高知工科大学

基礎数学ワークブック

(2001年度版)

Series A

No. 5

解答

＜１ページ. 極座標＞

解答

(1) $(-\sqrt{3},\ 1) = \left(2\cos\left(\frac{5}{6}\pi\right),\ 2\sin\left(\frac{5}{6}\pi\right)\right)$

$= (2\cos(150^\circ),\ 2\sin(150^\circ))$

(2) $(-1,\ -\sqrt{3}) = \left(2\cos\left(\frac{4}{3}\pi\right),\ 2\sin\left(\frac{4}{3}\pi\right)\right)$

$= (2\cos(240^\circ),\ 2\sin(240^\circ))$

(3) $\left(\frac{\sqrt{3}}{2},\ \frac{1}{2}\right) = \left(\cos\left(\frac{\pi}{6}\right),\ \sin\left(\frac{\pi}{6}\right)\right)$

$= (\cos(30^\circ),\ \sin(30^\circ))$

(4) $(\sqrt{2},\ \sqrt{2}) = \left(2\cos\left(\frac{\pi}{4}\right),\ 2\sin\left(\frac{\pi}{4}\right)\right)$

$= (2\cos(45^\circ),\ 2\sin(45^\circ))$

< 2ページ. 余弦定理 1 >

解答

(1) $\mathrm{P}(b\cos\theta,\ b\sin\theta)$

$\mathrm{Q}(a,\ 0)$

(2) $\mathrm{PQ}^2 = (b\cos\theta - a)^2 + (b\sin\theta)^2$

(3) $$\begin{aligned}\mathrm{PQ}^2 &= b^2\cos^2\theta - 2ab\cos\theta + a^2 + b^2\sin^2\theta \\ &= a^2 + b^2 - 2ab\cos\theta\end{aligned}$$

(4) $c^2 = a^2 + b^2 - 2ab\cos\theta$

＜３ページ. 余弦定理 ２＞

問１の解答

$c^2 = a^2 + b^2 - 2ab\cos\theta$

問２の解答

(1) $c^2 = 2^2 + (\sqrt{2})^2 - 2 \times 2 \times \sqrt{2}\cos(45^\circ)$

$= 4 + 2 - 4\sqrt{2} \times \dfrac{1}{\sqrt{2}}$

$= 2$

$\Downarrow$

$c = \sqrt{2}$

(2) $c^2 = 3^2 + (2\sqrt{3})^2 - 2 \times 3 \times 2\sqrt{3}\cos(150^\circ)$

$= 9 + 12 - 12\sqrt{3} \times \left(-\dfrac{\sqrt{3}}{2}\right)$

$= 21 + 18$

$= 39$

$\Downarrow$

$c = \sqrt{39}$

＜ 4ページ. 加法定理 1 ＞

解答

(1) $\mathrm{P}(\cos\beta,\ \sin\beta)$

(2) $\mathrm{Q}(\cos\alpha,\ -\sin\alpha)$

(3) $$\begin{aligned}\mathrm{PQ}^2 &= (\cos\alpha - \cos\beta)^2 + (-\sin\alpha - \sin\beta)^2 \\ &= \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta\end{aligned}$$

(4) $$\begin{aligned}\mathrm{PQ}^2 &= (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) \\ &\qquad -2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta \\ &= 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta\end{aligned}$$

(5) $$\begin{aligned}\mathrm{PQ}^2 &= 1^2 + 1^2 - 2\cos(\alpha + \beta) \\ &= 2 - 2\cos(\alpha + \beta)\end{aligned}$$

(6) $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

＜ 5 ページ. 加法定理 2 ＞

解答

$$
\begin{aligned}
\cos 105^\circ &= \cos(60^\circ + 45^\circ) \\
&= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\
&= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \\
&= \frac{\sqrt{2} - \sqrt{6}}{4}
\end{aligned}
$$

$$
\begin{aligned}
\sin 105^\circ &= \cos(90^\circ - 105^\circ) \\
&= \cos\bigl(90^\circ - 60^\circ + (-45^\circ)\bigr) \\
&= \cos(90^\circ - 60^\circ)\cos(-45^\circ) - \sin(90^\circ - 60^\circ)\sin(-45^\circ) \\
&= \cos 30^\circ \cos 45^\circ + \sin 30^\circ \sin 45^\circ \\
&= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} \\
&= \frac{\sqrt{6} + \sqrt{2}}{4}
\end{aligned}
$$

< 6ページ. 加法定理 3 >

問1の解答

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

問2の解答

(1) $\cos 165^\circ = \cos 105^\circ \cos 60^\circ - \sin 105^\circ \sin 60^\circ$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}\times\frac{1}{2}-\frac{\sqrt{6}+\sqrt{2}}{4}\times\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}-\sqrt{6}-3\sqrt{2}-\sqrt{6}}{8}$$

$$= -\frac{\sqrt{6}+\sqrt{2}}{4}$$

(2) $\sin 165^\circ = \sin 105^\circ \cos 60^\circ - \cos 105^\circ \sin 60^\circ$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}\times\frac{1}{2}+\frac{\sqrt{2}-\sqrt{6}}{4}\times\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

問3の解答

(1) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

(2) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

＜ 7 ページ. 加法定理 4 ＞

問 1 の解答

$$\tan 105^\circ = \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$$
$$= \frac{\sqrt{3}+1}{1-\sqrt{3}\times 1}$$
$$= \frac{(1+\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})}$$
$$= \frac{1+2\sqrt{3}+3}{1-3}$$
$$= \frac{4+2\sqrt{3}}{-2}$$
$$= -2-\sqrt{3}$$

