$<$1ページ　平面の一次変数3$>$

解答

問1　$BA = \begin{pmatrix} \cos 75^\circ & -\sin 75^\circ \\ \sin 75^\circ & \cos 75^\circ \end{pmatrix}$

問2

$$\begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix}\begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}-\sqrt{6}}{4} & -\frac{\sqrt{6}+\sqrt{2}}{4} \\ \frac{\sqrt{2}+\sqrt{6}}{4} & \frac{-\sqrt{6}+\sqrt{2}}{4} \end{pmatrix}$$

$\cos 105^\circ = \frac{\sqrt{2}-\sqrt{6}}{4}$　,　$\sin 105^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$

<2 ページ　平面の一次変数 4>

解答

問 1　$A^{-1}=\begin{pmatrix}\cos(-45^\circ) & -\sin(-45^\circ)\\ \sin(-45^\circ) & \cos(-45^\circ)\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}$

問 2

$$\begin{pmatrix}\cos 45^\circ & -\sin 45^\circ\\ \sin 45^\circ & \cos 45^\circ\end{pmatrix}\begin{pmatrix}\cos(-30^\circ) & -\sin(-30^\circ)\\ \sin(-30^\circ) & \cos(-30^\circ)\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}\begin{pmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{6}+\sqrt{2}}{4} & \frac{\sqrt{2}-\sqrt{6}}{4}\\ \frac{\sqrt{6}-\sqrt{2}}{4} & \frac{\sqrt{2}+\sqrt{6}}{4}\end{pmatrix}$$

$\cos 15^\circ=\frac{\sqrt{6}+\sqrt{2}}{4}$　，$\sin 15^\circ=\frac{\sqrt{6}-\sqrt{2}}{4}$

＜3 ページ　空間の一次変換＞

解答

問 1

(1) $\begin{pmatrix} 0 & -2\pi & 0 \\ 2\pi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$　　(2) $\begin{pmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{pmatrix}$

問 2

(1) $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(2) $A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{pmatrix}$, $A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$$= \begin{pmatrix} 0-2+6 \\ 1+0-9 \\ -2+6+0 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 4 \end{pmatrix}$$

<4 ページ　空間における等速円運動 1>

解答

問

$$* \begin{cases} x' = -\theta y \\ y' = \theta x \\ z' = 0 \end{cases} \qquad \begin{pmatrix} 0 & -\theta & 0 \\ \theta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

<5 ページ　空間における等速円運動 2>

解答

問

$$\left\{\begin{array}{l} v_1 = \omega_2 z - \omega_3 y \\ v_2 = \omega_3 x - \omega_1 z \\ v_3 = \omega_1 y - \omega_2 x \end{array}\right. \quad \Leftrightarrow \quad \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$<$6 ページ　微分の復習 $>$

