＜ 1 ページ. 定積分 3 ＞

問 1

(1) $\displaystyle\int dx = x + C$

(2) $\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + C$

(3) $\displaystyle\int \frac{dx}{x} = \log|x| + C$

(4) $\displaystyle\int e^x dx = e^x + C$

(5) $\displaystyle\int \cos x dx = \sin x + C$

(6) $\displaystyle\int \sin x dx = -\cos x + C$

(7) $\displaystyle\int \frac{1}{\cos^2 x}dx = \tan x + C$

(8) $\displaystyle\int \frac{1}{x^3}dx = -\frac{1}{2x^2} + C$

(9) $\displaystyle\int \sqrt{x}dx = \frac{2}{3}x\sqrt{x} + C$

(10) $\displaystyle\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + C$

問 2

(1) $\displaystyle\int_{-2}^{4} dx = \Big[\ x\ \Big]_{-2}^{4} = 6$

(2) $\displaystyle\int_{-1}^{2} x^7 dx = \left[\ \frac{1}{8}x^8\ \right]_{-1}^{2} = \frac{2^8}{8} - \frac{1}{8} = \frac{255}{8}$

(3) $\displaystyle\int_{1}^{e} \frac{dx}{x} = \Big[\log|x|\Big]_{1}^{e} = \log e - \log 1 = 1$

(4) $\displaystyle\int_{0}^{2} e^x dx = \Big[\ e^x\ \Big]_{0}^{2} = e^2 - 1$

(5) $\displaystyle\int_{0}^{\frac{\pi}{3}} \cos x dx = \Big[\sin x\Big]_{0}^{\frac{\pi}{3}} = \frac{\sqrt{3}}{2}$

(6) $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin x dx = \Big[-\cos x\Big]_{0}^{\frac{\pi}{2}} = 1$

(7) $\displaystyle\int_{0}^{\frac{\pi}{3}} \frac{dx}{\cos^2 x} = \Big[\tan x\Big]_{0}^{\frac{x}{3}} = \sqrt{3}$

(8) $\displaystyle\int_{1}^{3} \frac{1}{x^3}dx = \left[-\frac{1}{2x^2}\right]_{1}^{3} = -\frac{1}{18} + \frac{1}{2} = \frac{4}{9}$

(9) $\displaystyle\int_{0}^{9} \sqrt{x}dx = \left[\frac{2}{3}x\sqrt{x}\right]_{0}^{9} = \frac{2}{3}9\sqrt{9} = 18$

(10) $\displaystyle\int_{1}^{4} \frac{1}{\sqrt{x}}dx = \Big[2\sqrt{x}\Big]_{1}^{4} = 2\sqrt{4} - 2 = 2$

＜ 2ページ. 定積分4 ＞

問1

(1) $\displaystyle\int (ax+b)^n dx = \frac{1}{(n+1)a}(ax+b)^{n+1} + C$

(2) $\displaystyle\int \frac{1}{(ax+b)^2}dx = -\frac{1}{a(ax+b)} + C$

(3) $\displaystyle\int \sqrt{ax+b}\,dx = \frac{2}{3a}(ax+b)\sqrt{ax+b} + C$

(4) $\displaystyle\int \frac{1}{\sqrt{ax+b}}dx = \frac{2}{a}\sqrt{ax+b} + C$

(5) $\displaystyle\int \frac{1}{ax+b}dx = \frac{1}{a}\log|ax+b| + C$

(6) $\displaystyle\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$

(7) $\displaystyle\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$

(8) $\displaystyle\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + C$

問2

(1) $\displaystyle\int_0^2 (3x-1)^4 dx = \left[\frac{1}{15}(3x-1)^5\right]_0^2$

$\displaystyle = \frac{5^5}{15} - \frac{(1)^5}{15} = \frac{1042}{5}$

(2) $\displaystyle\int_1^3 \frac{dx}{(3x+2)^2} = \left[-\frac{1}{3(3x+2)}\right]_1^3$

$\displaystyle = -\frac{1}{3\times 11} + \frac{1}{3\times 5} = \frac{1}{3}\times\frac{6}{55} = \frac{2}{55}$

(3) $\displaystyle\int_1^6 \sqrt{3x-2}\,dx = \left[\frac{2}{9}(3x-2)\sqrt{3x-2}\right]_1^6$