問 2 の解答

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

< 8ページ. 円周率 >

問1の解答

(1) $\underline{\ell = 4\pi}$(cm)　　(2) $\underline{\ell = 2\pi r}$

問2の解答

(1) πr　　(2) $\dfrac{\pi}{2}r$　　(3) $\dfrac{\pi}{3}r$

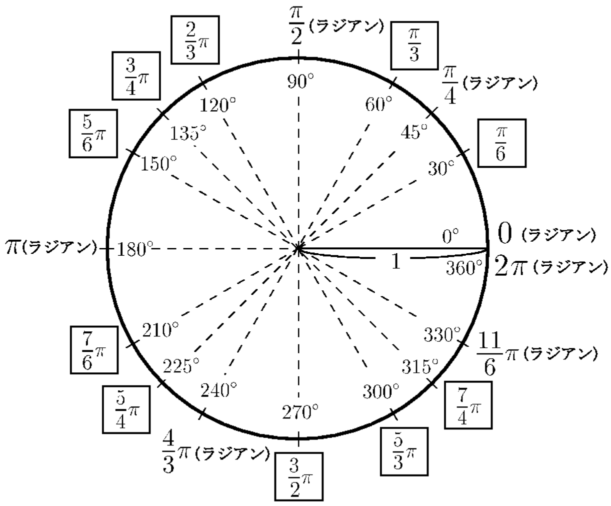
＜ 9ページ. 弧度法 1 ＞

解答

度数法	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
弧度法	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π

＜ 10 ページ. 弧度法 2 ＞

問 1 の解答



問 2 の解答

(1) $540^\circ = 360^\circ + 180^\circ$

$= 2\pi + \pi$

$= 3\pi$

(2) $-270^\circ = -\dfrac{3}{2}\pi$

(3) $630^\circ = 360^\circ + 270^\circ$

$= 2\pi + \dfrac{3}{2}\pi$

$= \dfrac{7}{2}\pi$

(4) $-405^\circ = -360^\circ - 45^\circ$

$= -2\pi - \dfrac{\pi}{4}$

$= -\dfrac{9}{4}\pi$

(5) $750^\circ = 720^\circ + 30^\circ$

$= 4\pi + \dfrac{\pi}{6}$

$= \dfrac{25}{6}\pi$

(6) $-855^\circ = -720^\circ - 135^\circ$

$= -4\pi - \dfrac{3}{4}\pi$

$= -\dfrac{19}{4}\pi$

問 3 の解答

(1) $\ell = 2\pi r$

(2) $S = \pi r^2$

＜ 11 ページ. 弧度法 3 ＞

問 1 の解答

度数法	45°	60°	90°	120°	180°	360°
弧度法 θ	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	π	2π
弧の長さ ℓ	$\frac{1}{4}\pi r$	$\frac{\pi}{3}r$	$\frac{\pi}{2}r$	$\frac{2}{3}\pi r$	πr	$2\pi r$
面積 S	$\frac{\pi}{8}\pi r^2$	$\frac{\pi}{6}\pi r^2$	$\frac{1}{4}\pi r^2$	$\frac{\pi}{3}r^2$	$\frac{\pi}{2}r^2$	πr^2

問 2 の解答

$\ell = \theta r$

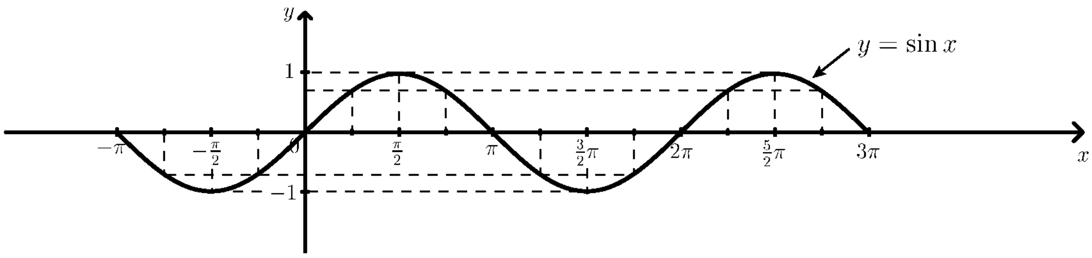
$S = \dfrac{1}{2}\theta r^2$

＜ 12 ページ. 三角関数のグラフ ＞

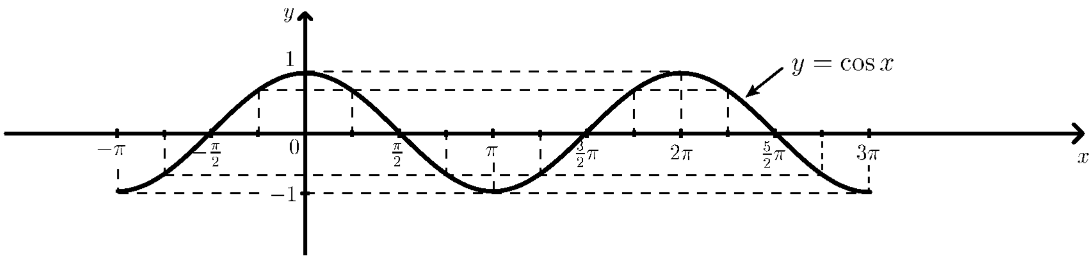
解答

x																		
x	度数法	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$	$405°$	$450°$	$495°$	$540°$
	弧度法	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	$\frac{9}{4}\pi$	$\frac{5}{2}\pi$	$\frac{11}{4}\pi$	3π
$\sin x$		0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\cos x$		-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1

