高知工科大学

基礎数学ワークブック

(2001年度版)

Series A

No. 6

解答

＜１ページ. 合成関数の不定積分 １＞

問１の解答

(微分) ⟺ (不定積分)

(1) $\left(e^{f(x)}\right)' = e^{f(x)} \times f'(x) \iff \int e^{f(x)} \times f'(x)dx = e^{f(x)} + C$

(2) $\left(\sin(f(x))\right)' = \cos(f(x)) \times f'(x) \iff \int \cos\big(f(x)\big) \times f'(x)dx = \sin\big(f(x)\big) + C$

(3) $\left(\cos(f(x))\right)' = -\sin(f(x)) \times f'(x) \iff \int \left\{-\sin\big(f(x)\big)f'(x)\right\} dx = \cos\big(f(x)\big) + C$

問２の解答

(1) $\int \frac{4x^3+5}{x^4+5x}dx = \log|x^4+5x| + C$

(2) $\int \frac{\cos x}{\sin x}dx = \log|\sin x| + C$

(3) $\int (2x+3)e^{x^2+3x}dx = e^{x^2+3x} + C$

(4) $\int (-x)e^{-\frac{x^2}{2}}dx = e^{-\frac{x^2}{2}} + C$

(5) $\int 4x^3\cos(x^4+3)dx = \sin(x^4+3) + C$

(6) $\int (-3x+2)\sin\left(\frac{3}{2}x^2-2x\right)dx$

$= \cos\left(\frac{3}{2}x^2-2x\right) + C$

＜ 2 ページ. 合成関数の不定積分 2 ＞

問 1 の解答

(1) $\left(\frac{1}{8}(f(x))^8\right)' = \big(f(x)\big)^7 \times f'(x) \iff \int (f(x))^7 \times f'(x)dx = \frac{1}{8}\big(f(x)\big)^8 + C$

(2) $\left(\frac{1}{n+1}(f(x))^{n+1}\right)' = \big(f(x)\big)^n \times f'(x) \iff \int (f(x))^n \times f'(x)dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + C$

問 2 の解答

(1) $\int (x^3+5x^2)^7(3x^2+10x)dx$

$= \frac{1}{8}(x^3+5x^2)^8 + C$

(2) $\int (2+\sin x)^5 \cos x dx$

$= \frac{1}{6}(2+\sin x)^6 + C$

(3) $\int (2x+1)\sqrt{x^2+x}dx$

$= \frac{1}{\frac{1}{2}+1}\left(x^2+x\right)^{\frac{1}{2}+1} + C$

$= \frac{2}{3}(x^2+x)\sqrt{x^2+x} + C$

(4) $\int (3x^2+5)\sqrt[4]{x^3+5x}dx$

$= \frac{1}{\frac{1}{4}+1}\left(x^3+5x\right)^{\frac{1}{4}+1} + C$

$= \frac{4}{5}(x^3+5x)\sqrt[4]{x^3+5x} + C$

(5) $\int \frac{5x^4+7}{(x^5+7x)^4}dx$

$= \frac{1}{-4+1}\left(x^5+7x\right)^{-3} + C$

$= -\frac{1}{3(x^5+7x)^3} + C$

(6) $\int \frac{6x}{\sqrt{3x^2+5}}dx$

$= \int 6x(3x^2+5)^{-\frac{1}{2}}dx$

$= \frac{1}{-\frac{1}{2}+1}\left(3x^2+5\right)^{\frac{1}{2}} + C$

$= 2\sqrt{3x^2+5} + C$

$<$ 3 ページ. 合成関数の定積分 $>$

問 1 の解答

(1) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(2x)dx = \left[\frac{1}{2}\sin(2x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$
$$= \frac{1}{2}\sin(\pi) - \frac{1}{2}\sin(-\pi)$$
$$= 0$$

(2) $$\int_{0}^{\frac{\pi}{2}} \sin(4x-\pi)dx = \left[-\frac{1}{4}\cos(4x-\pi)\right]_{0}^{\frac{\pi}{2}}$$
$$= -\frac{1}{4}\cos(\pi) + \frac{1}{4}\cos(-\pi)$$
$$= 0$$

(3) $$\int_{0}^{1} (3x-1)^4 dx = \left[\frac{1}{15}(3x-1)^5\right]_{0}^{1}$$
$$= \frac{32}{15} - \frac{-1}{15}$$
$$= \frac{33}{15}$$
$$= \frac{11}{5}$$

(4) $$\int_{0}^{3} \frac{1}{2x+1}dx = \left[\frac{1}{2}\log 2x+1||\right]_{0}^{3}$$
$$= \frac{1}{2}\log 7$$

(5) $$\int_{0}^{2} e^{4x-1}dx = \left[\frac{1}{4}e^{4x-1}\right]_{0}^{2}$$
$$= \frac{1}{4}e^7 - \frac{1}{4e}$$

