) = -1$
$f^{(8)}(x) = \sin x = f^{(4)}(x)$	$f^{(8)}(0) = f^{(4)}(0) = 0$

< 25 ページ. 関数の n 次近似 2 >

問の解答

$f^{(n)}(x) = e^x$, $f^{(n)}(a) = e^a$

$x \fallingdotseq a$ のとき

$$e^x \fallingdotseq e^a + e^a(x-a) + \frac{e^a}{2!}(x-a)^2 + \frac{e^a}{3!}(x-a)^3 + \cdots + \frac{e^a}{n!}(x-a)^n$$

< 26 ページ. テーラー展開 >

問 1 の解答

$$e^x = e^a + e^a(x-a) + \frac{1}{2!}e^a(x-a)^2 + \frac{1}{3!}e^a(x-a)^3 + \frac{1}{4!}e^a(x-a)^4 + \cdots$$
$$\cdots + \frac{1}{n!}e^a(x-a)^n + \cdots$$

問 2 の解答

(1) $a = 1$

$$e^x = e + e(x-1) + \frac{1}{2!}e(x-1)^2 + \frac{1}{3!}e(x-1)^3 + \frac{1}{4!}e(x-1)^4 + \cdots$$
$$\cdots + \frac{1}{n!}e(x-1)^n + \cdots$$

(2) $a = 0$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots + \frac{1}{n!}x^n + \cdots$$

< 27 ページ. マクローリン展開 1 >

問の解答

$f^{(1)}(0) = 1$, $f^{(2)}(0) = 0$, $f^{(3)}(0) = -1$, $f^{(4)}(0) = 0$

$f^{(5)}(0) = 1$, $f^{(6)}(0) = 0$, $f^{(7)}(0) = -1$, $f^{(8)}(0) = 0$

$f(0) = \sin 0 = 0$ より

(答) $\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} + \cdots$

< 29 ページ. マクローリン展開 3 >

問の解答

$$e \fallingdotseq 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = 2 + \frac{12 + 4 + 1}{24} = 2 + \frac{17}{24} \fallingdotseq 2.708$$

< 30ページ. 有理数 >

問の解答

(1) $\dfrac{1}{4} = 0.25$

(2) $\dfrac{2}{3} = 0.6666\cdots = 0.\dot{6}$

(3) $\dfrac{1}{15} = 0.0666\cdots = 0.0\dot{6}$

< 31 ページ. 実数 >

問の解答

(1) $\sqrt{3}$ 無理数

(2) $\dfrac{e}{2.18}$ 無理数

(3) $\dfrac{1}{3}$ 有理数

(4) $\sin\left(\dfrac{\pi}{2}\right)$ 有理数

(5) $\dfrac{13}{100}$ 有理数

< 32ページ. 虚数の導入 1 >

問の解答

(1) $x^2 = -25$

$$x = \pm 5i$$

(2) $4x^2 = -9$

$$x = \pm \frac{3}{2} i$$

(3) $6x^2 = -2$

$$x^2 = -\frac{1}{3}$$

$$x = \pm \frac{\sqrt{3}}{3} i$$

< 33 ページ. 虚数の導入 2 >

問の解答

(1) $\left(x-\dfrac{1}{2}\right)^2=-2$

$x-\dfrac{1}{2}=\pm\sqrt{2}i$

(答) $x=\dfrac{1}{2}\pm\sqrt{2}i$

(2) $x^2-6x+12=0$

$(x-3)^2+3=0$

$x-3=\pm\sqrt{3}i$

(答) $x=3\pm\sqrt{3}i$

(3) $(x-a)^2=-\dfrac{c^2}{b^2}$

$x-a=\pm\dfrac{c}{b}i$

(答) $x=a\pm\dfrac{c}{b}i$

＜ 34 ページ. 複素数の定義 ＞

問の解答

(1) $a + bi = \dfrac{1 - 3i}{2}$

$a = \dfrac{1}{2}$

$b = -\dfrac{3}{2}$

(2) $a + bi = \dfrac{1 + \sqrt{2}}{3} i$

$a = 0$

$b = \dfrac{1 + \sqrt{2}}{3}$

< 35 ページ. 複素数の四則演算 1 >

問 1 の解答

(1) $(1+i)+(1-i)=2$

(2) $(2-i)-(2-3i)=2i$

(3) $\left(0.25+\frac{1}{2}i\right)+\left(\frac{3}{4}+0.5i\right)=1+i$

(4) $\left(\frac{1}{2}-\frac{1}{3}i\right)-\left(\frac{1}{2}-\frac{1}{3}i\right)=0$

(5) $\left(\sqrt{2}-i\right)+(1+2i)=\sqrt{2}+1+i$

(6) $\left(\frac{1}{2}-\sqrt{2}i\right)-\left(\frac{1}{3}-\sqrt{3}i\right)=\frac{1}{6}+\left(\sqrt{3}-\sqrt{2}\right)i$

問 2 の解答

(1) $3(2-i)=6-3i$

(2) $\sqrt{2}\left(\frac{1}{4}-\frac{1}{2}i\right)=\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{2}i$

(3) $2(6-2i)-5(2-i)=2+i$

(4) $\sqrt{3}\left(\frac{1}{\sqrt{3}}-\sqrt{3}i\right)-\left(\frac{1}{3}-2i\right)=\frac{2}{3}-i$