(1) $y = \sin x$

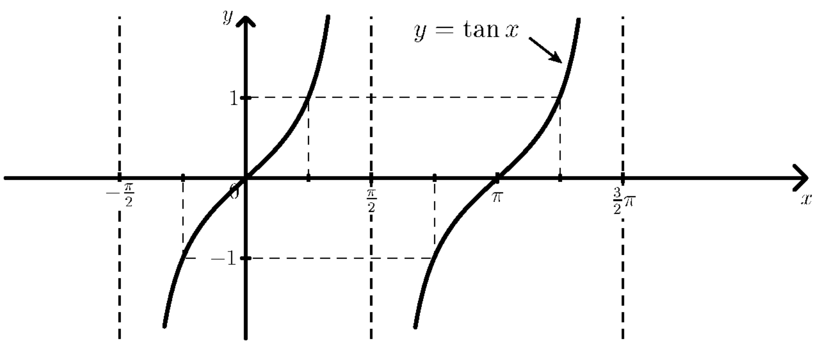


(2) $y = \cos x$



x	度数法	$-90°$	$-60°$	$-45°$	$-30°$	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$
	弧度法	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$
$\tan x$		×	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	×	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	×

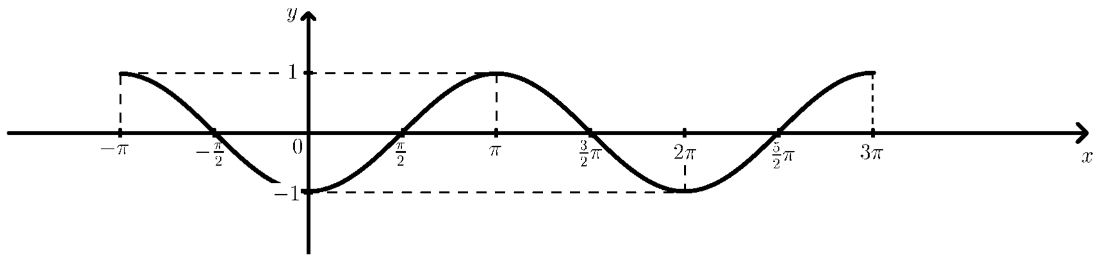
(3) $y = \tan x$



＜ 13 ページ. 正弦波 1 ＞

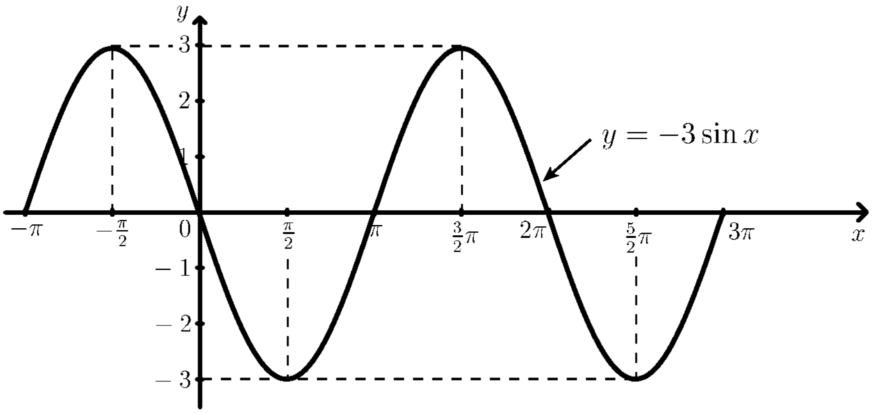
解答

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π	$\frac{5}{2}\pi$	3π
$\sin x$	0	-1	0	1	0	-1	0	1	0
$\sin\left(x-\frac{\pi}{2}\right)$	1	0	-1	0	1	0	-1	0	1



＜ 14 ページ. 正弦波 2 ＞

解答

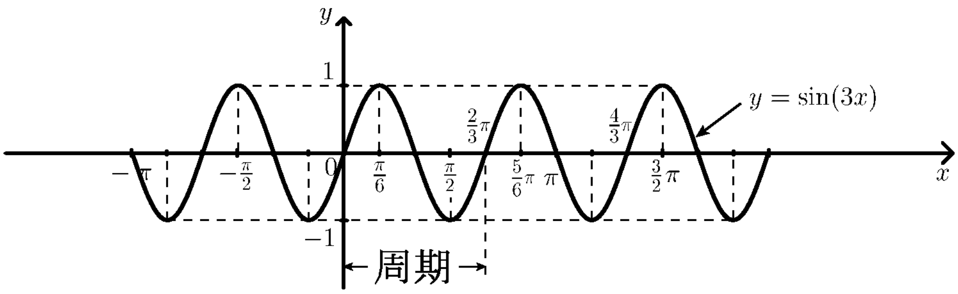


振幅 3

＜ 15 ページ. 正弦波 3 ＞

解答

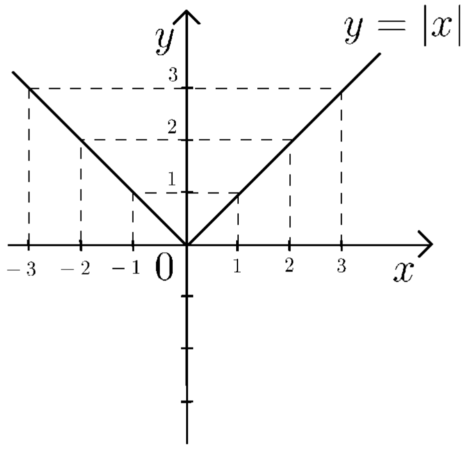
x	$-\frac{2}{3}\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{4}{3}\pi$
$3x$	-2π	$-\frac{3}{2}\pi$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π	$\frac{5}{2}\pi$	3π	$\frac{7}{2}\pi$	4π
$\sin(3x)$	0	1	0	-1	0	1	0	-1	0	1	0	-1	0