(6) $$\int_{0}^{1} \frac{1}{(2x+1)^3}dx = \left[-\frac{1}{4(2x+1)^2}\right]_{0}^{1}$$
$$= -\frac{1}{4\times 9} + \frac{1}{4}$$
$$= \frac{-1+9}{36}$$
$$= \frac{2}{9}$$

問 2 の解答

(1) $$\int_{0}^{1} \frac{3x^2+2x}{x^3+x^2+1}dx = \left[\log|x^3+x^2+1|\right]_{0}^{1}$$
$$= \log 3$$

(2) $$\int_{0}^{3} 2xe^{x^2}dx = \left[e^{x^2}\right]_{0}^{3}$$
$$= e^9 - 1$$

(3) $$\int_{0}^{1} (x^3+x)^4(3x^2+1)dx = \left[\frac{1}{5}(x^3+x)^5\right]_{0}^{1}$$
$$= \frac{32}{5}$$

(4) $$\int_{0}^{2} 3x^2\sqrt{1+x^3}dx = \left[\frac{2}{3}(1+x^3)\sqrt{1+x^3}\right]_{0}^{2}$$
$$= \frac{2}{3}(9\sqrt{9}-1)$$
$$= \frac{2\times 26}{3}$$
$$= \frac{52}{3}$$

< 4ページ. 積分記号 >

解答

(1) $\displaystyle\int (10 - 9.8t)dt = 10t - 4.9t^2 + C$

(2) $\displaystyle\int 4\pi r^2 dr = \frac{4\pi}{3} r^3 + C$

(3) $\displaystyle\int e^u du = e^u + C$

(4) $\displaystyle\int \frac{1}{y} dy = \log|y| + C$

(5) $\displaystyle\int \cos u du = \sin u + C$

＜ 5 ページ. 置換積分法 1 ＞

解答

(1) $\displaystyle\int \frac{f'(x)}{f(x)}dx = \log\bigl|f(x)\bigr| + C$

(2) $\displaystyle\int \sin\left(f(x)\right)f'(x)dx = -\cos\bigl(f(x)\bigr) + C$

(3) $\displaystyle\int (f(x))^n f'(x)dx = \frac{1}{n+1}\bigl(f(x)\bigr)^{n+1} + C$

＜ 6 ページ. 置換積分法 2 ＞

問 1 の解答

(1) $\displaystyle\int \frac{1}{ax+b}dx = \int \frac{1}{u} \times \frac{1}{a}du = \frac{1}{a}\int \frac{1}{u}du = \frac{1}{a}\log|u| + C = \frac{1}{a}\log|ax+b| + C$

(2) $\displaystyle\int \sin(ax+b)dx = \int \sin(u)\frac{1}{a}du = -\frac{1}{a}\cos(u) + C = -\frac{1}{a}\cos(ax+b) + C$

(3) $\displaystyle\int (ax+b)^n dx = \int u^n \times \frac{1}{a}du = \frac{1}{a} \times \frac{1}{n+1}u^{n+1} + C = \frac{1}{(n+1)a}(ax+b)^{n+1} + C$

問 2 の解答

(1) $\displaystyle\int e^{4x+5}dx = \frac{1}{4}e^{4x+5} + C$

(2) $\displaystyle\int \cos(3x-5)dx = \frac{1}{3}\sin(3x-5) + C$

(3) $\displaystyle\int \frac{1}{5x+6}dx = \frac{1}{5}\log|5x+6| + C$

(4) $\displaystyle\int \sin(2x+\pi)dx = -\frac{1}{2}\cos(2x+\pi) + C$

(5) $\displaystyle\int (8x+7)^5 dx = \frac{1}{48}(8x+7)^6 + C$

(6) $\displaystyle\int \frac{1}{(5x+6)^2}dx = -\frac{1}{5(5x+6)} + C$

＜ 7ページ. 置換積分 3 ＞

解答

(1) $\displaystyle\int xe^{x^2+1}dx = \frac{1}{2}e^{x^2+1} + C$

(2) $\displaystyle\int x^3e^{x^4}dx = \frac{1}{4}e^{x^4} + C$

(3) $\displaystyle\int x^2\cos(x^3+2)dx = \frac{1}{3}\sin(x^3+2) + C$

(4) $\displaystyle\int x\sin(x^2+3)dx = -\frac{1}{2}\cos(x^2+3) + C$

(5) $\displaystyle\int \frac{x}{x^2+3}dx = \frac{1}{2}\log|x^2+3| + C$

(6) $\displaystyle\int x(x^2+1)^5dx = \frac{1}{12}(x^2+1)^6 + C$

＜ 8 ページ. 定積分の積分変数 ＞

解答

(1) $\displaystyle\int_1^3 (4-9.8t)dt = \left[4t-4.9t^2\right]_1^3 = 4\times(3-1) - 4.9\times(9-1) = -31.2$

(2) $\displaystyle\int_0^R 2\pi r dr = \left[\pi r^2\right]_0^R = \pi R^2$

(3) $\displaystyle\int_0^\pi \sin\theta d\theta = [-\cos\theta]_0^\pi = -\cos\pi + \cos 0 = 2$