$\displaystyle = \frac{2}{9}(16\sqrt{16} - 1\sqrt{1}) = 14$

(4) $\displaystyle\int_1^3 \frac{1}{\sqrt{2x-2}}dx = \left[\sqrt{2x-2}\right]_1^3$

$= \sqrt{4} - \sqrt{0} = 2$

(5) $\displaystyle\int_0^1 \frac{1}{3x+4}dx = \left[\frac{1}{3}\log|3x+4|\right]_0^1$

$\displaystyle = \frac{1}{3}\log 7 - \frac{1}{3}\log 4$

(6) $\displaystyle\int_0^{\frac{\pi}{3}} \cos(2x)dx = \left[\frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{3}}$

$\displaystyle = \frac{1}{2}\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{4}$

(7) $\displaystyle\int_0^{\frac{\pi}{2}} \sin\left(\frac{1}{3}x\right)dx = \left[-3\cos\left(\frac{1}{3}x\right)\right]_0^{\frac{\pi}{2}}$

$\displaystyle = -3\cos\left(\frac{\pi}{6}\right) + 3\cos 0 = 3 - \frac{3\sqrt{3}}{2}$

(8) $\displaystyle\int_1^2 e^{3x-3}dx = \left[\frac{1}{3}e^{3x-3}\right]_1^2$

$\displaystyle = \frac{1}{3}e^3 - \frac{1}{3}$

＜ 3 ページ. 定積分 5 ＞

解答

(1) $\displaystyle \int_1^4 (5t - 3t^2)dt = \left[\frac{5}{2}t^2 - t^3\right]_1^4 = \left(\frac{80}{2} - 64\right) - \left(\frac{5}{2} - 1\right)$

$\displaystyle = \frac{75}{2} - 63 = -\frac{51}{2}$

(2) $\displaystyle \int_0^{10} (4\pi r^2)dr = \left[\frac{4}{3}\pi r^3\right]_0^{10} = \frac{4000}{3}\pi$

(3) $\displaystyle \int_0^{\frac{\pi}{2}} \sin^2\theta d\theta = \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} - \frac{1}{2}\cos(2\theta)\right) d\theta = \left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\right]_0^{\frac{\pi}{2}}$

$\displaystyle = \frac{\pi}{4}$

(4) $\displaystyle \int_a^b t^n dt = \left[\frac{1}{n+1}t^{n+1}\right]_a^b = \frac{1}{n+1}b^{n+1} - \frac{1}{n+1}a^{n+1}$

(5) $\displaystyle \int_1^9 t\sqrt{t}dt = \int_1^9 t^{\frac{3}{2}}dt = \left[\frac{2}{5}t^{\frac{5}{2}}\right]_1^9 = \frac{2}{5} \times \left(9^{\frac{5}{2}} - 1^{\frac{5}{2}}\right) = \frac{2}{5}(243 - 1)$

$\displaystyle = \frac{484}{5}$

< 4ページ. 定積分6 >

解答

$$\int (2x-1)\sqrt{x^2-x+4}\,dx = \frac{2}{3}(x^2-x+4)^{\frac{3}{2}} + C$$

$$= \frac{3}{2}(x^2-x+4)\sqrt{x^2-x+4} + C$$

$$\int_1^4 (2x-1)\sqrt{x^2-x+4}\,dx = \left[\frac{2}{3}(x^2-x+4)\sqrt{x^2-x+4}\right]_1^4$$

$$= \frac{2}{3}(16\sqrt{16} - 4\sqrt{4}) = \frac{112}{3}$$

＜ 5 ページ. 定積分の置換積分法 1 ＞

解答

$$\int_2^3 (2x-3)\sqrt{x^2-3x+4}dx$$

$$= \int_2^4 \sqrt{u}\,du = \left[\frac{2}{3}u\sqrt{u}\right]_2^4 = \frac{2}{3}4\sqrt{4} - \frac{2}{3}2\sqrt{2} = \frac{16}{3} - \frac{4\sqrt{2}}{3}$$

＜ 6 ページ. 定積分の置換積分法 2 ＞

解答

(1) $\displaystyle\int_{-1}^{1}(x^3+x+2)^4(3x^2+1)dx$

$$= \int_0^4 u^4 du = \left[\ \frac{1}{5}u^5\ \right]_0^4 = \frac{4^5}{5} = \frac{1024}{5}$$