解答

問

(1) $\dfrac{d}{dx}(x^2 - 3x + 5) = 2x - 3$

(2) $\dfrac{d}{dy}(y - 2y^2 + 5y^3) = 1 - 4y + 15y^2$

(3) $\dfrac{d}{dx}\left((2x-1)^5\right) = 10(2x-1)^4$

(4) $\dfrac{d}{dt}\left(e^{2t}\right) = 2e^{2t}$

(5) $\dfrac{d}{dx}\left(\sin(x^3)\right) = 3x^2\cos(x^3)$

(6) $\dfrac{d}{dy}(\cos(1-2y)) = 2\sin(1-2y)$

<7 ページ　2 変数関数 >

解答

問

(1) $f(2,1)=4-6+5=3$

(2) $f(7,6)=\dfrac{7-3}{6+4}=\dfrac{4}{10}=\dfrac{2}{5}$

(3) $f(\pi,e)=\sin(\pi)\log e=0$

(4) $f(3,4)=3^4=81$

<8ページ　偏導関数1>

解答

問

(1) $f(x,y)=5-2x+xy-3y^2$

$f(x,0)=5-2x$, $f_x(x,0)=-2$

$f(x,1)=5-2x+x-3=2-x$, $f_x(x,1)=-1$

$f(x,2)=5-2x+2x-12=-7$, $f_x(x,2)=0$

$f(x,b)=5-2x+bx-3b^2$, $f_x(x,b)=-2+b$

(2) $f(x,y)=x^5-4x^2y^3+3xy^4-10x$

$f(x,0)=x^5-10x$, $f_x(x,0)=5x^4-10$

$f(x,1)=x^5-4x^2-7x$, $f_x(x,1)=5x^4-8x-7$

$f(x,2)=x^5-32x^2-38x$, $f_x(x,2)=5x^4-64x+38$

$f(x,b)=x^5-4b^3x^2+3b^4x-10x$, $f_x(x,b)=5x^4-8b^3x+3b^4-10$

$<$9 ページ　偏導関数 2$>$

解答

問

(1) $f(x,y) = 3x^2 + x + 4xy - 2y^2 + y - 2$

$f(x,b) = 3x^2 + x + 4bx - 2b^2 + b - 2$

$f_x(x,b) = 6x + 1 + 4b$

$f_x(x,y) = 6x + 1 + 4y$

(2) $f(x,y) = x^5 - 5x^4y^2 + 7x^2y^3 - 11xy^4 + y^5$

$f(x,b) = x^5 - 5b^2x^4 + 7b^3x^2 - 11b^4x + b^5$

$f_x(x,b) = 5x^4 - 20b^2x^3 + 14b^3x - 11b^4$

$f_x(x,y) = 5x^4 - 20x^3y^2 + 14xy^3 - 11y^4$

<10 ページ　偏導関数 3>

解答

問

(1) $f(x,y) = 5 - 2x + xy - 3y^2$

$f(0,y) = 5 - 3y^2$, $f_y(0,y) = -6y$

$f(1,y) = 5 - 2 + y - 3y^2 = 3 + y - 3y^2$, $f_y(1,y) = 1 - 6y$

$$\begin{aligned} f(2,y) &= 5 - 4 + 2y - 3y^2 \\ &= 1 + 2y - 3y^2 \end{aligned}$$, $f_y(2,y) = 2 - 6y$

$f(a,y) = 5 - 2a + ay - 3y^2$, $f_y(a,y) = a - 6y$

(2) $f(x,y) = x^5 - 4x^2y^3 + 3xy^4 - 10x$

$f(0,y) = 0$, $f_y(0,y) = 0$

$$\begin{aligned} f(1,y) &= 1 - 4y^3 + 3y^4 - 10 \\ &= -9 - 4y^3 + 3y^4 \end{aligned}$$, $f_y(1,y) = -12y^2 + 12y^3$