(4) $\displaystyle\int_a^b u^n du = \left[\frac{1}{n+1}u^{n+1}\right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$

(5) $\displaystyle\int_1^9 \sqrt{u} du = \left[\frac{2}{3}u\sqrt{u}\right]_1^9 = \frac{2}{3}(9\sqrt{9}-1) = \frac{52}{3}$

＜ 9 ページ. 定積分の置換積分法 1 ＞

解答

(1) $u=x^3+1$ とおくと

$$\begin{cases} x=-1 & \Longleftrightarrow \quad u=0 \\ x=1 & \Longleftrightarrow \quad u=2 \end{cases}$$

$$\int_{-1}^{1} 3x^2(x^3+1)^4 dx = \int_0^2 u^4 du = \left[\frac{1}{5}u^5\right]_0^2 = \frac{32}{5}$$

(2) $u=x^2+1$ とおくと

$$\begin{cases} x=2 & \Longleftrightarrow \quad u=5 \\ x=0 & \Longleftrightarrow \quad u=1 \end{cases}$$

$$\int_0^2 2x\sqrt{x^2+1}dx = \int_1^5 \sqrt{u}du = \left[\frac{2}{3}u\sqrt{u}\right]_1^5 = \frac{2}{3}(5\sqrt{5}-1)$$

(3) $u=x^4+1$ とおくと

$$\begin{cases} x=0 & \Longleftrightarrow \quad u=1 \\ x=1 & \Longleftrightarrow \quad u=2 \end{cases}$$

$$\int_0^1 \frac{4x^3}{(x^4+1)^2}dx = \int_1^2 \frac{1}{u^2}du = \left[-\frac{1}{u}\right]_1^2 = -\frac{1}{2}+1 = \frac{1}{2}$$

＜ 10 ページ. 定積分の置換積分法 2 ＞

解答

(1) $u = x^2 + 2$ とおくと

$$\begin{cases} x = 0 \iff u = 2 \\ x = 1 \iff u = 3 \end{cases}$$

$$\int_0^1 x(x^2+2)^3 dx = \int_2^3 \frac{1}{2}u^3 du = \left[\frac{1}{8}u^4\right]_2^3 = \frac{1}{8}(3^4 - 2^4) = \frac{65}{8}$$

(2) $u = x^2$ とおくと

$$\int_0^3 xe^{x^2} dx = \int_0^9 \frac{1}{2}e^u du = \left[\frac{1}{2}e^u\right]_0^9 = \frac{1}{2}e^9 - \frac{1}{2}$$

(3) $u = x^3 + 2$ とおくと

$$\int_{-1}^2 \frac{x^2}{x^3+2} dx = \int_1^{10} \frac{1}{3u} du = \left[\frac{1}{3}\log|u|\right]_1^{10} = \frac{1}{3}\log 10$$

(4) $u = x^2 + 1$ とおくと

$$\int_0^2 \frac{x}{(x^2+1)^3} dx = \int_1^5 \frac{1}{2u^3} du = \left[-\frac{1}{4u^2}\right]_1^5 = -\frac{1}{100} + \frac{1}{4} = \frac{6}{25}$$

＜ 11 ページ. 積の微分 1 ＞

解答

$$\big(f(x) \times g(x)\big)' = f'(x) \times g(x) + f(x) \times g'(x)$$

＜ 12 ページ. 積の微分 2 ＞

解答

(1) $(x\cos x)' = \cos x - x\sin x$

(2) $\left(x^5\sin x\right)' = 5x^4\sin x + x^5\cos x$

(3) $$\begin{aligned}\left(\sin^2 x\right)' &= (\sin x \times \sin x)' \\ &= \cos x \times \sin x + \sin x \times \cos x \\ &= 2\sin x\cos x\end{aligned}$$

$<$ 13ページ. 商の微分 1 $>$

解答

$$\left(\frac{1}{g(x)}\right)' = -\frac{g'(x)}{\big(g(x)\big)^2}$$

< 14 ページ. 商の微分 2 >

解答

(1) $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$

(2) $\left(\dfrac{1}{x^2}\right)' = -\dfrac{2}{x^3}$

(3) $\left(\dfrac{1}{x^3}\right)' = -\dfrac{3}{x^4}$

(4) $\left(\dfrac{1}{\cos x}\right)' = \dfrac{\sin x}{\cos^2 x}$

＜ 15 ページ. 分数関数の微分 ＞

解答

(1)
$$\left(\frac{x}{\cos x}\right)' = \left(x \times \frac{1}{\cos x}\right)'$$
$$= (x)' \times \frac{1}{\cos x} + x \times \left(\frac{1}{\cos x}\right)'$$
$$= 1 \times \frac{1}{\cos x} + x \times \left(-\frac{-\sin x}{\cos^2 x}\right)$$
$$= \frac{\cos x + x \sin x}{\cos^2 x}$$

