

<1 ページ 微分の復習 1>

解答

問 1

$$(1) \frac{d}{dx}(k) = 0$$

$$(2) \frac{d}{dx}(x^r) = rx^{r-1}$$

$$(3) \frac{d}{dx}(\sin x) = \cos x$$

$$(4) \frac{d}{dx}(\cos x) = -\sin x$$

$$(5) \frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}$$

$$(6) \frac{d}{dx}(\log x) = \frac{1}{x}$$

$$(7) \frac{d}{dx}(a^x) = a^x \log a$$

$$(8) \frac{d}{dx}(e^x) = e^x$$

問 2

$$(1) \frac{d}{dx}(2e^x + 3x^5) = 2e^x + 15x^4$$

$$(2) \frac{d}{dx}(3 \log x - 4 \sin x) = \frac{3}{x} - 4 \cos x$$

$$(3) \frac{d}{dx}(e^x \cos x) = e^x \cos x - e^x \sin x$$

$$(4) \frac{d}{dx}(x^4 \log x) = 4x^3 \log x + x^3$$

<2 ページ 微分の復習 2>

解答

問 1

$$\frac{d}{dx} \left(\frac{k}{g(x)} \right) = -\frac{kg'(x)}{\{g(x)\}^2} \left(= -\frac{k\left(\frac{d}{dx}g(x)\right)}{\{g(x)\}^2} \right)$$

問 2

$$(1) \frac{d}{dx} \left(\frac{3}{x^2} \right) = -\frac{6}{x^6} \quad (2) \frac{d}{dx} \left(\frac{5}{e^x} \right) = -\frac{5}{e^x}$$

$$(3) \frac{d}{dx} \left(\frac{x+1}{x^2} \right) = \frac{x^2 - (x+1)2x}{x^4} = -\frac{x^2 + 2x}{x^4} = -\frac{x+2}{x^3}$$

$$(4) \frac{d}{dx} \left(\frac{\cos x}{2 + \sin x} \right) = \frac{-\sin x(2 + \sin x) - \cos^2 x}{(2 + \sin x)^2} = \frac{-2\sin x - 1}{(2 + \sin x)^2}$$

問 3

$$(1) \frac{d}{dx} (\cos(x^5 - x^2)) \quad (2) \frac{d}{dx} (e^{-x^2+1}) = -2xe^{-x^2+1}$$
$$= -(5x^4 - 2x) \sin(x^5 - x^2)$$

$$(3) \frac{d}{dx} (1 + \sin x)^3 \quad (4) \frac{d}{dx} (\sqrt{x^2 + 4}) = \frac{2x}{2\sqrt{x^2+4}} = \frac{x}{\sqrt{x^2+4}}$$
$$= 3(1 + \sin x)^2 \cos x$$

$$(5) \frac{d}{dx} \log(3 + \cos x) = \frac{-\sin x}{3 + \cos x} \quad (6) \frac{d}{dx} (\sin(\sqrt{x})) = \frac{\cos(\sqrt{x})}{2\sqrt{x}}$$