$f(2,y) = 12 - 16y^3 + 6y^4$, $f_y(2,y) = -48y^2 + 24y^3$

$f(a,y) = a^5 - 4a^2y^3 + 3ay^4 - 10a$, $f_y(a,y) = -12a^2y^2 + 12ay^3$

<11 ページ　偏導関数 4>

解答

問

(1) $f(x,y) = 3x^2 + x + 4xy - 2y^2 + y - 2$

$f(a,y) = 3a^2 + a + 4ay - 2y^2 + y - 2$

$f_y(a,y) = 4a - 4y + 1$

$f_y(x,y) = 4x - 4y + 1$

(2) $f(x,y) = x^5 - 5x^4y^2 + 7x^2y^3 - 11xy^4 + y^5$

$f(a,y) = a^5 - 5a^4y^2 + 7a^2y^3 - 11ay^4 + y^5$

$f_y(a,y) = -10a^4y + 21a^2y^2 - 44ay^3 + 5y^4$

$f_y(x,y) = -10x^4y + 21x^2y^2 - 44xy^3 + 5y^4$

<12 ページ　偏微分 1>

解答

問

(1) $f(x,y) = 2x^2 - xy + 4y^2 - 3x + 5y + 11$

$$f_x(x,y) = 4x - y + 3$$

(2) $f(x,y) = 3x^5 - 9x^4y + 8x^3y^2 + 7x^2y^3 - 5xy^4 + y^5$

$$f_x(x,y) = 15x^4 - 36x^3y + 24x^2y^2 + 14xy^3 - 5y^4$$

(3) $f(x,y) = e^x + \cos x \sin y - x \log y + \dfrac{y}{x} - 2y$

$$f_x(x,y) = e^x - \sin x \sin y - \log y - \frac{y}{x^2}$$

<13 ページ　偏微分 2>

解答

問

(1) $f(x,y) = 2x^2 - xy + 4y^2 - 3x + 5y + 11$

$$f_y(x,y) = -x + 8y + 5$$

(2) $f(x,y) = 3x^5 - 9x^4y + 8x^3y^2 + 7x^2y^3 - 5xy^4 + y^5$

$$f_y(x,y) = -9x^4 + 16x^3y + 21x^2y^2 - 20xy^3 + 5y^4$$

(3) $f(x,y) = e^x + \cos x \sin y - x \log y + \dfrac{y}{x} - 2y$

$$f_y(x,y) = \cos x \cos y - \frac{x}{y} + \frac{1}{x} - 2$$

＜14ページ　偏微分3＞

解答

問

(1) $\dfrac{\partial}{\partial x}(x^3 - 3x^2y + 4xy^2)$ $= 3x^2 - 6xy + 4y^2$, $\dfrac{\partial}{\partial y}(x^3 - 3x^2y + 4xy^2)$ $= -3x^2 + 8xy$

(2) $\dfrac{\partial}{\partial x}\left(\dfrac{x^2}{y^2}\right) = \dfrac{2x}{y^2}$, $\dfrac{\partial}{\partial y}\left(\dfrac{x^2}{y^2}\right) = -\dfrac{2x^2}{y^3}$

(3) $\dfrac{\partial}{\partial x}(x^y) = yx^{y-1}$, $\dfrac{\partial}{\partial y}(x^y) = x^y \log x$

<15 ページ　偏微分 4>

解答

問

(1) $z = (3x-y)^4$

$$\frac{\partial z}{\partial x} = 12(3x-y)^3 \quad , \quad \frac{\partial z}{\partial y} = -4(3x-y)^3$$

(2) $z = \sqrt{1+x^2-y^2}$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{1+x^2-y^2}} \quad , \quad \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{1+x^2-y^2}}$$

(3) $z = e^{x^2+2xy-y^2}$

$$\frac{\partial z}{\partial x} = (2x+2y)e^{x^2+2xy-y^2} \quad , \quad \frac{\partial z}{\partial y} = (2x-2y)e^{x^2+2xy-y^2}$$

(4) $z = \log(1-\cos x \sin y)$

$$\frac{\partial z}{\partial x} = \frac{\sin x \sin y}{1-\cos x \sin y} \quad , \quad \frac{\partial z}{\partial y} = \frac{-\cos x \cos y}{1-\cos x \sin y}$$

<16 ページ　偏微分 5>

解答

問

(1) $f(x,y)=x^3-3x^2y^2-2xy^4$

$f_x(x,y)=3x^2-6xy^2-2y^4$ 　, $f_y(x,y)=-6x^2y-8xy^3$

(2) $f(x,y)=\sin(2x-3y)$

$f_x(x,y)=2\cos(2x-3y)$ 　, $f_y(x,y)=-3\cos(2x-3y)$

(3) $z=\dfrac{1}{xy-y^2}$

$\dfrac{\partial z}{\partial x}=-\dfrac{y}{(xy-y^2)^2}$ 　, $\dfrac{\partial z}{\partial y}=-\dfrac{x-2y}{(xy-y^2)^2}$

(4) $z=\dfrac{1}{\sqrt{x^2-y^2}}$

$$z_x=-\frac{1}{2}(x^2-y^2)^{-\frac{3}{2}}\times 2x$$
$$=-\frac{x}{(x^2-y^2)\sqrt{x^2-y^2}}$$

　, $z_y=\dfrac{y}{(x^2-y^2)\sqrt{x^2-y^2}}$

(5) $z=e^{x^2-y^2}$

$z_x=2xe^{x^2-y^2}$ 　, $z_y=-2ye^{x^2-y^2}$

<17ページ　2階偏導関数1>

解答

問

(1) $f(x,y) = x^3 - 2x^2y + 3xy^2 - 2y^3$

$$f_{xx}(x,y) = 6x - 4y \quad , \quad f_{yy}(x,y) = 6x - 12y$$

(2) $z = \cos(xy^2)$

$$\frac{\partial^2 z}{\partial x^2} = -y^4\cos(xy^2) \quad , \quad \frac{\partial^2 z}{\partial y^2} = -2x\sin(xy^2) - 4x^2y^2\cos(xy^2)$$