(2) $\displaystyle\int_0^{\pi} \cos\left(3x - \frac{\pi}{2}\right) \times 3dx$

$$= \int_{-\frac{\pi}{2}}^{\frac{5}{2}\pi} \cos u du = \left[\ \sin u\ \right]_{-\frac{\pi}{2}}^{\frac{5}{2}\pi} = \sin\left(\frac{5}{2}\pi\right) - \sin\left(-\frac{\pi}{2}\right)$$

$$= 1 - (-1) = 2$$

＜ 7 ページ. 定積分の置換積分法 3 ＞

解答

(1) $\displaystyle\int_0^{\frac{\pi}{2}} \cos(4x-\pi)\ dx$

$\displaystyle= \frac{1}{4}\int_{-\pi}^{\pi} \cos u du = \frac{1}{4}\Big[\sin u\Big]_{-\pi}^{\pi} = \frac{1}{4}\left(\sin\pi - \sin(-\pi)\right) = 0$

(2) $\displaystyle\int_0^{\pi} \sin\left(3x-\frac{\pi}{2}\right)\ dx$

$\displaystyle= \frac{1}{3}\int_{-\frac{\pi}{2}}^{\frac{5}{2}\pi} \sin u du = \frac{1}{3}\Big[-\cos u\Big]_{-\frac{\pi}{2}}^{\frac{5}{2}\pi} = \frac{1}{3}\left\{-\cos\left(\frac{5}{2}\pi\right) + \cos\left(-\frac{\pi}{2}\right)\right\}$

$= 0$

(3) $\displaystyle\int_{-1}^{2} (2x-1)^3\ dx$

$\displaystyle= \frac{1}{2}\int_{-3}^{3} u^3 du = \frac{1}{2}\left[\frac{1}{4}u^4\right]_{-3}^{3} = \frac{1}{8}\left(3^4 - (-3)^4\right) = 0$

＜ 8 ページ. 定積分の置換積分法 4 ＞

解答

(1) $\displaystyle\int_{\pi}^{3\pi} \cos\left(\frac{1}{2}x - \frac{\pi}{2}\right) dx$

$$= \int_0^{\pi} (\cos u) 2du = \Big[2\sin u\Big]_0^{\pi} = 2\sin\pi - 2\sin 0 = 0$$

(2) $\displaystyle\int_0^{\pi} \sin(4x - 3\pi) dx$

$$= \int_{-3\pi}^{\pi} \sin(u)\frac{1}{4}du = \left[-\frac{1}{4}\cos u\right]_{-3\pi}^{\pi}$$

$$= -\frac{1}{4}\cos\pi - \left(-\frac{1}{4}\cos(-3\pi)\right) = -\frac{1}{4}\times(-1) + \frac{1}{4}\times(-1) = 0$$

＜ 9ページ. 定積分の置換積分法 5 ＞

解答

$$\int_0^{\frac{1}{2}} \sqrt{1-x^2}dx$$

$$= \int_0^{\frac{\pi}{6}} \left(\sqrt{1-\sin^2 u}\right) \cos u du = \int_0^{\frac{\pi}{6}} \cos^2(u) du$$

$$= \int_0^{\frac{\pi}{6}} \left\{\frac{1}{2} + \frac{1}{2}\cos(2u)\right\} du = \left[\frac{u}{2} + \frac{1}{4}\sin(2u)\right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{12} + \frac{1}{4}\sin(\frac{\pi}{3}) - 0 = \frac{\pi}{12} + \frac{\sqrt{3}}{8}$$

＜ 10 ページ. 定積分の部分積分 ＞

解答

(1) $\displaystyle\int_0^1 x(x-1)^5 dx$

$$= \left[x \times \frac{1}{6}(x-1)^6\right]_0^1 - \int_0^1 1 \times \frac{1}{6}(x-1)^6 dx$$

$$= 0 - 0 - \left[\frac{1}{42}(x-1)^7\right]_0^1 = -\left(\frac{0}{42} - \left(\frac{1}{42} \times (-1)\right)\right) = -\frac{1}{42}$$

(2) $\displaystyle\int_1^e x \log x dx$

$$= \left[\frac{x^2}{2}\log x\right]_1^e - \int_1^e \frac{x^2}{2} \times \frac{1}{x} dx = \frac{e^2}{2}\log e - \frac{1}{2}\log 1 - \left[\frac{x^2}{4}\right]_1^e$$

$$= \frac{e^2}{2} - \left(\frac{e^2}{4} - \frac{1}{4}\right) = \frac{e^2}{4} + \frac{1}{4}$$