<3 ページ 高階導関数>

解答

問1

$$(1) f(x) = 5x^3 - 4x^2 + 3x$$

$$f'(x) = 15x^2 - 8x + 3$$

$$f''(x) = 30x - 8$$

$$(2) f(x) = \cos(2x)$$

$$f'(x) = -2 \sin(2x)$$

$$f''(x) = -4 \cos(2x)$$

$$(3) f(x) = \log(3x)$$

$$f'(x) = \frac{3}{3x} = \frac{1}{x}$$

$$f''(x) = -\frac{1}{x^2}$$

問2

$$(1) f(x) = x^4 - 3x^2 + 5x$$

$$f'(x) = 4x^3 - 6x + 5$$

$$f''(x) = 12x^2 - 6$$

$$f'''(x) = 24x$$

$$(2) f(x) = \sin x$$

$$f'(x) = \cos x$$

$$f''(x) = -\sin x$$

$$f'''(x) = -\cos x$$

$$(3) f(x) = e^{3x}$$

$$f'(x) = 3e^{3x}$$

$$f''(x) = 9e^{3x}$$

$$f'''(x) = 27e^{3x}$$

$$(4) f(x) = x^2 \log x$$

$$f'(x) = 2x \log x + x$$

$$f''(x) = 2 \log x + 2 + 1 = 2 \log x + 3$$

$$f'''(x) = \frac{2}{x}$$

<4 ページ グラフの凹凸 1>

解答

問

(1) $y = x^3 + 3x + 2$

$$y' = 3x^2 + 3$$

$$y'' = 6x$$

x	...	0	...
y''	-	0	+
y	凸	2	凹

(2) $y = x^4 + 4x^3 + 10$

$$y' = 4x^3 + 12x^2$$

$$y'' = 12x^2 + 24x$$

$$= 12x(x + 2)$$

x	...	-2	...	0	...
y''	+	0	-	0	+
y	凹	-6	凸	10	凹

<5 ページ グラフの凹凸2>

解答

問

(1) $y = x^4 + 4x^3 + 10$

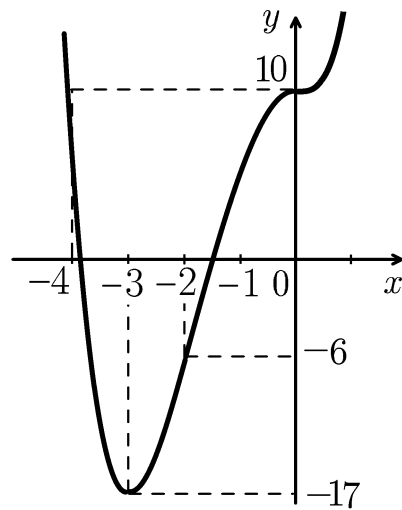
$$y' = 4x^3 + 12x^2$$

$$= 4x^2(x + 3)$$

$$y'' = 12x^2 + 24x$$

$$= 12x(x + 2)$$

x	...	-3	...	-2	...	0	...
y'	-	0	+	+	+	0	+
y''	+	+	+	0	-	0	+
y	↘	-17	↗	-6	↗	10	↗



<6 ページ 加速度 >

解答

問1

$$(\text{解}) v = \frac{dx}{dt} = -9.8t$$

$$a = \frac{dv}{dt} = -9.8$$

問2

$$(\text{解}) v = \frac{dx}{dt} = -2\pi \sin(2\pi t)$$

$$a = \frac{dv}{dt} = -4\pi^2 \cos(2\pi t) = -4\pi^2 x$$

<7ページ 関数の一次近似>

解答

問

$x \doteq a$ のとき

$$\sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a)$$

$$\sqrt{1.1} \doteq \sqrt{1} + \frac{1}{2\sqrt{1}}(1.1 - 1) = 1 + \frac{1}{2} \times 0.1 = 1 + \frac{1}{20} (= 1.05)$$

<8 ページ ロピタルの定理 1>

解答

問

$$(1) \lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{4x^3}{1} = 4$$

$$(2) \lim_{x \rightarrow 2} \frac{\log x - \log 2}{x - 2} = \lim_{x \rightarrow 2} \frac{\frac{1}{x}}{1} = \frac{1}{2}$$

$$(3) \lim_{x \rightarrow 1} \frac{x^3 - 1 - 3(x - 1)}{(x - 1)^2} = \lim_{x \rightarrow 1} \frac{3x^2 - 3}{2(x - 1)} = \lim_{x \rightarrow 1} \frac{6x}{2} = \frac{6}{2} = 3$$

$$(4) \lim_{x \rightarrow 2} \frac{x^4 - 16 - 32(x - 2) - 24(x - 2)^2}{(x - 2)^3} = \lim_{x \rightarrow 2} \frac{4x^3 - 32 - 48(x - 2)}{3(x - 2)^2}$$
$$= \lim_{x \rightarrow 2} \frac{12x^2 - 48}{6(x - 2)} = \lim_{x \rightarrow 2} \frac{24x}{6} = \frac{48}{6} = 8$$