<18ページ　2階偏導関数2>

解答

問

(1) $f(x,y) = x^3 - 2x^2y + 3xy^2 - 4y^5$

$$f_{xy}(x,y) = -4x + 9y^2 \quad , \quad f_{yx}(x,y) = -4x + 9y^2$$

(2) $z = \cos(2x)\sin(3y)$

$$\frac{\partial^2 z}{\partial y \partial x} = -6\sin(2x)\cos(3y) \quad , \quad \frac{\partial^2 z}{\partial x \partial y} = -6\sin(2x)\cos(3y)$$

<19 ページ　偏微分係数>

解答

問

(1) $f(x,y) = x^2y - 3xy^2 + y^3$

$f_x(x,y) = 2xy - 3y^2$ ， $f_y(x,y) = x^2 - 6xy + 3y^2$

$f_x(2,1) = 2 \times 2 \times 1 - 3 \times 1 = 1$ ， $f_y(2,1) = 4 - 12 + 3 = -5$

(2) $z = \cos x \sin y$

$f_x(x,y) = -\sin x \sin y$ ， $f_y(x,y) = \cos x \cos y$

$f_x\left(\frac{\pi}{2}, \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ ， $f_y\left(\frac{\pi}{2}, \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{3}\right) = 0$

(3) $f(x,y) = e^x \log y$

$f_x(x,y) = e^x \log y$ ， $f_y(x,y) = \frac{e^x}{y}$

$f_x(1,e) = e^1 \log e = e$ ， $f_y(1,e) = \frac{e^1}{e} = 1$

<20ページ　2変数関数のグラフ>

解答

問1

(1) $f(x,y)=2x+3y+5$

x 軸方向の傾き $=2$

y 軸方向の傾き $=3$

z 切片 $=5$

(2) $f(x,y)=mx+ny+k$

x 軸方向の傾き $=m$

y 軸方向の傾き $=n$

z 切片 $=k$

問2

(1) ℓ_1の方程式

$y=1$

$z=3$

$\big(f(x,1)=4-x+x-1\big)$

(2) ℓ_2の方程式

$y=2$

$z=x$

$\big(f(x,2)=4-x+2x-4\big)$

(3) ℓ_3の方程式

$x=2$

$z=-y^2+2y+2$

$\big(f(2,y)=4-2+2y-y^2\big)$

<21 ページ　偏微分係数の幾何学的意味>

解答

問

$$f_x(3,2) = -2 \times 3 + 7 = 1$$

$$y = 2$$

$$\begin{aligned} z &= f_x(3,2)(x-3) + f(3,2) \\ &= 1(x-3) + 3 = x \end{aligned}$$

$$f_y(3,2) = -2 \times 2 + 3 = -1$$

$$x = 3$$

$$\begin{aligned} z &= f_y(3,2)(y-x) + f(3,2) \\ &= -1(y-2) + 3 = -y + 5 \end{aligned}$$

<22 ページ　接平面 >

解答

問

$f_x(x, y) = 2x - y, \quad f_y(x, y) = -x - 3y^3$

$f_x(1, -1) = 2 + 1 = 3, \quad f_y(1, -1) = -1 - 3(-1)^2 = -4, \quad f(1, -1) = 1 + 1 - (-1) = 3$

$$
\begin{aligned}
z &= 3(x-1) - 4(y+1) + 3 \\
&= 3x - 3 - 4y - 4 + 3 \qquad \underline{(\text{答})\ z = 3x - 4y - 4}
\end{aligned}
$$

＜23ページ　2変数関数の一次近似＞

解答

問

(1) $f(x,y)=x^2y^3$

$$\underline{(a+\Delta x)^2(b+\Delta y)^3 \fallingdotseq 2ab^3\Delta x+3a^2b^2\Delta y+a^2b^3}$$

$$\left(\begin{array}{ccc} f_x(x,y)=2xy^3 & , & f_y(x,y)=3x^2y^2 \\ f_x(a,b)=2ab^3 & , & f_y(a,b)=3a^2b^2 \end{array}\right)$$

(2) $f(x,y)=(\cos x)\sqrt{y}$

$$\underline{\cos(a+\Delta x)\sqrt{b+\Delta y} \fallingdotseq -(\sin a)\sqrt{b}\Delta x+\frac{\cos a}{2\sqrt{b}}\Delta y+(\cos a)\sqrt{b}}$$

$$\left(f_x=-(\sin x)\sqrt{y} \quad , \quad f_y=\tfrac{\cos x}{2\sqrt{y}} \right)$$