＜ 11 ページ. 面積 1 ＞

解答

$$S = \int_{-1}^{1} \Big\{(-x^2+2x+3)-(x^2+2x+1)\Big\}dx$$

$$= \int_{-1}^{1} \Big\{(-2x^2+2)\Big\}dx = \left[-\frac{2}{3}x^3+2x\right]_{-1}^{1}$$

$$= \left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right) = 4-\frac{4}{3} = \frac{8}{3}$$

＜ 12ページ. 面積2 ＞

解答

問1

$$S = \int_a^b \Big\{ f(x) - g(x) \Big\} dx$$

問2

$$S = \int_{-1}^{2} \Big\{ (-x^2 + 2x + 3) - (x^2 - 1) \Big\} dx = \int_{-1}^{2} \Big\{ -2x^2 + 2x + 4 \Big\} dx$$

$$= \left[-\frac{2}{3}x^3 + x^2 + 4x \right]_{-1}^{2} = (-\frac{16}{3} + 4 + 8) - (\frac{2}{3} + 1 - 4)$$

$$= -\frac{18}{3} + 12 + 3 = -6 + 15 = 9$$

＜ 13 ページ. 体積 1 ＞

解答

$$
\begin{aligned}
V &= \lim_{n\to\infty} V_n \\
&= \lim_{n\to\infty} \frac{25\times 7}{12} \times \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) \\
&= \frac{25\times 7}{12} \times 1 \times 2 = \frac{25\times 7}{6} = \frac{175}{6}
\end{aligned}
$$

＜ 14 ページ. 体積 2 ＞

解答

$$V = \int_0^7 f(x)dx$$

$$= \int_0^7 \frac{25}{98}x^2 dx = \left[\frac{25}{3 \times 98} \times x^3\right]_0^7 = \frac{25 \times 7^3}{3 \times 98} = \frac{25 \times 7 \times 49}{3 \times 2 \times 49}$$

$$= \frac{175}{6}$$

＜ 15 ページ. 体積 3 ＞

解答

(1) $f(x) = \dfrac{1}{2} \times \dfrac{4}{9}x \times \dfrac{x}{3} \times \sin 30^\circ = \dfrac{2x^2}{27} \times \dfrac{1}{2} = \dfrac{x^2}{27}$

(2) $V = \displaystyle\int_0^9 f(x)dx = \int_0^9 \frac{x^2}{27}dx = \left[\frac{x^3}{81}\right]_0^9 = \frac{9^3}{81} = 9$

＜ 16ページ. 体積4 ＞

解答

問1

$$f(x) = \pi\left(\frac{x}{4}\right)^2 = \frac{\pi x^2}{16}$$

$$V = \int_0^8 \frac{\pi}{16}x^2 dx = \left[\frac{\pi x^3}{3\times 16}\right]_0^8 = \frac{\pi\times 8^3}{3\times 16} = \frac{\pi\times 64}{3\times 2} = \frac{32}{3}\pi$$

問2

$$f(x) = \left(\frac{1}{3}x\right)^2 = \frac{x^2}{9}$$

$$V = \int_0^6 \frac{x^2}{9}dx = \left[\frac{x^3}{27}\right]_0^6 = \frac{6^3}{27} = 8$$

＜ 17 ページ. 体積 5 ＞

解答

$$
\begin{aligned}
V &= \int_{-r}^{r} \pi \left(\sqrt{r^2 - x^2}\right)^2 dx \\
&= \pi \left[r^2 x - \frac{x^3}{3}\right]_{-r}^{r} = \pi \left\{ \left(r^3 - \frac{r^3}{3}\right) - \left(-r^3 + \frac{r^3}{3}\right) \right\} \\
&= \pi r^3 \left\{ 1 - \frac{1}{3} + 1 - \frac{1}{3} \right\} = \frac{4}{3}\pi r^3
\end{aligned}
$$