<9 ページ ロピタルの定理 2>

解答

問

$$\begin{aligned}(1) \quad & \lim_{x \rightarrow a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2}{(x-a)^3} \\ &= \lim_{x \rightarrow a} \frac{f'(x) - f'(a) - f''(a)(x-a)}{3(x-a)^2} = \lim_{x \rightarrow a} \frac{f''(x) - f''(a)}{6(x-a)} = \lim_{x \rightarrow a} \frac{f'''(x)}{6} \\ &= \frac{f'''(a)}{6}\end{aligned}$$

$$\begin{aligned}(2) \quad & \lim_{x \rightarrow a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2 - \frac{1}{6}f'''(a)(x-a)^3}{(x-a)^4} \\ &= \frac{f''''(a)}{24}\end{aligned}$$

<10 ページ 関数の高次近似 >

解答

問

4次近似式

$x \doteq a$ のとき

$$f(x) \doteq f(a) + f'(a)(x - a) + \frac{1}{2}f''(a)(x - a)^2 \\ + \frac{1}{6}f'''(a)(x - a)^3 + \frac{1}{24}f^{(4)}(a)(x - a)^4$$

<11 ページ 高階微分係数 >

解答

問

$$(1) f(x) = e^x$$

$$f^{(4)}(x) = e^x \quad , \quad f^{(4)}(0) = 1$$

$$(2) f(x) = e^x$$

$$f^{(n)}(x) = e^x \quad , \quad f^{(n)}(0) = 1$$

$$(3) f(x) = \cos x$$

$$f^{(1)}(x) = -\sin x \quad f^{(1)}(0) = 0$$

$$f^{(2)}(x) = -\cos x \quad f^{(2)}(0) = -1$$

$$f^{(3)}(x) = \sin x \quad f^{(3)}(0) = 0$$

$$f^{(4)}(x) = \cos x \quad f^{(4)}(0) = 1$$

$$f^{(5)}(x) = -\sin x \quad f^{(5)}(0) = 0$$

$$f^{(6)}(x) = -\cos x \quad f^{(6)}(0) = -1$$

$$f^{(7)}(x) = \sin x \quad f^{(7)}(0) = 0$$

$$f^{(8)}(x) = \cos x \quad f^{(8)}(0) = 1$$

<12 ページ 関数の n 次近似 >

解答

問

$x \doteq a$ のとき

$$e^x \doteq e^a + e^a(x - a) + \frac{e^a}{2!}(x - a)^2 + \frac{e^a}{3!}(x - a)^3 + \cdots + \frac{e^a}{n!}(x - a)^n$$

<13 ページ テーラー展開>

解答

問

(1) $a = 3$

$$e^x = e^3 + e^3(x-3) + \frac{1}{2!}e^3(x-3)^2 + \frac{1}{3!}e^3(x-3)^3 + \cdots + \frac{1}{n!}e^3(x-3)^n + \cdots$$

(2) $a = 0$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

<14 ページ マクローリン展開>

解答

問

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10} + \dots$$

<15 ページ 近似の練習 1>

解答

問 1

$$(1) f(x) = \sqrt{x}$$

$$\sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a)$$

$$\sqrt{4.1} \doteq \sqrt{4} + \frac{1}{2\sqrt{4}}(4.1 - 4) = 2 + \frac{1}{4}(0.1) = 2 + \frac{1}{40} = 2.025$$

$$(2) f(x) = \log x$$

$$\log x \doteq \log a + \frac{1}{a}(x - a)$$

$$\log 1.1 \doteq \log 1 + \frac{1}{1}(1.1 - 1) = 0 + 0.1 = 0.1$$

問 2

$$x \doteq a \text{ のとき } f(x) \doteq f'(a)(x - a) + \frac{1}{2}f''(a)(x - a)^2$$

$$(1) f(x) = \sqrt{x}$$

$$\sqrt{x} \doteq \sqrt{a} + \frac{1}{2\sqrt{a}}(x - a) - \frac{1}{8a\sqrt{a}}(x - a)^2$$

$$(2) f(x) = \log x$$

$$\log x \doteq \log a + \frac{1}{a}(x - a) - \frac{1}{2a^2}(x - a)^2$$

$$(3) f(x) = \cos x$$

$$\log x \doteq \cos a - \sin a(x - a) - \frac{1}{2}\cos a(x - a)^2$$

<16 ページ 近似の練習 2>

解答

問 1

$$\begin{aligned} e &= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \\ &= 2 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = 2 + \frac{12 + 4 + 1}{24} = 2 + \frac{17}{24} = 2.70833\dots \\ &\approx 2.7083 \end{aligned}$$