(2)
$$\left(\frac{\cos x}{\sin x}\right)' = \left(\cos x \times \frac{1}{\sin x}\right)'$$
$$= (\cos x)' \times \frac{1}{\sin x} + \cos x \times \left(\frac{1}{\sin x}\right)'$$
$$= -\sin x \times \frac{1}{\sin x} + \cos x \times \left(-\frac{1}{\sin^2 x}\right)$$
$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$
$$= -\frac{1}{\sin^2 x}$$

< 16 ページ. 部分積分法 1 >

解答

$$\int f(x) \times g'(x)dx = f(x) \times g(x) - \int f'(x) \times g(x)dx$$

< 17 ページ. 部分積分法 2 >

解答

(1)
$$\begin{aligned}\int (3x-2)\sin x dx &= (3x-2)(-\cos x) - \int (3x-1)'(-\cos x)dx \\ &= -(3x-2)\cos x + \int 3\cos x dx \\ &= -(3x-2)\cos x + 3\sin x + C\end{aligned}$$

(2)
$$\begin{aligned}\int (x^2+1)\cos x dx &= (x^2+1)\sin x - \int 2x \sin x dx \\ &= (x^2+1)\sin x - \left\{2x(-\cos x) - \int 2(-\cos x)dx\right\} \\ &= (x^2+1)\sin x + 2x\cos x - 2\sin x + C\end{aligned}$$

(3)
$$\begin{aligned}\int xe^x dx &= xe^x - \int 1e^x dx \\ &= xe^x - e^x + C\end{aligned}$$

＜ 18 ページ. 部分積分法 3 ＞

解答

$$
\begin{aligned}
\int (\log x) \times x dx &= (\log x) \times \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2} dx \\
&= \frac{x^2}{2} \log x - \int \frac{1}{2} x dx \\
&= \frac{x^2}{2} \log x - \frac{1}{4} x^2 + C
\end{aligned}
$$

＜ 19 ページ. 不定積分の検証 ＞

解答

(1) $\left(\dfrac{1}{4}(x^4-1)^4\right)' = (x^4-1)^3 \times 4x^3$ より正しくない。

(2) $\left(\dfrac{1}{2}\log|x^2-1|\right)' = \dfrac{1}{2} \times \dfrac{2x}{x^2-1} = \dfrac{x}{x^2-1}$ より正しい。

(3) $\left(x^2e^x - 2xe^x + 2e^x\right)' = 2xe^x + x^2e^x - (2e^x + 2xe^x) + 2e^x = x^2e^x$ より正しい。

＜ 20 ページ. 定積分の部分積分 ＞

解答

(1)
$$\int_{-1}^{1}(x+1)(x-1)^3dx = \left[(x+1)\frac{(x-1)^4}{4}\right]_{-1}^{1} - \int_{-1}^{1}\frac{(x-1)^4}{4}dx$$
$$= 0 - \left[\frac{1}{20}(x-1)^5\right]_{-1}^{1}$$
$$= -\left(0 - \frac{(-2)^5}{20}\right)$$
$$= -\frac{32}{20}$$
$$= -\frac{8}{5}$$

(2)
$$\int_{0}^{\frac{\pi}{2}} x\sin x dx = \left[x(-\cos x)\right]_{0}^{\frac{\pi}{2}} - \int_{0}^{\frac{\pi}{2}}(-\cos x)dx$$
$$= 0 + \left[\sin x\right]_{0}^{\frac{\pi}{2}}$$
$$= 1$$

(3)
$$\int_{0}^{1} xe^x dx = [xe^x]_0^1 - \int_{0}^{1} e^x dx$$
$$= e - \left[e^x\right]_{-1}^{1}$$
$$= e - (e-1)$$
$$= 1$$

< 21 ページ. 関数の定義域と値域 1 >

解答

(1) $f(x) = 2 - \sqrt{x-1}$

<u>定義域 $x \geqq 1$</u>

<u>値域 $y \leqq 2$</u>

(2) $f(x) = 1 + \sqrt{1-x}$

<u>定義域 $x \leqq 1$</u>

<u>値域 $y \geqq 1$</u>

(3) $f(x) = \dfrac{1}{x+1} - 1$

<u>定義域 $x \neq -1$</u>

<u>値域 $y \neq -1$</u>

＜ 22 ページ. 関数の定義域と値域 2 ＞

解答

(1) $f(x) = \log_3(1-x)$

定義域 $x < 1$

値域　実数全体

(2) $f(x) = 2^x - 1$

定義域 実数全体

値域　$y > -1$

(3) $f(x) = 1 - 5^x$

定義域 実数全体

値域　$y < 1$

＜ 23 ページ. 関数の定義域と値域 3 ＞

解答

(1) $f(x) = 1 + \sin x$

定義域　実数全体

値域　$0 \leqq y \leqq 2$

(2) $f(x) = 2 - 2\cos x$

定義域　実数全体

値域　$0 \leqq y \leqq 4$

(3) $f(x) = 1 + 3\sin(2x)$

定義域　実数全体

値域　$-2 \leqq y \leqq 4$

(4) $f(x) = 3 + 4\cos(-x)$

定義域　実数全体

値域　$-1 \leqq y \leqq 7$

< 24 ページ. 関数の定義域と値域 4 >

解答

(1) $f(x) = \tan(3x)$

定義域 $x \neq \pm\frac{\pi}{6} \pm \frac{n}{3}\pi$ (n は整数)