＜ 18 ページ. 体積 6 ＞

解答

$$V = \int_0^{r\cos\theta} \pi\big((\tan\theta)x\big)^2 dx + \int_{r\cos\theta}^{r} \pi\left(\sqrt{r^2 - x^2}\right)^2 dx$$

$$= \pi\tan^2\theta\left[\ \frac{x^3}{3}\ \right]_0^{r\cos\theta} + \pi\left[\ r^2x - \frac{x^3}{3}\ \right]_{r\cos\theta}^{r}$$

$$= \pi\tan^2\theta\frac{r^3\cos^3\theta}{3} + \pi\left\{\frac{2}{3}r^3 - \left(r^3\cos\theta - \frac{r^3\cos^3\theta}{3}\right)\right\}$$

$$= \frac{\pi r^3}{3}\Big\{\tan^2\theta\cos^3\theta + 2 - 3\cos\theta + \cos^3\theta\Big\}$$

$$= \frac{\pi r^3}{3}\Big\{\big(\tan^2\theta + 1\big)\cos^3\theta + 2 - 3\cos\theta\Big\}$$

$$= \frac{\pi r^3}{3}\Big\{\cos\theta + 2 - 3\cos\theta\Big\}$$

$$= \frac{\pi r^3}{3}\big(2 - 2\cos\theta\big)$$

$$= \frac{2}{3}\pi r^3\big(1 - \cos\theta\big)$$

< 19ページ. 体積7 >

解答

問1

$a = \frac{1}{3}x + 1$

問2

$b = \frac{1}{3}x + 2$

問3

$$S(x) = \frac{\left(\frac{1}{3} + 1\right) + \left(\frac{1}{3}x + 2\right)}{2} \times 4$$

$$= 2\left\{\frac{2}{3}x + 3\right\} = \frac{4}{3}x + 6$$

問4

$$V = \int_0^3 S(x)dx = \int_0^3 \left(\frac{4}{3}x + 6\right) dx = \left[\frac{2}{3}x^2 + 6x\right]_0^3$$

$$= 6 + 18 = 24$$

＜ 20 ページ. 体積 8 ＞

解答

$$
\begin{aligned}
S(x) &= \int_0^3 (5 - x + 0.2y)\,dy \\
&= \Big[5y - xy + 0.1y^2\Big]_{y=0}^{y=3} \\
&= 15 - 3x + 0.9 = 15.9 - 3x
\end{aligned}
$$

$$
\begin{aligned}
V &= \int_0^2 S(x)dx \\
&= \int_0^2 (15.9 - 3x)dx = \Big[15.9x - \frac{3}{2}x^2\Big]_0^2 \\
&= 31.8 - 6 = 25.8
\end{aligned}
$$

＜ 21 ページ. 体積 9 ＞

解答

$$S(x) = \int_0^3 \left(3 - \frac{x^2}{3} + \frac{xy}{2} + 2y - y^2\right) dy$$

$$= \left[3y - \frac{x^2}{2}y + \frac{xy^2}{4} + y^2 - \frac{y^3}{3}\right]_{y=0}^{y=3}$$

$$= 9 - \frac{3}{2}x^2 + \frac{9}{4}x + 9 - 9 = 9 - \frac{3}{2}x^2 + \frac{9}{4}x$$

$$V = \int_0^2 S(x)dx$$

$$= \int_0^2 \left(9 - \frac{3}{2}x^2 + \frac{9}{4}x\right) dx$$

$$= \left[9x - \frac{1}{2}x^3 + \frac{9}{8}x^2\right]_0^2 = 18 - \frac{8}{2} + \frac{9}{2} = 18 + \frac{1}{2}$$

$$= 18.5 \qquad \left(= \frac{37}{2}\right)$$

$<$ 22ページ. 体積10 $>$

解答

$$S(y) = \int_0^3 \left(\frac{1}{3}x - \frac{1}{4}y + 2\right) dx$$

$$= \left[\frac{x^2}{6} - \frac{y}{4}x + 2x\right]_{x=0}^{x=3} = \frac{3}{2} - \frac{3y}{4} + 6 = \frac{15}{2} - \frac{3}{4}y$$

$$V = \int_0^4 S(y)dy$$

$$= \int_0^4 \left(\frac{15}{2} - \frac{3}{4}y\right) dy = \left[\frac{15}{2}y - \frac{3y^2}{8}\right]_0^4 = 30 - 6 = 24$$