問 2

$$\begin{aligned} \sin 1 &\approx 1 - \frac{1}{3!} + \frac{1}{5!} \\ &= 1 - \frac{1}{6} + \frac{1}{120} = \frac{120 - 20 + 1}{120} = \frac{101}{120} \\ &= 0.841666\dots \\ &\approx 0.8417 \end{aligned}$$

<17ページ 2変数関数>

解答

問

$$(1) f(2, 1) = 2^2 + 3 \times 2 \times 1 + 5 \times 1^2 = 4 + 6 + 5 = 15$$

$$(2) f(5, 7) = \frac{5-3}{7+1} = \frac{2}{8} = \frac{1}{4}$$

$$(3) f(\pi, e) = \cos \pi \log e = -1 \times 1 = -1$$

$$(4) f(3, 2) = 3^2 = 9$$

<18 ページ 偏導関数 1>

解答

問

$$(1) f(x, y) = 7 - 2x + xy - 4y$$

$$f(x, 0) = 7 - 2x, \quad f_x(x, 0) = -2$$

$$f(x, 1) = 7 - 2x + x - 4 = 3 - x, \quad f_x(x, 1) = -1$$

$$f(x, 2) = 7 - 2x + 2x - 8 = -1, \quad f_x(x, 2) = 0$$

$$f(x, b) = 7 - 2x + bx - 4b, \quad f_x(x, b) = -2 + b$$

$$(2) f(x, y) = x^5 - 4x^3y + 5x^2y^2 - 7xy^3$$

$$f(x, 0) = x^5, \quad f_x(x, 0) = 5x^4$$

$$f(x, 1) = x^5 - 4x^3 + 5x^2 - 7x, \quad f_x(x, 1) = 5x^4 - 12x^2 + 10x - 7$$

$$f(x, 2) = x^5 - 8x^3 + 20x^2 - 56x, \quad f_x(x, 2) = 5x^4 - 24x^2 + 40x - 56$$

$$f(x, b) = x^5 - 4bx^3 + 5b^2x^2 - 7b^3x, \quad f_x(x, b) = 5x^4 - 12bx^2 + 10b^2x - 7b^3$$