周期 $\dfrac{2}{3}\pi$

＜ 16 ページ. 絶対値 ＞

問１の解答

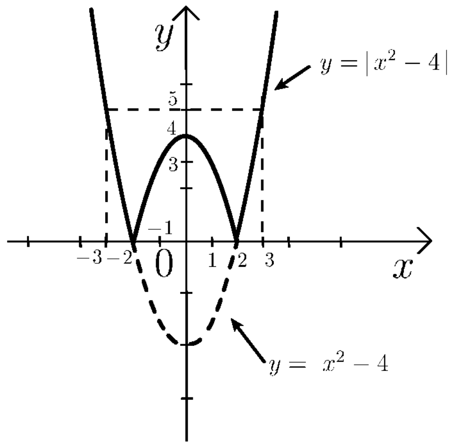


x	-3	-2	-1	0	1	2	3
y	3	2	1	0	1	2	3

$x \geqq 0$ の範囲では、直線 $y=$ [x] であり
$x<0$ の範囲では、直線 $y=$ [$-x$] であることから、

$$y=|x|=\begin{cases} \boxed{x} & (x \geqq 0) \\ \boxed{-x} & (x<0) \end{cases}$$ が分かる。」

問２の解答



x	-3	-2	-1	0	1	2	3
y	5	0	3	4	3	0	5

$$y=|x^2-4|=\begin{cases} \boxed{x^2-4} & (\boxed{2} \leqq x) \\ \boxed{-x^2+4} & (\boxed{-2}<x<\boxed{2}) \\ \boxed{x^2-4} & (x \leqq \boxed{-2}) \end{cases}$$

$y=\boxed{x^2-4}$

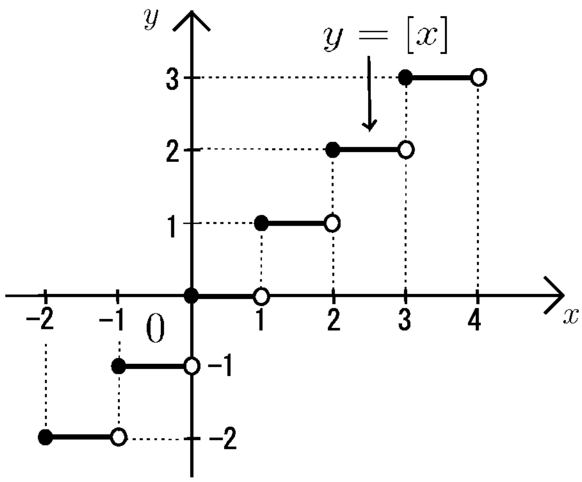
< 17ページ. ガウス記号 >

問 1 の解答

(1) $[1.23] = 1$　　(2) $[9.87] = 9$　　(3) $[\ 0.9999\ ] = 0$

(4) $[-0.1] = -1$　　(5) $[-3.69] = -4$　　(6) $[-9.5\ ] = -10$

問 2 の解答



< 18ページ. 左極限・右極限 1 >

解答

(1) $\lim_{x\to 3-0}[x]=2$

(2) $\lim_{x\to 3+0}[x]=3$

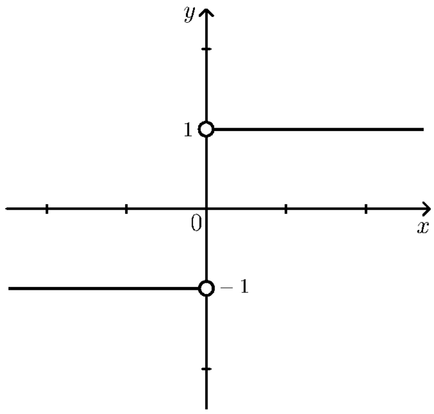
(3) $\lim_{x\to -0}[x]=-1$

(4) $\lim_{x\to +0}[x]=0$

$<$ 19ページ. 左極限・右極限 2 $>$

解答

(1) $\displaystyle\lim_{x\to -0}\frac{|x|}{x}=-1$　　　(2) $\displaystyle\lim_{x\to +0}\frac{|x|}{x}=1$



＜ 20 ページ. 三角関数の極限 1 ＞

問 1 の解答

$\ell_1 = \sin\theta \qquad , \qquad \ell_3 = \tan\theta$

問 2 の解答

$\ell_2 = \theta r = \theta$

問 3 の解答

$\sin\theta < \theta < \tan\theta$

問 4 の解答

$\sin\theta < \theta < \dfrac{\sin\theta}{\cos\theta}$

＜ 21 ページ. 三角関数の極限 2 ＞

問 1 の解答

$$\cos\theta < \frac{\sin\theta}{\theta} < 1$$

問 2 の解答

$$\lim_{\theta\to+0}\frac{\sin\theta}{\theta} = 1$$

問 3 の解答

$$\lim_{\theta\to-0}\frac{\sin\theta}{\theta} = \lim_{\theta_1\to+0}\frac{\sin(-\theta_1)}{-\theta_1} = \lim_{\theta_1\to+0}\frac{-\sin\theta_1}{-\theta_1} = \lim_{\theta_1\to+0}\frac{\sin\theta_1}{\theta_1} = 1$$

＜ 22 ページ. 三角関数の極限 3 ＞

解答

$$
\begin{aligned}
\lim_{\theta \to 0} \frac{\cos(x+\theta) - \cos x}{\theta} &= \lim_{\theta \to 0} \frac{\cos x \cos\theta - \sin x \sin\theta - \cos x}{\theta} \\
&= \lim_{\theta \to 0} \frac{\cos x(\cos\theta - 1) - \sin x \sin\theta}{\theta} \\
&= \lim_{\theta \to 0} \left\{ -\left(\frac{1-\cos\theta}{\theta} \right) \times \cos x - \sin x \times \left(\frac{\sin\theta}{\theta} \right) \right\} \\
&= -0 \times \cos x - \sin x \times 1 \\
&= -\sin x
\end{aligned}
$$

＜ 23 ページ. 三角関数の導関数 ＞

問 1 の解答

$$(\cos x)' = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{\theta \to 0} \frac{\cos(x+\theta) - \cos x}{\theta} = -\sin x$$