＜ 23 ページ. 累次積分 1 ＞

解答

$$\int_1^4 \left\{ \int_1^2 \left(3 - 6x^2 + 4xy\right) dy \right\} dx$$

$$= \int_1^4 \left\{ \left[3y - 6x^2 y + 2xy^2 \right]_{y=1}^{y=2} \right\} dx$$

$$= \int_1^4 \left\{ (6 - 12x^2 + 8x) - (3 - 6x^2 + 2x) \right\} dx$$

$$= \int_1^4 \left(3 - 6x^2 + 6x\right) dx = \left[3x - 2x^3 + 3x^2 \right]_1^4$$

$$= (12 - 128 + 48) - (3 - 2 + 3) = -68 - 4 = -72$$

＜ 24 ページ. 累次積分 2 ＞

解答

$$\int_2^3 \left\{ \int_1^2 \left(4 - x + xy + y^2\right) dx \right\} dy$$

$$= \int_2^3 \left\{ \left[4x - \frac{x^2}{2} + \frac{x^2}{2}y + y^2 x \right]_{x=1}^{x=2} \right\} dy$$

$$= \int_2^3 \left\{ \left(8 - 2 + 2y + 2y^2\right) - \left(4 - \frac{1}{2} + \frac{1}{2}y + y^2\right) \right\} dy$$

$$= \int_2^3 \left\{ \frac{5}{2} + \frac{3}{2}y + y^2 \right\} dy = \left[\frac{5}{2}y + \frac{3}{4}y^2 + \frac{1}{3}y^3 \right]_{y=2}^{y=3}$$

$$= \left(\frac{15}{2} + \frac{27}{4} + \frac{27}{3} \right) - \left(\frac{10}{2} + \frac{12}{4} + \frac{8}{3} \right)$$

$$= \frac{5}{2} + \frac{15}{4} + \frac{19}{3} = \frac{25}{4} + \frac{19}{3} = \frac{75 + 76}{12} = \frac{151}{12}$$

＜ 25 ページ. 長方形領域の 2 重積分 1 ＞

解答

$$\iint_D \left(2xy + 3y^2\right) dxdy$$

$$= \int_{-1}^{1} \left\{ \int_0^2 \left(2xy + 3y^2\right) dy \right\} dx = \int_{-1}^{1} \left\{ \left[xy^2 + y^3 \right]_{y=0}^{y=2} \right\} dx$$

$$= \int_{-1}^{1} \left\{ 4x + 8 \right\} dx = \left[2x^2 + 8x \right]_{-1}^{1}$$

$$= (2 + 8) - (2 - 8) = 16$$

＜ 26 ページ. 長方形領域の 2 重積分 2 ＞

解答

(1) $\displaystyle\iint_D x^3 \sin y\, dxdy$

$$= \left(\int_0^2 x^3 dx\right)\left(\int_0^\pi \sin y\, dy\right) = \left(\left[\ \frac{1}{4}x^4\ \right]_0^2\right)\left(\left[-\cos y\right]_0^\pi\right)$$

$$= \left(\ \frac{16}{4}\ \right) \times \left(-(-1) - (-1)\right) = 4 \times 2 = 8$$

(2) $\displaystyle\iint_D e^{-2x-3y} dxdy$

$$= \left(\int_0^1 e^{-2x} dx\right)\left(\int_0^1 e^{-3y} dy\right) = \left(\left[\ \frac{1}{-2}e^{-2x}\ \right]_0^1\right)\left(\left[\frac{1}{-3}e^{-3y}\right]_0^1\right)$$

$$= \left(-\frac{1}{2}e^{-2} + \frac{1}{2}\right)\left(-\frac{1}{3}e^{-3} + \frac{1}{3}\right)$$

$$= \frac{1}{6} - \frac{1}{6}e^{-3} - \frac{1}{6}e^{-2} + \frac{1}{6}e^{-5}$$

＜ 27 ページ. 一般領域の 2 重積分 1 ＞

解答

$$\iint_D (x+y)\,dxdy$$

$$= \int_1^2 \left\{ \int_0^1 (x+y)dy \right\} dx + \int_2^3 \left\{ \int_1^2 (x+y)dy \right\} dx$$

$$= \int_1^2 \left(\left[xy + \frac{1}{2}y^2 \right]_{y=0}^{y=1} \right) dx + \int_2^3 \left(\left[xy + \frac{1}{2}y^2 \right]_{y=1}^{y=2} \right) dx$$

$$= \int_1^2 \left(x + \frac{1}{2} \right) dx + \int_2^3 \left\{ (2x+2) - \left(x + \frac{1}{2} \right) \right\} dx$$

$$= \left[\frac{1}{2}x^2 + \frac{1}{2}x \right]_1^2 + \left[\frac{1}{2}x^2 + \frac{3}{2}x \right]_2^3$$