値域 実数全体

(2) $f(x) = \tan(\pi x)$

定義域 $x \neq \pm\frac{1}{2} \pm n$ (n は整数)

値域 実数全体

$<$ 25ページ.1対1関数 $>$

解答

(1) $y = 2 - x$

<u>（答）　1対1である。</u>

(2) $y = x^3 - 4x$

<u>（答）　1対1でない。</u>

(3) $y = \dfrac{1}{x}\ (x \neq 0)$

<u>（答）　1対1である。</u>

＜ 26 ページ. 逆関数 1 ＞

解答

(1) $f(x) = 2 - x$

(解) $b = f(a) = 2 - a$

$$\Downarrow$$

$$a = 2 - b = f^{-1}(b)$$

$$\Downarrow$$

$$f^{-1}(b) = 2 - b$$

(2) $f(x) = \dfrac{1}{x+1}$

(解) $b = f(a) = \dfrac{1}{a+1}$

$$\Downarrow$$

$$a + 1 = \frac{1}{b}$$

$$\Downarrow$$

$$a = \frac{1}{b} - 1$$

$$\Downarrow$$

$$f^{-1}(b) = \frac{1}{b} - 1$$

(3) $f(x) = \sqrt{x}$

(解) $b = f(a) = \sqrt{a}$

$$\Downarrow$$

$$b^2 = a$$

$$\Downarrow$$

$$f^{-1}(b) = b^2$$

＜ 27ページ. 逆関数 2 ＞

解答

(1) $f(x) = 3x - 1$

(解) $y = 3x - 1$

$$\Downarrow$$

$$3x = y + 1$$

$$\Downarrow$$

$$x = \frac{y+1}{3}$$

$$\Downarrow$$

$$f^{-1}(x) = \frac{x+1}{3}$$

(2) $f(x) = \dfrac{1}{x-1} + 1$

(解) $y = \dfrac{1}{x-1} + 1$

$$\Downarrow$$

$$y - 1 = \frac{1}{x-1}$$

$$\Downarrow$$

$$x - 1 = \frac{1}{y-1}$$

$$\Downarrow$$

$$x = \frac{1}{y-1} + 1$$

$$\Downarrow$$

$$f^{-1}(x) = \frac{1}{x-1} + 1$$

または$f^{-1}(x) = \dfrac{x}{x-1}$

(3) $f(x) = \sqrt{x+1}$

(解) $y = \sqrt{x+1}$

$$\Downarrow$$

$$y^2 = x + 1$$

$$\Downarrow$$

$$x = y^2 - 1$$

$$\Downarrow$$

$$f^{-1}(x) = x^2 - 1$$

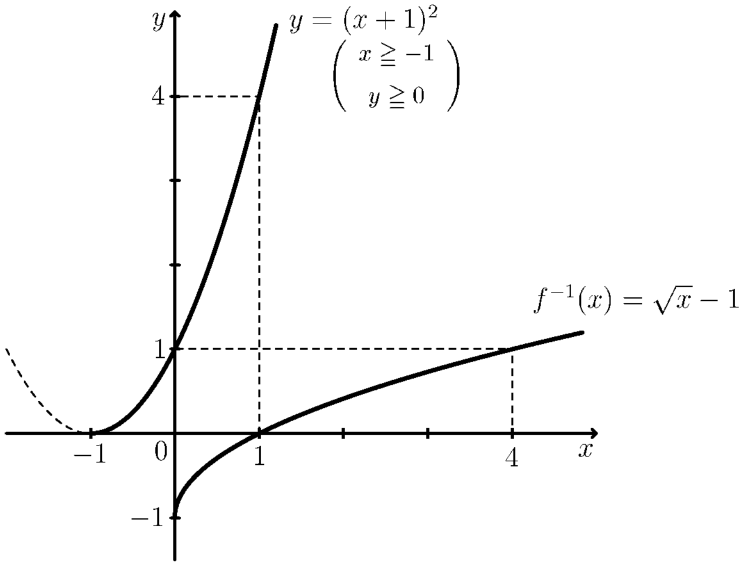
＜ 28 ページ. 逆関数 3 ＞

解答

(解)