<19 ページ 偏導関数 2>

解答

問

$$(1) f(x, y) = 3x^2 - 4x + 2xy + 5y^2 - 7y + 6$$

$$f(x, b) = 3x^2 - 4x + 2bx + 5b^2 - 7b + 6$$

$$f_x(x, b) = 6x - 4 + 2b$$

$$f_x(x, y) = 6x - 4 + 2y$$

$$(2) f(x, y) = x^6 - 7x^5y^3 + 8x^3y^4 - 5xy^5 + y^7$$

$$f(x, b) = x^6 - 7b^3x^5 + 8b^4x^3 - 5b^5x + b^7$$

$$f_x(x, b) = 6x^5 - 35b^3x^4 + 24b^4x^2 - 5b^5$$

$$f_x(x, y) = 6x^5 - 35x^4y^3 + 24x^2y^4 - 5y^5$$

<20 ページ 偏導関数 3>

解答

問

$$(1) f(x, y) = 7 - 2x + xy - 4y$$

$$f(0, y) = 7 - 4y, \quad f_y(0, y) = -4$$

$$f(1, y) = 7 - 2 + y - 4y = 5 - 3y, \quad f_y(1, y) = -3$$

$$f(2, y) = 7 - 4 + 2y - 4y \\ = 3 - 2y, \quad f_y(2, y) = -2$$

$$f(a, y) = 7 - 2a + ay - 4y, \quad f_y(a, y) = a - 4$$

$$(2) f(x, y) = x^5 - 4x^3y + 5x^2y^2 - 7xy^3$$

$$f(0, y) = 0, \quad f_y(0, y) = 0$$

$$f(1, y) = 1 - 4y + 5y^2 - 7y^3, \quad f_y(1, y) = -4 + 10y - 21y^2$$

$$f(2, y) = 32 - 32y + 20y^2 - 14y^3, \quad f_y(2, y) = -32 + 40y - 42y^2$$

$$f(a, y) = a^5 - 4a^3y + 5a^2y^2 - 7ay^3, \quad f_y(a, y) = -4a^3 + 10a^2y - 21ay^2$$

<21 ページ 偏導関数 4>

解答

問

$$(1) f(x, y) = 2x^3 - 5x + 3xy - 3y^2 + 6y - 13$$

$$f(a, y) = 2a^3 - 5a + 3ay - 3y^2 + 6y - 13$$

$$f_y(a, y) = 3a - 6y + 6$$

$$f_y(x, y) = 3x - 6y + 6$$

$$(2) f(x, y) = x^7 - 6x^4y^3 + 5x^2y^5 - 4x^2y + y^6$$

$$f(a, y) = a^7 - 6a^4y^3 + 5a^2y^5 - 4a^2y + y^6$$

$$f_y(a, y) = -18a^4y^2 + 25a^2y^4 - 4a^2 + 6y^5$$

$$f_y(x, y) = -18x^4y^2 + 25x^2y^4 - 4x^2 + 6y^5$$

<22 ページ 偏微分 1>

解答

問

$$(1) f(x, y) = x^3 - 5xy + y^2 - 3x + 4y - 6$$

$$f_x(x, y) = 3x^2 - 5y - 3$$

$$(2) f(x, y) = x^6 - 3x^4y + 6x^3y^2 - 7x^2y^3 + 10xy^4 - 4y^5$$

$$f_x(x, y) = 6x^5 - 12x^3y + 18x^2y^2 - 14xy^3 + 10y^4$$

$$(3) f(x, y) = 3e^x - \cos x \sin y + x \log y + \frac{x}{y}$$

$$f_x(x, y) = 3e^x + \sin x \sin y + \log y + \frac{1}{y}$$

<23 ページ 偏微分 2>

解答

問

$$(1) f(x, y) = x^3 - 5xy + y^2 - 3x + 4y - 6$$

$$f_y(x, y) = -5x + 2y + 4$$

$$(2) f(x, y) = x^6 - 3x^4y + 6x^3y^2 - 7x^2y^3 + 10xy^4 - 4y^5$$

$$f_y(x, y) = -3x^4 + 12x^3y - 21x^2y^2 + 40xy^3 - 20y^4$$

$$(3) f(x, y) = 3e^x - \cos x \sin y + x \log y + \frac{x}{y}$$

$$f_y(x, y) = -\cos x \cos y + \frac{x}{y} - \frac{x}{y^2}$$

<24 ページ 偏微分 3>

解答

問

$$(1) \frac{\partial}{\partial x}(x^4 - 5x^3y + 7xy^3) = 4x^3 - 15x^2y + 7y^3, \quad \frac{\partial}{\partial y}(x^4 - 5x^3y + 7xy^3) = -5x^3 + 21xy^2$$

$$(2) \frac{\partial}{\partial x} \left(\frac{y}{x^2} \right) = -\frac{2y}{x^3}, \quad \frac{\partial}{\partial y} \left(\frac{y}{x^2} \right) = \frac{1}{x^2}$$

$$(3) \frac{\partial}{\partial x}(y^x) = y^x \log y, \quad \frac{\partial}{\partial y}(y^x) = xy^{x-1}$$