< 36 ページ. 複素数の四則演算 2 >

問の解答

(1) $i^3 = -i$

(2) $i^4 = 1$

(3) $i^5 = i$

(4) $i^6 = -1$

(5) $i^7 = -i$

(6) $i^8 = 1$

(7) $(1+i)(1-i) = 1 - i^2 = 2$

(8) $(2+\sqrt{3}i)(2-\sqrt{3}i) = 4 + 3 = 7$

(9) $\left(\frac{\sqrt{2}+i}{3}\right)\left(\frac{\sqrt{2}-i}{3}\right) = \frac{1}{3}$

(10) $(1+i)^2 = 1 + 2i + i^2 = 2i$

(11) $(1-i)^2 = 1 - 2i + i^2 = -2i$

(12) $(5+2i)(3-i) = 15 + 6i - 5i - 2i^2 = 17 + i$

(13) $(3-2i)(1-i) = 3 - 2i - 3i + 2i^2 = 1 - 5i$

(14) $(2-i)^3 = 8 - 12i + 6i^2 - i^3 = 8 - 12i - 6 + i = 2 - 11i$

< 37ページ. 複素数の四則演算 3 >

問の解答

(1) $\dfrac{i}{1+i} = \dfrac{1-i}{1^2-i^2} = \dfrac{1-i}{2}$

(2) $\dfrac{1}{1-i} = \dfrac{1+i}{1^2-i^2} = \dfrac{1+i}{2}$

(3) $\dfrac{i}{1-i} = \dfrac{i(1+i)}{1^2-i^2} = \dfrac{i-1}{2} = -\dfrac{1}{2} + \dfrac{i}{2}$

(4) $\dfrac{3}{\sqrt{2}-i} = \dfrac{3(\sqrt{2}+i)}{2-i^2} = \sqrt{2}+i$

(5) $\dfrac{7}{2+\sqrt{3}i} = \dfrac{7(2-\sqrt{3}i)}{4-3i^2} = 2-\sqrt{3}i$

(6) $\dfrac{i}{1+i} = \dfrac{i(1-i)}{1^2-i^2} = \dfrac{i+1}{2} \quad \left(= \dfrac{1}{2} + \dfrac{i}{2}\right)$

(7) $\dfrac{1}{\sqrt{2}i(\sqrt{2}+i)} = \dfrac{i(\sqrt{2}-i)}{\sqrt{2}i^2(2-i^2)} = \dfrac{\sqrt{2}i-i^2}{-\sqrt{2}(2+1)} = -\dfrac{\sqrt{2}i+1}{3\sqrt{2}} = -\dfrac{\sqrt{2}}{6} - \dfrac{1}{3}i$

(8) $\dfrac{\sqrt{3}}{\sqrt{3}-i} = \dfrac{\sqrt{3}(\sqrt{3}+i)}{3-i^2} = \dfrac{3}{4} + \dfrac{\sqrt{3}}{4}i$

(9) $\dfrac{1}{(1-i)^2} = \dfrac{1}{1-2i+i^2} = -\dfrac{1}{2i} = -\dfrac{i}{2i^2} = \dfrac{i}{2}$

(10) $\dfrac{4}{(1+i)^4} = \dfrac{4}{(1+2i+i^2)^2} = \dfrac{4}{(+2i)^2} = \dfrac{4}{4i^2} = -1$

< 38 ページ. 負の数の平方根 >

問の解答

(1) $\sqrt{(-1)\times(-2)\times(-3)} = \sqrt{-6} = \sqrt{6}i$

(2) $\sqrt{-1}\times\sqrt{-2}\times\sqrt{-3} = i\times\sqrt{2}i\times\sqrt{3}i = \sqrt{6}i^3 = -\sqrt{6}i$

(3) $\dfrac{\sqrt{5}}{\sqrt{-4}} = \dfrac{\sqrt{5}}{2i} = \dfrac{\sqrt{5}i}{2i^2} = -\dfrac{\sqrt{5}}{2}i$

(4) $\sqrt{\dfrac{5}{-4}} = \sqrt{\dfrac{5}{4}}i = \dfrac{\sqrt{5}}{2}i$

< 39 ページ.2 次方程式 >

問の解答

(1) $x^2+3x+3=0$

$$x=\frac{-3\pm\sqrt{9-12}}{2}=\frac{-3\pm\sqrt{3}i}{2}$$

(2) $x^2+3x+6=0$

$$x=\frac{-3\pm\sqrt{9-24}}{2}=\frac{-3\pm\sqrt{15}i}{2}$$

(3) $2x^2-x+3=0$

$$x=\frac{1\pm\sqrt{1-24}}{4}=\frac{1\pm\sqrt{23}i}{4}$$

< 40 ページ.2 次式の因数分解 >

問の解答

(1) $x^2 - 2x + 3 = \left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$

(2) $-2x^2 + 4x - 3 = -2\left(x - 1 - \dfrac{\sqrt{2}}{2}i\right)\left(x - 1 + \dfrac{\sqrt{2}}{2}i\right)$

(3) $3x^2 + 3x + 3 = 3(x^2 + x + 1)$

$$= 3\left(x - \frac{-1 + \sqrt{3}i}{2}\right)\left(x - \frac{-1 - \sqrt{3}i}{2}\right)$$

$$= 3\left(x + \frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\left(x + \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$