(3) $f(x,y)=\dfrac{x}{y}$

$$\underline{\frac{a+\Delta x}{b+\Delta y} \fallingdotseq \frac{1}{b}\Delta x-\frac{a}{b^2}\Delta y+\frac{a}{b}}$$

$$\left(f_x=\tfrac{1}{y} \quad , \quad f_y=-\tfrac{x}{y^2} \right)$$

$<$24 ページ　2 変数合成関数の微分 1 $>$

解答

問　$\dfrac{d}{dt}f(x(t),y(t)) = f_x(x,y)\dfrac{dx}{dt} + f_y(x,y)\dfrac{dy}{dt}$

<25 ページ　2 変数合成関数の微分 2>

解答

問

(1) $\dfrac{d}{dt}f(3+2t, 2+3t)$

$= 2f_x(3+2t, 2+3t) + 3f_y(3+2t, 2+3t)$

(2) $\dfrac{d}{dr}f(r\cos\theta, r\sin\theta)$

$= \cos\theta f_x(r\cos\theta, r\sin\theta) + \sin\theta f_y(r\cos\theta, r\sin\theta)$

$<$26 ページ　全微分 $>$

解答

問 1

(1) $z = x^3 - y^2$

$dz = 3x^2 dx - 2y dy$

(2) $z = x \cos y$

$dz = (\cos y) dx - x(\sin y) dy$

問 2

(1) $x = 2u + 3v$

$dx = 2du + 3dv$

(2) $y = u(u^2 - v) = u^3 - uv$

$dy = (3u^2 - v) du - u dv$

＜27 ページ　ヤコビアン＞

解答

問

(1) $\begin{cases} x = u + 2v \\ y = 3u - v \end{cases}$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= -1 - 6 = -7$$

(2) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta = r$$

<28 ページ　積分の復習>

解答

問 1

(1) $\displaystyle\int dx = x + C$

(2) $\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + C$

(3) $\displaystyle\int \frac{1}{x}dx = \log|x| + C$

(4) $\displaystyle\int \sin x dx = -\cos x + C$

(5) $\displaystyle\int \cos x dx = \sin x + C$

(6) $\displaystyle\int e^x dx = e^x + C$

(7) $\displaystyle\int (ax+b)^4 dx$

$\displaystyle= \frac{1}{5a}(ax+b)^5 + C$

(8) $\displaystyle\int \cos(ax+b)dx$

$\displaystyle= \frac{1}{a}\sin(ax+b) + C$

(9) $\displaystyle\int e^{ax+b}dx$

$\displaystyle= \frac{1}{a}e^{ax+b} + C$

問 2

(1) $\displaystyle\int_1^2 (x^2-3x)dx = \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_1^2$

$\displaystyle= \left(\frac{8}{3} - \frac{12}{2}\right) - \left(\frac{1}{3} - \frac{3}{2}\right) = \frac{7}{3} - \frac{9}{2}$

$\displaystyle= \frac{14-27}{6} = -\frac{13}{6}$

(2) $\displaystyle\int_{-1}^1 (3y-y^3)dy = \left[\frac{3x^2}{2} - \frac{x^3}{4}\right]_{-1}^1$

$\displaystyle= \left(\frac{3}{2} - \frac{1}{4}\right) - \left(\frac{3}{2} - \frac{1}{4}\right) = 0$

(3) $\displaystyle\int_0^2 (ax^2+bx)dx = \left[\frac{ax^3}{3} + \frac{bx^2}{2}\right]_0^2 = \frac{8}{3}a + 2b$

(4) $\displaystyle\int_{-1}^1 (ay+by^2+cy^3)dy$

$\displaystyle= \left[\frac{ay^2}{2} + \frac{by^3}{3} + \frac{cy^4}{4}\right]_{-1}^1 = \frac{2}{3}b$

問 3

(1) $\displaystyle S = \int_0^2 (-x^2+2x)dx$

$\displaystyle= \left[-\frac{x^3}{3} + x^2\right]_0^2$

$\displaystyle= -\frac{8}{3} + 4 = \frac{-8+12}{3} = \frac{4}{3}$

(2) $\displaystyle S = 2 + \int_0^1 (x^3-x+2)dx$

$\displaystyle= 2 + \left[\frac{x^4}{4} - \frac{x^2}{2} + 2x\right]_0^1$

$\displaystyle= 2 + \frac{1}{4} - \frac{1}{2} + 2 = 4 - \frac{1}{4} = \frac{15}{4}$