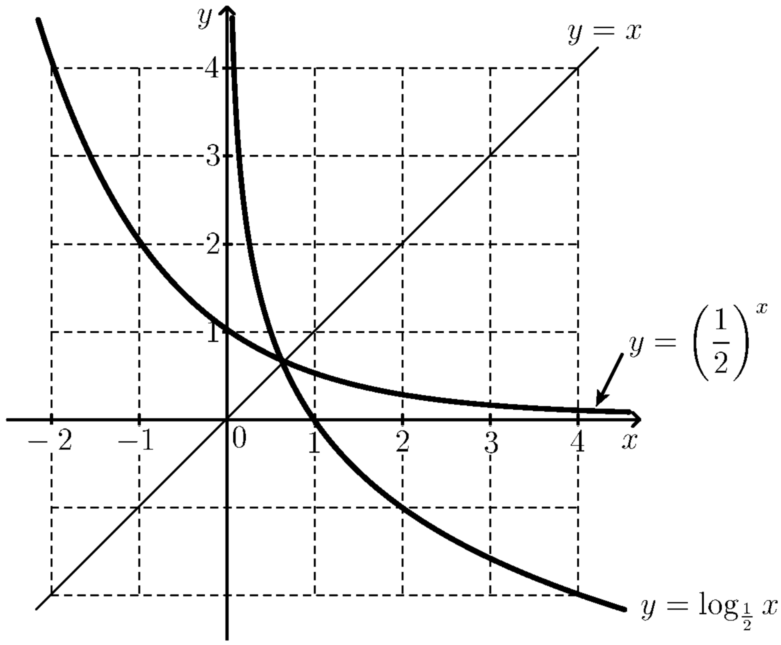
$$y = (x+1)^2$$
$$\Downarrow$$
$$\sqrt{y} = x+1$$

$$x = \sqrt{y} - 1$$
$$\Downarrow$$
$$f^{-1}(x) = \sqrt{x} - 1$$



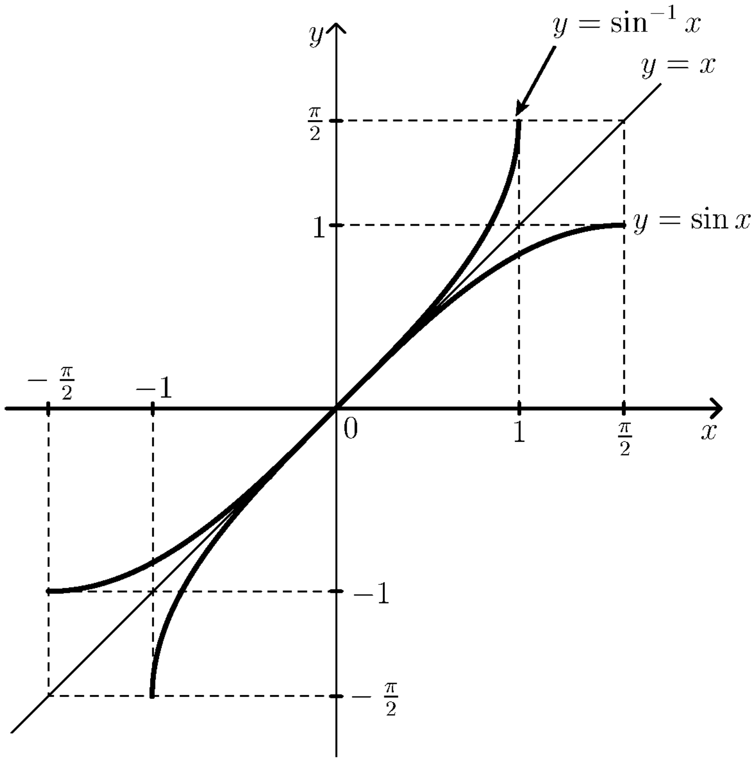
＜ 29 ページ. 逆関数 4 ＞

解答



< 30 ページ. 逆三角関数 1 >

問 1 の解答



問 2 の解答

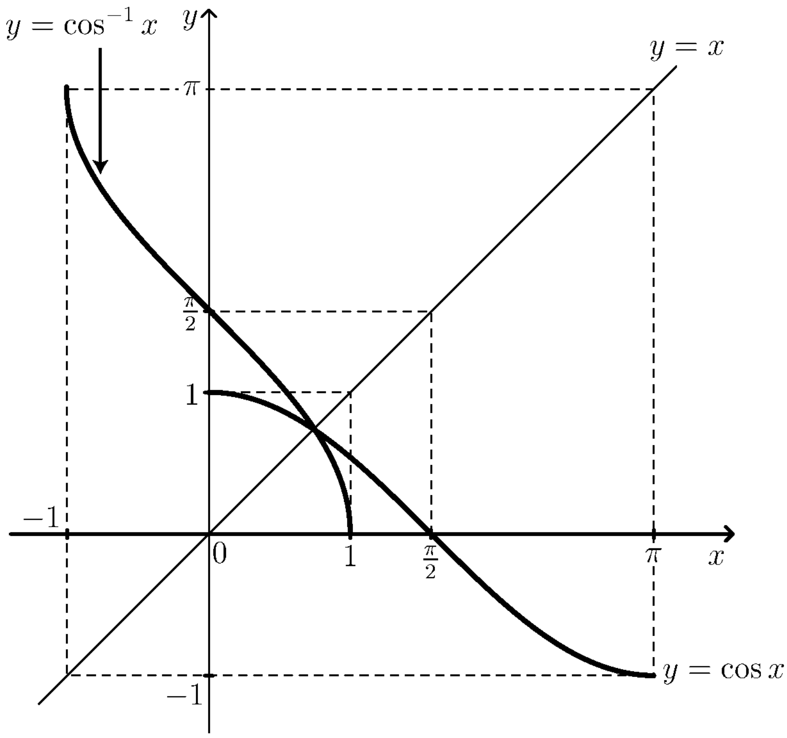
θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