<25 ページ 偏微分 4>

解答

問

$$(1) z = (3x + 4y)^6$$

$$\frac{\partial z}{\partial x} = 18(3x + 4y)^5, \quad \frac{\partial z}{\partial y} = 24(3x + 4y)^5$$

$$(2) z = \sqrt{1 + x^2 + 4y^2}$$

$$\frac{\partial z}{\partial x} = \frac{2x}{2\sqrt{1 + x^2 + 4y^2}} = \frac{x}{\sqrt{1 + x^2 + 4y^2}}, \quad \frac{\partial z}{\partial y} = \frac{4y}{\sqrt{1 + x^2 + 4y^2}}$$

$$(3) z = e^{-x^2 - y^2}$$

$$\frac{\partial z}{\partial x} = -2xe^{-x^2 - y^2}, \quad \frac{\partial z}{\partial y} = -2ye^{-x^2 - y^2}$$

$$(4) z = \log(2 - \sin y \cos x)$$

$$\frac{\partial z}{\partial x} = \frac{\sin y \sin x}{2 - \sin y \sin x}, \quad \frac{\partial z}{\partial y} = \frac{-\cos y \cos x}{2 - \sin y \sin x}$$

<26 ページ 偏微分 5>

解答

問

(1) $f(x, y) = x^3 - 5x^2y + 6xy^4 - 5y^6$

$$f_x(x, y) = 3x^2 - 10xy + 6y^4, \quad f_y(x, y) = -5x^2 + 24xy^3 - 30y^5$$

(2) $f(x, y) = \sin(2x + 3y)$

$$f_x(x, y) = 2 \cos(2x + 3y), \quad f_y(x, y) = 3 \cos(2x + 3y)$$

(3) $z = \frac{1}{x^2 - 2xy}$

$$\frac{\partial z}{\partial x} = -\frac{2x - 2y}{(x^2 - 2xy)^2}, \quad \frac{\partial z}{\partial y} = -\frac{-2x}{(x^2 - 2xy)^2} = \frac{2x}{(x^2 - 2xy)^2}$$

(4) $z = \frac{1}{\sqrt{x^2 + xy + y^2}}$

$$z_x = -\frac{2x + y}{2(x^2 + xy + y^2)\sqrt{x^2 + xy + y^2}}$$

$$z_y = -\frac{x + 2y}{2(x^2 + xy + y^2)\sqrt{x^2 + xy + y^2}}$$

(5) $z = e^{-x^2+xy}$

$$z_x = (-2x + y)e^{-x^2+xy}, \quad z_y = xe^{-x^2+xy}$$

<27 ページ 2階偏導関数 1>

解答

問

$$(1) f(x, y) = x^4 - 5x^3y + 6x^2y^2 - 4xy^3$$

$$f_{xx}(x, y) = 12x^2 - 30xy + 12y^2 \quad , \quad f_{yy}(x, y) = 12x^2 - 24xy$$

$$(2) z = \sin(2x) \cos(3y)$$

$$\frac{\partial^2 z}{\partial x^2} = -4 \sin(2x) \cos(3y) \quad , \quad \frac{\partial^2 z}{\partial y^2} = -9 \sin(2x) \cos(3y)$$

<28 ページ 2階偏導関数 2>

解答

問

$$(1) f(x, y) = x^5 - 6x^3y + 8xy^3 + 4y^5$$

$$f_{xy}(x, y) = -18x^2 + 24y^2 \quad , \quad f_{yx}(x, y) = -18x^2 + 24y^2$$

$$(2) z = \sin(3x + 2y)$$

$$\frac{\partial^2 z}{\partial y \partial x} = -6 \sin(3x + 2y) \quad , \quad \frac{\partial^2 z}{\partial x \partial y} = -6 \sin(3x + 2y)$$

<29 ページ 偏微分係数>

解答

問

$$(1) f(x, y) = x^4 - 5x^3y + 6x^2y^2 - 4y^3$$

$$f_x(x, y) = 4x^3 - 15x^2y + 12xy^2, \quad f_y(x, y) = -5x^3 + 12x^2y - 12y^2$$

$$f_x(2, 1) = 32 - 60 + 24 = -4, \quad f_y(2, 1) = -40 + 48 - 12 = -4$$

$$(2) z = \cos x \sin y$$

$$f_x(x, y) = -\sin x \sin y, \quad f_y(x, y) = \cos x \cos y$$

$$f_x\left(\frac{\pi}{2}, \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \quad f_y\left(\frac{\pi}{2}, \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{3}\right) = 0$$