問 2 の解答

$x = 0$ のとき　$y = \cos x$ の傾き$= -\sin 0 = 0$

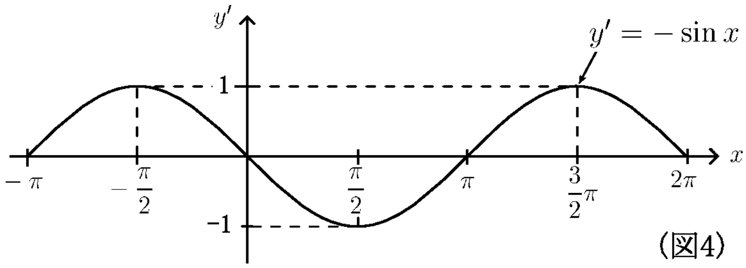
$x = \dfrac{\pi}{2}$ のとき　$y = \cos x$ の傾き$= -\sin\left(\dfrac{\pi}{2}\right) = -1$

$x = \pi$ のとき　$y = \cos x$ の傾き$= -\sin(\pi) = 0$

$x = \dfrac{3}{2}\pi$ のとき　$y = \cos x$ の傾き$= -\sin\left(\dfrac{3}{2}\pi\right) = 1$

$x = 2\pi$ のとき　$y = \cos x$ の傾き$= -\sin(2\pi) = 0$

問 3 の解答



（図4）

＜ 24 ページ. 三角関数の微分・積分 ＞

問１の解答

(1) $(4\sin x + 6\cos x)' = 4\cos x - 6\sin x$

(2) $(5 + 3\sin x - 4\cos x)' = 3\cos x + 4\sin x$

問２の解答

(1) $\displaystyle\int (3\cos x + 4\sin x)dx = 3\sin x - 4\cos x + C$

(2) $\displaystyle\int (5\cos x - 6\sin x)dx = 5\sin x + 6\cos x + C$

問３の解答

(1) $\displaystyle\int_0^{\pi} 2\cos x dx = \big[2\sin x\big]_0^{\pi} = 0$

(2) $\displaystyle\int_0^{\frac{\pi}{2}} 3\sin x dx = \big[-3\cos x\big]_0^{\frac{\pi}{2}} = 3$

(3)
$$\begin{aligned}\int_0^{\frac{\pi}{2}} (\cos x + 2\sin x)dx &= \big[\sin x - 2\cos x\big]_0^{\frac{\pi}{2}} \\ &= \sin\left(\frac{\pi}{2}\right) - 2\cos\left(\frac{\pi}{2}\right) - (\sin 0 - 2\cos 0) \\ &= 1 - (-2) \\ &= 3\end{aligned}$$

(4) $\displaystyle\int_0^{\pi} (3\cos x - \sin x)dx = \big[3\sin x + \cos x\big]_0^{\pi} = -2$

＜ 25 ページ. 微分・不定積分の練習 ＞

問 1 の解答

(1) $(1)' = 0$

(2) $(x^5)' = 5x^4$

(3) $(x^n)' = nx^{n-1}$

(4) $\left(\dfrac{1}{x^2}\right)' = -\dfrac{2}{x^3}$

(5) $\left(\dfrac{1}{x^3}\right)' = -\dfrac{3}{x^4}$

(6) $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$

(7) $(\sqrt[3]{x})' = \dfrac{1}{3\sqrt[3]{x^2}}$

(8) $(\sqrt[4]{x^5})' = \dfrac{5}{4}\sqrt[4]{x}$

(9) $(x\sqrt{x})' = \frac{3}{2}\sqrt{x}$

(10) $\left(\dfrac{1}{\sqrt{x}}\right)' = -\dfrac{1}{2x\sqrt{x}}$

(11) $\left(\dfrac{1}{\sqrt[3]{x^2}}\right)' = -\dfrac{2}{3x\sqrt[3]{x^2}}$

(12) $\left(\dfrac{1}{x\sqrt{x}}\right)' = -\dfrac{3}{2x^2\sqrt{x}}$

(13) $(2^x)' = 2^x \log 2$

(14) $(e^x)' = e^x$

(15) $(\log x)' = \dfrac{1}{x}$

(16) $(\sin x)' = \cos x$

(17) $(\cos x)' = -\sin x$

(18) $\left(\dfrac{1}{n+1}x^{n+1}\right)' = x^n$

(19) $(5x^4 - 6x^3 + 7x + 8)' = 20x^3 - 18x^2 + 7$

(20) $\left(\dfrac{3}{x^2} + \dfrac{4}{x}\right)' = -\dfrac{6}{x^3} - \dfrac{4}{x^2}$

(21) $\left(3\sqrt{x} + \dfrac{5}{\sqrt{x}}\right)' = \dfrac{3}{2\sqrt{x}} - \dfrac{5}{2x\sqrt{x}}$

(22) $(3e^x + 4\log x)' = 3e^x + \dfrac{4}{x}$

(23) $(3\sin x + 4\cos x)' = 3\cos x - 4\sin x$

(24) $\left(\dfrac{x^2+1}{x}\right)' = \left(x + \dfrac{1}{x}\right)' = 1 - \dfrac{1}{x^2}$

問 2 の解答

(1) $\displaystyle\int dx = x + C$

(2) $\displaystyle\int x^3 dx = \frac{1}{4}x^4 + C$

(3) $\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + C$

(4) $\displaystyle\int \frac{1}{x^2}dx = -\frac{1}{x} + C$

(5) $\displaystyle\int \frac{1}{x^3}dx = -\frac{1}{2x^2} + C$

(6) $\displaystyle\int \sqrt{x}dx = \frac{2}{3}x\sqrt{x} + C$

(7) $\displaystyle\int \sqrt[3]{x}dx = \frac{3}{4}x\sqrt[3]{x} + C$

(8) $\displaystyle\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + C$

(9) $\displaystyle\int \frac{1}{\sqrt[3]{x^4}}dx = -\frac{3}{\sqrt[3]{x}} + C$

(10) $\displaystyle\int 3^x dx = \frac{3^x}{\log 3} + C$

(11) $\displaystyle\int e^x dx = e^x + C$

(12) $\displaystyle\int \frac{1}{x}dx = \log x + C$

(13) $\displaystyle\int \cos x dx = \sin x + C$

(14) $\displaystyle\int \sin x dx = -\cos x + C$

(15) $\displaystyle\int (x+1)(x-1)dx = \int (x^2-1)dx$

$\displaystyle = \frac{1}{3}x^3 - x + C$

(16) $\displaystyle\int (3\cos x - 4\sin x + 5e^x)dx$

$= 3\sin x + 4\cos x + 5e^x + C$

(17) $\displaystyle\int \left(\frac{x^2+1}{x}\right)dx = \int \left(x + \frac{1}{x}\right)dx$