問 3 の解答

(1) $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$　　(2) $\sin^{-1}\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}$　　(3) $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = -\dfrac{\pi}{4}$

＜ 31 ページ. 逆三角関数 2 ＞

問 1 の解答



問 2 の解答

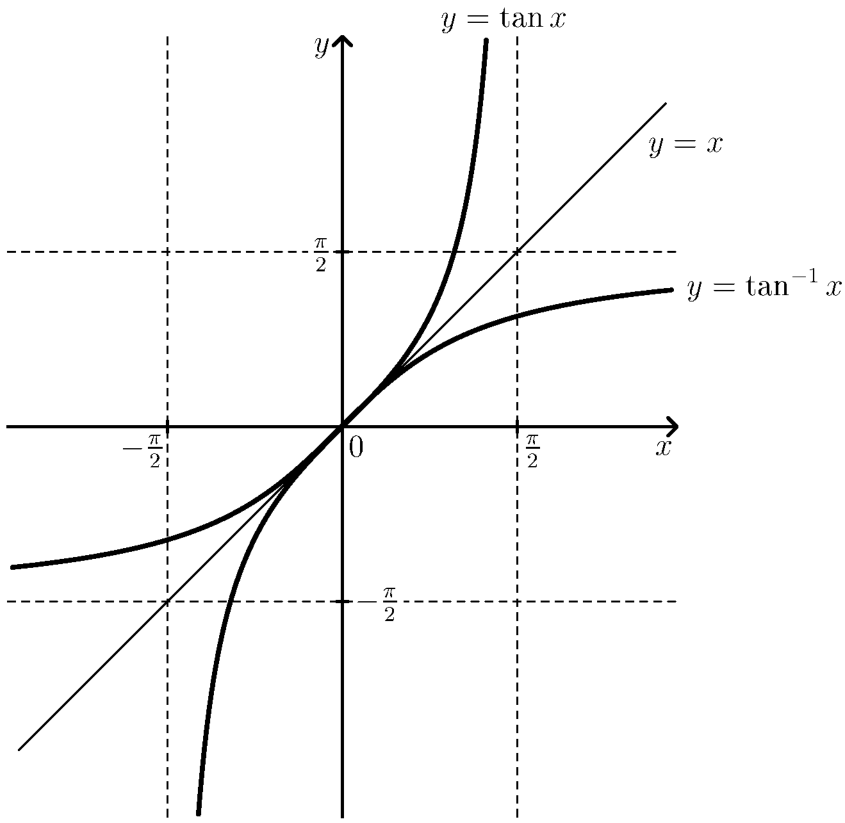
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

問 3 の解答

(1) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ (2) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ (3) $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2}{3}\pi$

＜32ページ. 逆三角関数 3 ＞

問 1 の解答



問 2 の解答

θ	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan\theta$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

問 3 の解答

(1) $\tan^{-1}(1) = \dfrac{\pi}{4}$　　(2) $\tan^{-1}(-1) = -\dfrac{\pi}{4}$　　(3) $\tan^{-1}\left(-\dfrac{\sqrt{3}}{3}\right) = -\dfrac{\pi}{6}$

＜ 33 ページ. 逆関数の微分 ＞

解答

(1) $y = \cos^{-1} x \quad \Leftrightarrow \quad x = \cos y$

$$\frac{dy}{dx} = \frac{1}{\dfrac{dy}{dx}} = \frac{1}{(\cos y)'} = \frac{1}{-\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$$

(2) $y = \tan^{-1} x \quad \Leftrightarrow \quad x = \tan y$

$$\frac{dy}{dx} = \frac{1}{\dfrac{dy}{dx}} = \frac{1}{(\tan y)'} = \frac{1}{\dfrac{1}{\cos^2 y}} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

< 34ページ. 逆三角関数の積分 >

問1の解答

$$x = au \ \Rightarrow \ dx = adu$$

$$\begin{aligned}
\int \frac{1}{\sqrt{a^2 - x^2}} dx &= \int \frac{1}{\sqrt{a^2 - a^2u^2}} adu \\
&= \int \frac{1}{\sqrt{1 - u^2}} du \\
&= \sin^{-1}(u) + C \\
&= \sin^{-1}\left(\frac{x}{a}\right) + C
\end{aligned}$$