$$(3) f(x, y) = x^2 \log y$$

$$f_x(x, y) = 2x \log y, \quad f_y(x, y) = \frac{x^2}{y}$$

$$f_x(1, e) = 2 \log e = 2, \quad f_y(1, e) = \frac{1}{e}$$

<30 ページ 2変数関数のグラフ>

解答

問1

$$(1) f(x, y) = 3x + 4y + 1$$

$$x \text{ 軸方向の傾き} = 3$$

$$y \text{ 軸方向の傾き} = 4$$

$$z \text{ 切片} = 1$$

$$(2) f(x, y) = mx + ny + k$$

$$x \text{ 軸方向の傾き} = m$$

$$y \text{ 軸方向の傾き} = n$$

$$z \text{ 切片} = k$$

問2

(1) l_1 の方程式

$$y = 1$$

$$z = 3$$

(2) l_2 の方程式

$$y = 2$$

$$z = x$$

(3) l_3 の方程式

$$x = 2$$

$$z = -y^2 + 2y + 2$$

<31 ページ 偏微分係数の幾何学的意味>

解答

問

$$f_x(x, y) = 2x - 2 + 4y$$

$$f_y(x, y) = 4x - 2y$$

$$f(3, 2) = 29$$

$$f_x(3, 2) = 12$$

$$f_y(3, 2) = 8$$

接線 L の方程式

$$y = 2$$

$$z = 12(x - 3) + 29$$

$$= 12x - 7$$

接線 l の方程式

$$x = 3$$

$$z = 8(y - 2) + 29$$

$$= 8y + 13$$

<32 ページ 接平面>

解答

問

$$f_x(x, y) = 6x + 2y, \quad f_y(x, y) = 2x - 4y, \quad f(1, 2) = 3 + 4 - 8 = -1$$

$$f_x(1, 2) = 6 + 4 = 10, \quad f_y(1, 2) = 2 - 8 = -6$$

$$z = 10(x - 1) + (-6)(y - 2) - 1 = 10x - 10 - 6y + 12 - 1$$

$$= 10x - 6y + 1 \quad \underline{\text{(答) } z = 10x - 6y + 1}$$

<33 ページ 2変数関数の一次近似>

解答

問

(1) $f(x, y) = x^4 y^5$

$$\frac{(a + \Delta x)^4 (b + \Delta y)^5 \doteq 4a^3 b^5 \Delta x + 5a^4 b^4 \Delta y + a^4 b^5}{}$$

$$\left(\begin{array}{l} f_x(x, y) = 4x^3 y^4 \quad , \quad f_x(a, b) = 4a^3 b^4 \\ f_y(x, y) = 5x^4 y^4 \quad , \quad f_y(a, b) = 5a^4 b^4 \end{array} \right)$$

(2) $f(x, y) = (\sin x)\sqrt{y}$

$$\frac{\sin(a + \Delta x)\sqrt{b + \Delta y} \doteq (\cos a)\sqrt{b}\Delta x + \frac{\sin a}{2\sqrt{b}}\Delta y + (\sin a)\sqrt{b}}{}$$

$$\left(f_x = (\cos x)\sqrt{y} \quad , \quad f_y = \frac{\sin x}{2\sqrt{y}} \right)$$

(3) $f(x, y) = \frac{x}{y}$

$$\frac{a + \Delta x}{b + \Delta y} \doteq \frac{1}{b}\Delta x - \frac{a}{b^2}\Delta y + \frac{a}{b}$$

$$\left(f_x = \frac{1}{y} \quad , \quad f_y = -\frac{x}{y^2} \right)$$

<34 ページ 2変数合成関数の微分1 >

解答

問
$$\frac{d}{dt}f(x(t), y(t)) = f_x(x, y)\frac{dx}{dt} + f_y(x, y)\frac{dy}{dt}$$