$\displaystyle = \frac{1}{2}x^2 + \log x + C$

＜ 26 ページ. 定積分の練習 ＞

問 1 の解答

(1) $\displaystyle\int_a^b dx = \int_a^b 1dx = \big[x\big]_a^b = b - a$

(2) $\displaystyle\int_a^b x^n dx = \left[\frac{1}{n+1}x^{n+1}\right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$

(3) $\displaystyle\int_a^b \frac{1}{x}dx = \big[\log x\big]_a^b = \log b - \log a = \log\left(\frac{b}{a}\right)$

(4) $\displaystyle\int_a^b e^x dx = \big[e^x\big]_a^b = e^b - e^a$

(5) $\displaystyle\int_a^b \cos x dx = \big[\sin x\big]_a^b = \sin b - \sin a$

(6) $\displaystyle\int_a^b \sin x dx = \big[-\cos x\big]_a^b = -\cos b + \cos a$

問 2 の解答

(1) $\displaystyle\int_4^{10} dx = \big[x\big]_4^{10} = 10 - 4 = 6$

(2) $\displaystyle\int_{-1}^1 (x^2 + x^3 + x^4)dx = \left[\frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5\right]_{-1}^1 = \frac{2}{3} + \frac{2}{5} = \frac{16}{15}$

(3) $\displaystyle\int_1^5 \frac{1}{x^2}dx = \left[-\frac{1}{x}\right]_1^5 = -\frac{1}{5} + \frac{1}{1} = \frac{4}{5}$

(4) $\displaystyle\int_1^2 \frac{1}{x^3}dx = \left[-\frac{1}{2x^2}\right]_1^2 = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$

(5) $\displaystyle\int_4^9 \sqrt{x}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9 = \frac{2}{3}(27 - 8) = \frac{38}{3}$

(6) $\displaystyle\int_1^8 \sqrt[3]{x}dx = \left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^8 = \frac{3}{4}(16 - 1) = \frac{45}{4}$

(7) $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}dx = \big[2\sqrt{x}\big]_0^9 = 6$

(8) $\displaystyle\int_1^{e^2} \frac{1}{x}dx = \big[\log x\big]_1^{e^2} = 2$

(9) $\displaystyle\int_2^4 \frac{3}{x}dx = \big[3\log x\big]_2^4 = 3\log 2$

(10) $\displaystyle\int_0^2 e^x dx = \big[e^x\big]_0^2 = e^2 - 1$

(11) $\displaystyle\int_{-1}^1 4e^x dx = \big[4e^x\big]_{-1}^1 = 4\left(e - \frac{1}{e}\right)$

(12) $\displaystyle\int_0^\pi \sin x dx = \big[-\cos x\big]_0^\pi = 2$

(13) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = \big[\sin x\big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$

(14) $\displaystyle\int_0^{\frac{\pi}{2}} 3\sin x dx = \big[-3\cos x\big]_0^{\frac{\pi}{2}} = 3$

< 27 ページ. 合成関数 >

問 1 の解答

(1) $g(f(x)) = 3(x^2+1) = 3x^2+3$, $f(g(x)) = (3x)^2+1 = 9x^2+1$

(2) $g(f(x)) = (\tan x)+2$, $f(g(x)) = \tan(x+2)$

(3) $g(f(x)) = x-1$, $f(g(x)) = \sqrt{x^2-1}$

(4) $g(f(x)) = \log_2(x^2+2)$, $f(g(x)) = (\log_2 x)^2+2$

問 2 の解答

(1) $f(x) = x^2-x+2$, $g(x) = x^7$

(2) $f(x) = 2x+3$, $g(x) = \cos x$

(3) $f(x) = 1-x^2$, $g(x) = \sqrt{x}$

< 28ページ. 微分記号 >

解答

(1) $\dfrac{dy}{dx} = 2x - 1$

(2) $\dfrac{dy}{dt} = -9.8$

(3) $\dfrac{d\ell}{dt} = 6t - 2$

(4) $\dfrac{dS}{dr} = 2\pi r$

(5) $\dfrac{dV}{dr} = 4\pi r^2$

< 29 ページ. 増分記号 $\varDelta$ (デルタ) >

解答

(1) $\displaystyle \lim_{\varDelta x \to 0} \frac{(x+\varDelta x)^5 - x^5}{\varDelta x} = (x^5)' = 5x^4$

(2) $\displaystyle \lim_{\varDelta t \to 0} \frac{\sin(t+\varDelta t) - \sin(t)}{\varDelta t} = (\sin t)' = \cos t$

(3) $\displaystyle \lim_{\varDelta u \to 0} \frac{\cos(u+\varDelta u) - \cos(u)}{\varDelta u} = (\cos u)' = -\sin u$

＜ 30 ページ. 合成関数の微分 1 ＞

解答

$$\frac{dy}{dx} = \left(\lim_{\Delta u \to 0} \frac{\cos(u + \Delta u) - \cos u}{\Delta u}\right) \times \left(\lim_{\Delta x \to 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x}\right)$$

$$= (\cos u)' \times (x^4)'$$

$$= -\sin(u) \times 4x^3$$

$$= -4x^3 \sin(x^4)$$

＜ 31 ページ. 合成関数の微分 2 ＞

問 1 の解答

(答) $\boxed{\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}}$

問 2 の解答

(1) $u = x^2 - 2x + 5$

$$\begin{aligned}\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} &= (u^3)' \times (x^2 - 2x + 5)' \\ &= 3u^2 \times (2x - 2) \\ &= 3(2x - 2)(x^2 - 2x + 5)^2\end{aligned}$$