問2の解答

$$x = au \ \Rightarrow \ dx = adu$$

$$\begin{aligned}
\int \frac{1}{a^2 - x^2} dx &= \int \frac{1}{a^2 - a^2u^2} adu \\
&= \frac{1}{a} \int \frac{1}{1 - u^2} du \\
&= \frac{1}{a} \tan^{-1}(u) + C \\
&= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C
\end{aligned}$$

＜ 35 ページ. 不定積分の特例 1 ＞

解答

(1) $$\int \sin^2 x dx = \int \frac{1-\cos(2x)}{2} dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

(2) $$\begin{aligned}\int \cos(3x)\cos(2x)dx &= \int \frac{1}{2}\left\{\cos(5x) + \cos x\right\} dx \\ &= \frac{1}{10}\sin(5x) + \frac{1}{2}\sin x + C\end{aligned}$$

(3) $$\begin{aligned}\int \sin(-x)\sin x dx &= \int \frac{1}{2}\left\{\cos(-2x) - \cos(0)\right\} dx \\ &= \int \frac{1}{2}\left\{\cos(2x) - 1\right\} dx \\ &= \frac{1}{4}\sin(2x) - \frac{1}{2}x + C\end{aligned}$$

＜ 36 ページ. 不定積分の特例 2 ＞

解答

$x = au$ とおくと $dx = adu$ より

$$
\begin{aligned}
\int \sqrt{a^2 - x^2}dx &= \int \sqrt{a^2 - a^2u^2}adu \\
&= a^2 \int \sqrt{1 - u^2}du \\
&= a^2 \left(\frac{1}{2}\sin^{-1}(u) + \frac{1}{2}u\sqrt{1 - u^2} \right) + C \\
&= \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{a^2}{2} \times \frac{x}{a}\sqrt{1 - \left(\frac{x}{a}\right)^2} + C \\
&= \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}x\sqrt{a^2 - x^2} + C
\end{aligned}
$$

＜ 37 ページ. 円の面積 ＞

解答

(1) $\displaystyle\int_0^a \sqrt{a^2 - x^2}dx = \left[\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2}\right]_0^a = \frac{a^2}{2}\sin^{-1}(1)$

(2) $\displaystyle\frac{S}{4} = \frac{a^2}{2}\sin^{-1}(1) = \frac{a^2}{2} \times \frac{\pi}{2} = \frac{a^2\pi}{4}$

$$\Downarrow$$

$$S = a^2\pi$$

< 38 ページ. 楕円の面積 >

解答

(1) $\dfrac{y^2}{b^2} = 1 - \dfrac{x^2}{a^2} \ \Rightarrow \ y^2 = b^2\left(1 - \dfrac{x^2}{a^2}\right) = \left(\dfrac{b}{a}\right)^2 (a^2 - x^2)$

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

(2) $\dfrac{S}{4} = \displaystyle\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}dx$

(3)
$$\begin{aligned}
\frac{S}{4} &= \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}dx \\
&= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1}\left(\frac{x}{4}\right) + \frac{x}{2}\sqrt{a^2 - x^2}\right]_0^a \\
&= \frac{b}{a} \times \frac{a^2}{2}\sin^{-1}(1) \\
&= \frac{b}{a} \times \frac{a^2}{2} \times \frac{\pi}{2} \\
&= \frac{ab\pi}{4}
\end{aligned}$$

$$\underline{S = \pi ab}$$

< 39ページ. 回転体の表面積 1 >

解答

(1) $S = S(b) - S(a)$

(2) $S = S(b) - S(a) = \pi m\sqrt{1+m^2}b^2 - \pi m\sqrt{1+m^2}a^2 = \pi m\sqrt{1+m^2}(b^2 - a^2)$

(3) $S(x) = \pi m\sqrt{1+m^2}\,x^2$, $S'(x) = 2\pi m\sqrt{1+m^2}\,x$

(4) $\pi m\sqrt{1+m^2}(b^2 - a^2) = \int_a^b 2\pi m\sqrt{1+m^2}\,x dx$

$<$ 40 ページ. 回転体の表面積 2 $>$

問 1 の解答

$$y = \sqrt{r^2 - y^2} \quad , \quad y' = -\frac{x}{\sqrt{r^2 - x^2}} \quad , \quad 1 + (y')^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}$$

$$\begin{aligned} S(r) &= \int_{-r}^{r} 2\pi y \sqrt{1 + (y')^2} dy \\ &= \int_{-r}^{r} 2\pi \sqrt{r^2 - x^2} \sqrt{\frac{r^2}{r^2 - x^2}} dx \\ &= \int_{-r}^{r} 2\pi r dx \\ &= [2\pi r x]_{-r}^{r} \\ &= 4\pi r^2 \end{aligned}$$

問 2 の解答

$$\int_0^r S(x)dx = \int_0^r 4\pi x^2 dx = \left[\frac{4\pi}{3}x^3\right]_0^r = \frac{4\pi}{3}r^3$$

$\displaystyle\int_0^r S(x)dx$ は半径 r の球の体積を意味する。