$$= \left\{ (2+1) - \left(\frac{1}{2} + \frac{1}{2} \right) \right\} + \left\{ \left(\frac{9}{2} + \frac{9}{2} \right) - (2+3) \right\}$$

$$= 2 + 9 - 5 = 6$$

＜ 28 ページ. 一般領域の 2 重積分 2 ＞

解答

$$\iint_D \left(2xy - y^2\right) dxdy$$

$$= \int_0^1 \left\{ \int_0^{-x+1} \left(2xy - y^2\right) dy \right\} dx$$

$$= \int_0^1 \left\{ \left[xy^2 - \frac{1}{3}y^3 \right]_{y=0}^{y=-x+1} \right\} dx$$

$$= \int_0^1 \left(x\left(-x+1\right)^2 - \frac{1}{3}\left(-x+1\right)^3 \right) dx$$

$$= \int_0^1 \left(\frac{4}{3}x^3 - 3x^2 + 2x - \frac{1}{3} \right) dx$$

$$= \left[\frac{1}{3}x^4 - x^3 + x^2 - \frac{1}{3}x \right]_0^1$$

$$= \frac{1}{3} - 1 + 1 - \frac{1}{3} = 0$$

＜ 29 ページ. 一般領域の 2 重積分 3 ＞

解答

$$D = \left\{ (x, y) : \begin{array}{c} 0 \leqq y \leqq 1 \\ 0 \leqq x \leqq y \end{array} \right\}$$

$$\iint_D (x + xy) dx dy$$

$$= \int_0^1 \left\{ \int_0^y (x + xy)\, dx \right\} dy = \int_0^1 \left\{ \left[\frac{x^2}{2} + \frac{x^2}{2} y \right]_{x=0}^{x=y} \right\} dy$$

$$= \int_0^1 \left(\frac{y^2}{2} + \frac{y^3}{2} \right) dy = \left[\frac{y^3}{6} + \frac{y^4}{8} \right]_{y=0}^{y=1}$$

$$= \frac{1}{6} + \frac{1}{8} = \frac{4 + 3}{24} = \frac{7}{24}$$

＜ 30ページ. 面積比 ＞

解答

(1) $\Delta(u,v)=4$

$$\Delta(x,y)=\begin{vmatrix} 6 & 4 \\ 2 & 8 \end{vmatrix}=48-8=40$$

$$\frac{\Delta(x,y)}{\Delta(u,v)}=\frac{40}{4}=10$$

(2) $\dfrac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}=12-2=10$

＜ 31 ページ. 重積分の変数変換１ ＞

解答

$$\iint_D (x-y)\,dxdy$$

$$= \iint_{\Omega} \Big\{(3u+2v)-(u+4v)\Big\} 10dudv$$

$$= 10 \times \int_0^2 \left\{ \int_0^2 (2u-2v)\,du \right\} dv = 10 \int_0^2 \left\{ \Big[u^2 - 2vu\Big]_{u=0}^{u=2} \right\} dv$$

$$= 10 \int_0^2 \Big\{4-4v\Big\} dv = 10 \Big[4v - 2v^2\Big]_{v=0}^{v=2} = 10 \times (8-8)$$

$$= 0$$

＜ 32 ページ. 重積分の変数変換 2 ＞

解答

$$\Omega = \left\{ (r\ ,\theta)\ :\ 0 \leqq r \leqq R\ ,\ \ 0 \leqq \theta \leqq \frac{\pi}{2}\ \right\}$$

$$\iint_D e^{-x^2-y^2}dxdy$$

$$= \iint_\Omega e^{-r^2} r\ drd\theta = \int_0^{\frac{\pi}{2}} \left\{ \left[-\frac{1}{2}e^{-r^2} \right]_{r=0}^{r=R} \right\} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(-\frac{1}{2}e^{-R^2} + \frac{1}{2} \right) d\theta = \frac{\pi}{4}\left(1 - e^{-R^2} \right)$$

＜ 33 ページ. 質量と重心 1 ＞

解答

$$g = \frac{1}{M}\Big\{m_1x_1 + m_2x_2 + \cdots + m_{n-1}x_{n-1} + m_nx_n\Big\}$$