(2) $u = 2x - 3$

$$\begin{aligned}\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} &= (\cos u)' \times (2x - 3)' \\ &= -\sin(u) \times 2 \\ &= -2\sin(2x - 3)\end{aligned}$$

(3) $u = x^5 - 2x^2$

$$\begin{aligned}\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} &= (\sin u)' \times (x^5 - 2x^2)' \\ &= \cos(u) \times (5x^4 - 4x) \\ &= (5x^4 - 4x)\cos(x^5 - 2x^2)\end{aligned}$$

$<$ 32 ページ. 合成関数の微分 3 $>$

問 1 の解答

(1) $\left((3x+5)^7\right)'$

$= 21(3x+5)^6$

(2) $\left((4x^2+9x)^6\right)'$

$= 6(8x+9)(4x^2+9x)^5$

(3) $\left((x^4-2x^3)^{10}\right)'$

$= 10(4x^3-6x^2)(x^4-2x^3)^9$

(4) $\left((x+e^x)^5\right)'$

$= 5(1+e^x)(x+e^x)^4$

(5) $\left((x^2+3\sin x)^5\right)'$

$= 5(2x+3\cos x)(x^2+3\sin x)^4$

(6) $\left((e^x-2\cos x)^8\right)'$

$= 8(e^x+2\sin x)(e^x-2\cos x)^7$

問 2 の解答

(1) $\left(\dfrac{1}{(x^4+7x^2)^3}\right)' = -\dfrac{3(4x^3+14x)}{(x^4+7x^2)^4}$

(2) $\left(\dfrac{1}{x^5+6x}\right)' = -\dfrac{5x^4+6}{(x^5+6x)^2}$

(3) $\left(\dfrac{1}{(2+\cos x)^3}\right)' = -\dfrac{-3\sin x}{(2+\cos x)^4}$

$= \dfrac{3\sin x}{(2+\cos x)^4}$

(4) $\left(\dfrac{1}{(1+e^x)^2}\right)' = -\dfrac{2e^x}{(1+e^x)^3}$

＜ 33 ページ. 合成関数の微分 4 ＞

問 1 の解答

(1) $\left(\sqrt[3]{(4x+6)^5}\right)' = \dfrac{20}{3}\sqrt[3]{(4x+6)^2}$

(2) $\left(\sqrt{(5x^2+6x)^3}\right)' = 3(5x+3)\sqrt{5x^2+6x}$

問 2 の解答

(1) $\left(\sqrt{x^3+4x^2}\right)' = \dfrac{3x^2+8x}{2\sqrt{x^3+4x^2}}$

(2) $\left(\sqrt[4]{2+\sin x}\right)' = \dfrac{\cos x}{4\sqrt[4]{(2+\sin x)^3}}$

問 3 の解答

(1) $$\left(\frac{1}{\sqrt{1+x^2}}\right)' = -\frac{2x}{2(1+x^2)\sqrt{1+x^2}} = -\frac{x}{(1+x^2)\sqrt{1+x^2}}$$

(2) $$\left(\frac{1}{x\sqrt{x}}\right)' = \left(x^{-\frac{3}{2}}\right)' = -\frac{3}{2}x^{-\frac{5}{2}} = -\frac{3}{2x^2\sqrt{x}}$$

＜ 34 ページ. 合成関数の微分 5 ＞

問 1 の解答

(1) $\left(\cos(4x+3)\right)'$
$= -4\sin(4x+3)$

(2) $\left(\cos(x^5-2x+1)\right)'$
$= -5(x^4-2)\sin(x^5-2x+1)$

問 2 の解答

(1) $\left(\sin(5x-4)\right)'$
$= 5\cos(5x-4)$

(2) $\left(\sin(x^6+7x^2-3)\right)'$
$= (6x^5+14x)\cos(x^6+7x^2-3)$

問 3 の解答

(1) $\left(e^{2x}\right)' = 2e^{2x}$

(2) $\left(e^{-\frac{x^2}{2}}\right)' = -xe^{-\frac{x^2}{2}}$

(3) $\left(e^{1+3x}\right)' = 3e^{1+3x}$

(4) $\left(e^{x+x^3}\right)' = (3x^2+1)e^{x+x^2}$

＜ 35 ページ. 合成関数の微分 6 ＞

問 1 の解答

(1) $y = \log(x^3 + 2x - 5)$

$$\frac{dy}{dx} = \frac{3x^2 + 2}{x^3 + 2x - 5}$$

(2) $y = \log(1 + \sin x)$

$$\frac{dy}{dx} = \frac{\cos x}{1 + \sin x}$$

(3) $y = \log(5 - \cos x)$

$$\frac{dy}{dx} = \frac{\sin x}{5 - \cos x}$$

問 2 の解答

(答)

$$\Big(\log\left(f(x)\right)\Big)' = \frac{f'(x)}{f(x)}$$

＜ 36 ページ. 合成関数の微分 7 ＞

問 1 の解答

(1) $\left(e^{ax+b}\right)' = ae^{ax+b}$　(2) $\left(\log(ax+b)\right)' = \dfrac{a}{ax+b}$

(3) $\left(\sin(ax+b)\right)' = a\cos(ax+b)$　(4) $\left(\cos(ax+b)\right)' = -a\sin(ax+b)$

問 2 の解答

(1) $\left(\dfrac{1}{7a}(ax+b)^7\right)' = (ax+b)^6$　(2) $\left(\dfrac{1}{(n+1)a}(ax+b)^{n+1}\right)' = (ax+b)^n$