(3) $\displaystyle S = \int_{-1}^2 (x-x^2+2)dx$

$\displaystyle= \left[-\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{-1}^2$

$\displaystyle= \left(\frac{4}{2} - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$

$\displaystyle= \frac{9}{2}$

<29ページ　体積1>

解答

問

$$V=\int_0^{r\cos\theta}\pi\left(\tan\theta\right)^2x^2dx+\int_{r\cos\theta}^{r}\pi\left(\sqrt{r^2-x^2}\right)^2dx$$

$$=\pi\tan^2\theta\left[\frac{x^3}{3}\right]_0^{r\cos\theta}+\pi\left[r^2x-\frac{x^3}{3}\right]_{r\cos\theta}^{r}$$

$$=\pi\tan^2\theta\times\frac{r^3\cos^3\theta}{3}+\pi\left(\left(r^3-\frac{r^3}{3}\right)-\left(r^3\cos\theta-\frac{r^3\cos^3\theta}{3}\right)\right)$$

$$=\frac{\pi r^3}{3}\{\tan^2\theta\cos^3\theta+2-3\cos\theta+\cos^3\theta\}$$

$$=\frac{\pi r^3}{3}\{\cos^3\left(1+\tan^2\theta\right)+2-3\cos\theta\}=\frac{\pi r^3}{3}(2-2\cos\theta)$$

<30ページ　体積2>

解答

問

$$V=\int_0^4\left(\int_0^5\left(\frac{1}{4}x-\frac{1}{5}y+5\right)dy\right)dx$$

$$=\int_0^4\left(\left[\frac{1}{4}xy-\frac{1}{10}y^2+5y\right]_{y=0}^{y=5}\right)dx=\int_0^4\left(\frac{5}{4}x-\frac{25}{10}+25\right)dx$$

$$=\left[\frac{5}{8}x^2+\frac{45}{2}x\right]_0^4=\frac{5}{8}\times16+\frac{45}{2}\times4=10+90=100$$

$<$31 ページ　体積 3$>$

解答

問

$$S(x) = \int_0^3 \left(4 - 2x + xy + 2y - y^2\right) dy$$

$$= \left[4y - 2xy + \frac{x}{2}y^2 + y^2 - \frac{y^3}{3}\right]_{y=0}^{y=3}$$

$$= 12 - 6x + \frac{9}{2}x + 9 - \frac{27}{3} = 12 - \frac{3}{2}x$$

$$V = \int_0^2 S(x)dx = \int_0^2 \left(12 - \frac{3}{2}x\right) dx$$

$$= \left[12x - \frac{3}{4}x^2\right]_0^2 = 24 - 3 = 21$$

<32ページ　体積4>

解答

問

$$S(y) = \int_0^3 \left(\frac{1}{3}x - \frac{1}{4}y + 2\right) dx$$

$$= \left[\frac{x^2}{6} - \frac{x}{4}y + 2x\right]_{x=0}^{x=3}$$

$$= \frac{9}{6} - \frac{3}{4}y + 6 = \frac{15}{2} - \frac{3}{4}y$$

$$V = \int_0^4 S(y)dy = \int_0^4 \left(\frac{15}{2} - \frac{3}{4}y\right) dy$$

$$= \left[\frac{15}{2}y - \frac{3}{8}y^2\right]_0^4 = \frac{15}{2} \times 4 - \frac{3}{8} \times 16 = 30 - 6 = 24$$

<33 ページ　累次積分 1>

解答

問

$$\int_2^3 \left\{ \int_1^2 (x^2 - 3xy^2) dy \right\} dx = \int_2^3 \left\{ \left[x^2 y - xy^3 \right]_{y=1}^{y=2} \right\} dx$$

$$= \int_2^3 \left\{ (2x^2 - 8x) - (x^2 - x) \right\} dx = \int_2^3 (x^2 - 7x) dx = \left[\frac{x^3}{3} - \frac{7}{2} x^2 \right]_2^3$$

$$= \left(\frac{27}{3} - \frac{63}{2} \right) - \left(\frac{8}{3} - \frac{28}{2} \right) = \frac{19}{3} - \frac{35}{2} = \frac{38 - 105}{6}$$