＜ 35 ページ. 質量と重心 3 ＞

解答

$$M = \int_0^2 f(x)dx$$

$$= \int_0^2 \left(-x^2 + 2x\right) dx = \left[-\frac{1}{3}x^3 + x^2\right]_0^2 = -\frac{8}{3} + 4 = \frac{4}{3}$$

$$g = \frac{1}{M}\int_0^2 xf(x)dx$$

$$= \frac{1}{\frac{4}{3}}\int_0^2 \left(-x^3 + 2x^2\right) dx = \frac{3}{4}\left[-\frac{x^4}{4} + \frac{2}{3}x^3\right]_0^2 = \frac{3}{4}\left\{-\frac{16}{4} + \frac{16}{3}\right\}$$

$$= \frac{3}{4} \times 16 \times \left(-\frac{1}{4} + \frac{1}{3}\right) = 12 \times \frac{-3+4}{12}$$

$$= 1$$

＜ 36 ページ. 質量と重心 4 ＞

解答

$$g_Y = \frac{1}{M}\iint_D y\, f(x,y)dxdy$$

$$= \frac{1}{3}\iint_{D_1} y\, dxdy + \frac{1}{3}\iint_{D_2} y\, dxdy$$

$$= \frac{1}{3}\int_0^1 \left\{\int_0^{2x} y\, dy\right\} dx + \frac{1}{3}\int_1^3 \left\{\int_0^{-x+3} y\, dy\right\} dx$$

$$= \frac{1}{3}\int_0^1 \left\{\left[\frac{1}{2}y^2\right]_{y=0}^{y=2x}\right\} dx + \frac{1}{3}\int_1^3 \left\{\left[\frac{1}{2}y^2\right]_{y=0}^{y=-x+3}\right\} dx$$

$$= \frac{1}{3}\int_0^1 \left(2x^2\right) dx + \frac{1}{3}\int_1^3 \frac{1}{2}\left(-x+3\right)^2 dx$$

$$= \frac{1}{3}\left\{\left[\frac{2}{3}x^3\right]_0^1 + \left[-\frac{1}{6}\left(-x+3\right)^3\right]_1^3\right\}$$

$$= \frac{1}{3}\left\{\frac{2}{3} + \left(-0 + \frac{8}{6}\right)\right\} = \frac{1}{3} \times \frac{2+4}{3} = \frac{6}{9}$$

$$= \frac{2}{3}$$

＜ 37 ページ. 広義積分 1 ＞

解答

(1) $$\int_0^\infty e^{-\lambda x}dx = \lim_{b\to\infty}\int_0^b e^{-\lambda x}dx = \lim_{b\to\infty}\left[-\frac{1}{\lambda}e^{-\lambda x}\right]_0^b$$

$$= \lim_{b\to\infty}\left(-\frac{1}{\lambda}e^{-\lambda b} + \frac{1}{\lambda}\right) = \frac{1}{\lambda}$$

(2) $$\int_1^\infty \frac{1}{x^2}dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^2}dx$$

$$= \lim_{b\to\infty}\left[-\frac{1}{x}\right]_1^b = \lim_{b\to\infty}\left(-\frac{1}{b} + \frac{1}{1}\right) = 1$$

(3) $$\int_1^\infty \frac{1}{x^r}dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^r}dx$$

$$= \lim_{b\to\infty}\left[\frac{1}{-r+1}x^{-r+1}\right]_1^b$$

$$= \lim_{b\to\infty}\left(\frac{1}{-r+1}b^{-r+1} - \frac{1}{-r+1}\right)$$

$$= \lim_{b\to\infty}\left(-\frac{1}{(r-1)b^{r-1}} + \frac{1}{r-1}\right) = \frac{1}{r-1}$$

＜ 38 ページ. 広義積分 2 ＞

解答

$$\int_0^\infty \int_0^\infty e^{-x^2-y^2} dxdy = \lim_{R\to\infty} \iint_{D_R} e^{-x^2-y^2} dxdy$$

$$= \lim_{R\to\infty} \frac{\pi}{4} \left(1 - e^{-R^2}\right) = \frac{\pi}{4}$$

＜ 40 ページ. 広義積分 4 ＞

解答

(1) $\displaystyle\int_{-\infty}^{\infty} e^{-\frac{x^2}{6}} dx$

$$= \int_{-\infty}^{\infty} e^{-t^2} \sqrt{6}\, dt = \sqrt{6} \int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{6\pi}$$

(2) $\displaystyle\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\lambda}} dx$

$$= \int_{-\infty}^{\infty} e^{-t^2} \sqrt{2\lambda}\, dt = \sqrt{2\lambda} \int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{2\lambda\pi}$$