<35 ページ 2変数合成関数の微分 2>

解答

問

$$(1) \frac{d}{dt} f(2-3t, 4-5t) \\ = -3f_x(2-3t, 4-5t) - 5f_y(2-3t, 4-5t)$$

$$(2) \frac{d}{dr} f(r \cos \theta, r \sin \theta) \\ = (\cos \theta) f_x(r \cos \theta, r \sin \theta) + (\sin \theta) f_y(r \cos \theta, r \sin \theta)$$

<36 ページ 全微分 >

解答

問1

$$(1) z = x^2 + 3xy$$

$$dz = (2x + 3y)dx + 3xdy$$

$$(2) z = x^2 \cos y$$

$$dz = 2x \cos y dx - x^2 \sin y dy$$

問2

$$(1) x = 2u + 3v$$

$$dx = 2du + 3dv$$

$$(2) y = 4u + 5v$$

$$dy = 4du + 5dv$$

<37ページ ヤコビアン>

解答

問

$$(1) \begin{cases} x = 5u + v \\ y = 3u + 2v \end{cases}$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} \\ = 10 - 3 = 7$$

$$(2) \begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} \\ = r \cos^2 \theta + r \sin^2 \theta = r$$

<38 ページ ラプラシアン 1>

解答

問

$$(1) f(x, y) = 2x^2y + 4y^3$$

$$\begin{aligned}\Delta f(x, y) &= f_{xx} + f_{yy} \\ &= 4y + 24y = 28y\end{aligned}$$

$$\left(\begin{array}{l} f_x = 4xy \quad , \quad f_{xx} = 4y \\ f_y = 2x^2 + 12y^2 \quad , \quad f_{yy} = 24y \end{array} \right)$$

$$(2) f(x, y) = \log(x^2 + y^2)$$

$$\Delta f(x, y) = \frac{-2x^2 + 2y^2}{(x^2 + y^2)^2} + \frac{2x^2 - 2y^2}{(x^2 + y^2)^2} = 0$$

$$\left(\begin{array}{l} f_x = \frac{2x}{x^2+y^2} \quad , \quad f_{xx} = \frac{2(x^2+y^2) - 2x \times 2x}{(x^2+y^2)^2} = \frac{-2x^2+2y^2}{(x^2+y^2)^2} \\ f_y = \frac{2y}{x^2+y^2} \quad , \quad f_{yy} = \frac{2(x^2+y^2) - 4y^2}{(x^2+y^2)^2} = \frac{2x^2-2y^2}{(x^2+y^2)^2} \end{array} \right)$$

<40 ページ ラプラシアン 3>

解答

問

$$(1) z = e^{-x^2-y^2} = e^{-r^2}$$

$$z_r = -2re^{-r^2}, \quad z_\theta = 0$$

$$z_{rr} = -2e^{-r^2} + 4r^2e^{-r^2}, \quad z_{\theta\theta} = 0$$

$$\begin{aligned} \Delta z &= z_{rr} + \frac{1}{r}z_r \\ &= (4r^2 - 2)e^{-r^2} + \frac{1}{r}(-2re^{-r^2}) \\ &= (4r^2 - 4)e^{-r^2} \\ &= 4(x^2 + y^2 - 1)e^{-x^2-y^2} \end{aligned}$$

$$(2) z = \frac{xy}{x^2 + y^2} = \frac{x}{\sqrt{x^2 + y^2}} \times \frac{y}{\sqrt{x^2 + y^2}} = \cos \theta \times \sin \theta$$

$$z_\theta = -\sin^2 \theta + \cos^2 \theta$$

$$z_{\theta\theta} = -2 \sin \theta \cos \theta - 2 \cos \theta \sin \theta = -4 \sin \theta \cos \theta$$

$$z_r = 0, \quad z_{rr} = 0$$

$$\begin{aligned} \Delta z &= \frac{1}{r^2}z_{\theta\theta} = -\frac{4}{r^2} \sin \theta \cos \theta \\ &= -\frac{4}{x^2 + y^2} \times \frac{y}{\sqrt{x^2 + y^2}} \times \frac{x}{\sqrt{x^2 + y^2}} \\ &= -\frac{4xy}{(x^2 + y^2)^2} \end{aligned}$$