(3) $\left(\dfrac{1}{a}e^{ax+b}\right)' = e^{ax+b}$　(4) $\left(\dfrac{1}{a}\log(ax+b)\right)' = \dfrac{1}{ax+b}$

(5) $\left(\dfrac{1}{a}\sin(ax+b)\right)' = \cos(ax+b)$　(6) $\left(\dfrac{1}{a}\cos(ax+b)\right)' = -\sin(ax+b)$

＜ 37 ページ. 対数微分法 ＞

問 1 の解答

(解) $\log y = \log 3^x = x \log 3$

$\dfrac{y'}{y} = \log 3$

$\underline{y' = y \times \log 3 = 3^x \log 3}$

問 2 の解答

(答) $(a^x)' = a^x \log a$

問 3 の解答

(答) $(e^x)' = e^x \log e = e^x$

＜ 38 ページ.$\log|x|$ の微分 ＞

解答

(1) $u = \sin x$ とおくと

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (\log|u|)' \times (\sin x)'$$
$$= \frac{1}{u} \times \cos x$$
$$= \frac{\cos x}{\sin x}$$

(2) $u = 2x^3 - 3x^2$ とおくと

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (\log|u|)' \times (2x^3 - 3x^2)'$$
$$= \frac{1}{u} \times (6x^2 - 6x)$$
$$= \frac{6x^2 - 6x}{2x^3 - 3x^2} \left(= \frac{6x - 6}{2x^2 - 3x} \right)$$

(3) $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

＜ 39 ページ. 不定積分の練習 1 ＞

問 1 の解答

(1) $(x^n)' = nx^{n-1} \quad \Leftrightarrow \quad \int nx^{n-1}dx = x^n + C$

(2) $(e^x)' = e^x \quad \Leftrightarrow \quad \int e^x dx = e^x + C$

(3) $(\log x)' = \dfrac{1}{x} \quad \Leftrightarrow \quad \int \dfrac{1}{x}dx = \log x + C$

(4) $(\sin x)' = \cos x \quad \Leftrightarrow \quad \int \cos x dx = \sin x + C$

(5) $(\cos x)' = -\sin x \quad \Leftrightarrow \quad \int (-\sin x)dx = \cos x + C$

問 2 の解答

(1) $\left(\dfrac{1}{3}e^{3x-2}\right)' = e^{3x-2} \quad \Leftrightarrow \int e^{3x-2}dx = \dfrac{1}{3}e^{3x-2} + C$

(2) $\left(\dfrac{1}{4}\sin(4x+5)\right)' = \cos(4x+5) \quad \Leftrightarrow \int \cos(4x+5)dx = \dfrac{1}{4}\sin(4x+5) + C$

(3) $\left(\dfrac{1}{5}\log(5x+6)\right)' = \dfrac{1}{5x+6} \quad \Leftrightarrow \int \dfrac{1}{5x+6}dx = \dfrac{1}{5}\log(5x+6) + C$

(4) $\left(-\dfrac{1}{7}\cos(7x-3)\right)' = \sin(7x-3) \quad \Leftrightarrow \int \sin(7x-3)dx = -\dfrac{1}{7}\cos(7x-3) + C$

問 3 の解答

(1) $\int e^{ax+b}dx = \dfrac{1}{a}e^{ax+b} + C$

(2) $\int \cos(ax+b)dx = \dfrac{1}{a}\sin(ax+b) + C$

(3) $\int \dfrac{1}{ax+b}dx = \dfrac{1}{a}\log(ax+b) + C$

(4) $\int \sin(ax+b)dx = -\dfrac{1}{a}\cos(ax+b) + C$

＜ 40 ページ. 不定積分の練習 2 ＞

問 1 の解答

(1) $\left(\frac{1}{7}x^7\right)' = x^6 \quad \Leftrightarrow \int x^6 dx = \frac{1}{7}x^7 + C$

(2) $\left(\frac{1}{21}(3x+5)^7\right)' = (3x+5)^6 \quad \Leftrightarrow \int (3x+5)^6 dx = \frac{1}{21}(3x+5)^7 + C$

(3) $\left(\frac{1}{7a}(ax+b)^7\right)' = (ax+b)^6 \quad \Leftrightarrow \int (ax+b)^6 dx = \frac{1}{7a}(ax+b)^7 + C$

(4) $\left(\frac{1}{(n+1)a}(ax+b)^{n+1}\right)' = (ax+b)^n \quad \Leftrightarrow \int (ax+b)^n dx = \frac{1}{(n+1)a}(ax+b)^{n+1} + C$

問 2 の解答

(1) $\int (4x+1)^2 \, dx = \frac{1}{12}(4x+1)^3 + C$

(2) $\int (5x-3)^6 \, dx = \frac{1}{35}(5x-3)^7 + C$

(3) $\int \sqrt{4x+3} dx = \frac{2}{3 \times 4}(4x+3)^{\frac{3}{2}} + C = \frac{1}{6}(4x+3)\sqrt{4x+3} + C$

(4) $\int \sqrt[3]{6x-5} dx = \frac{3}{4 \times 6}(6x-5)^{\frac{4}{3}} + C = \frac{1}{8}(6x-5)\sqrt[3]{6x-5} + C$

(5) $\int \frac{1}{(3x+1)^2} dx = -\frac{1}{3(3x+1)} + C$

(6) $\int \frac{1}{\sqrt{5x-2}} dx = \frac{2}{5}\sqrt{5x-2} + C$

問 3 の解答

(1) $\int e^{3x-2} dx = \frac{1}{3}e^{3x-2} + C$

(2) $\int \cos(5x+4) \, dx = \frac{1}{5}\sin(5x+4) + C$

(3) $\int \frac{1}{10x+13} dx = \frac{1}{10}\log(10x+13) + C$

(4) $\int \sin(6x-1) \, dx = -\frac{1}{6}\cos(6x-1) + C$