$$= -\frac{67}{6}$$

<34 ページ　累次積分 2>

解答

問

$$\int_1^2 \left\{ \int_2^3 (x^2 - 3xy^2) dx \right\} dy = \int_1^2 \left\{ \left[\frac{x^3}{3} - \frac{3x^2}{2} y^2 \right]_{x=2}^{x=3} \right\} dy$$

$$= \int_1^2 \left\{ \left(\frac{27}{3} - \frac{27}{2} y^2 \right) - \left(\frac{8}{3} - \frac{12}{2} y^2 \right) \right\} dy = \int_1^2 \left(\frac{19}{3} - \frac{15}{2} y^2 \right) dy$$

$$= \left[\frac{19}{3} y - \frac{5}{2} y^3 \right]_1^2 = \left(\frac{38}{3} - \frac{40}{2} \right) - \left(\frac{19}{3} - \frac{5}{2} \right) = \frac{19}{3} - \frac{35}{2}$$

$$= \frac{38 - 105}{6} = -\frac{67}{6}$$

<35 ページ　長方形領域の 2 重積分 >

解答

問

$$\iint_D (xy+y^2)dxdy$$

$$= \int_{-1}^{1} \left\{ \int_0^2 (xy+y^2)dx \right\} dy = \int_{-1}^{1} \left\{ \left[\frac{x^2}{2}y + y^2 x \right]_{x=0}^{x=2} \right\} dy$$

$$= \int_{-1}^{1} \left(2y + 2y^2 \right) dy = \left[y^2 + \frac{2}{3}y^3 \right]_{-1}^{1} = \left(1 + \frac{2}{3} \right) - \left(1 - \frac{2}{3} \right)$$

$$= \frac{4}{3}$$

<36 ページ　一般領域の 2 重積分 1>

解答

問

$$\iint_{D_1}(x+y)dxdy=\int_1^2\left\{\int_0^1(x+y)dy\right\}dx=\int_1^2\left\{\left[xy+\frac{y^2}{2}\right]_{y=0}^{y=1}\right\}dx$$

$$=\int_1^2\left(x+\frac{1}{2}\right)dx=\left[\frac{x^2}{2}+\frac{x}{2}\right]_1^2=\left(\frac{4}{2}+\frac{2}{2}\right)-\left(\frac{1}{2}+\frac{1}{2}\right)=2$$

$$\iint_{D_2}(x+y)dxdy=\int_2^3\left\{\int_1^2(x+y)dy\right\}dx=\int_2^3\left\{\left[xy+\frac{y^2}{2}\right]_{y=1}^{y=2}\right\}dx$$

$$=\int_2^3\left(x+\frac{3}{2}\right)dx=\left[\frac{x^2}{2}+\frac{3}{2}x\right]_2^3=\left(\frac{9}{2}+\frac{9}{2}\right)-\left(\frac{4}{2}+\frac{6}{2}\right)=9-5=4$$

(答) $\iint_D(x+y)dxdy=\iint_{D_1}(x+y)dxdy+\iint_{D_2}(x+y)dxdy=2+4=6$

<37 ページ　一般領域の 2 重積分 2>

解答

問

$$\iint_D (x^2 - xy)dxdy$$

$$= \int_0^1 \left\{ \int_0^{-x+1} (x^2 - xy)dy \right\} dx = \int_0^1 \left\{ \left[x^2 y - \frac{xy^2}{2} \right]_{y=0}^{y=-x+1} \right\} dx$$

$$= \int_0^1 \left\{ x^2(-x+1) - \frac{(-x+1)^2}{2} x \right\} dx = \int_0^1 \left\{ -\frac{3}{2}x^3 + 2x^2 - \frac{x}{2} \right\} dx$$

$$= \left[-\frac{3}{8}x^4 + \frac{2}{3}x^3 - \frac{x^2}{4} \right]_0^1 = -\frac{3}{8} + \frac{2}{3} - \frac{1}{4} = \frac{-9+16-6}{24}$$

$$= \frac{1}{24}$$

<38ページ　一般領域の2重積分3>

解答

問

$$\iint_D (y-x)dxdy \qquad \left(D=\{(x,y)\ :\ 0 \leqq x \leqq y\ ,\ 0 \leqq y \leqq 1\} \right)$$

$$= \int_0^1 \left\{ \int_0^y (y-x)dx \right\} dy = \int_0^1 \left\{ \left[yx - \frac{1}{2}x^2 \right]_{x=0}^{x=y} \right\} dy$$

$$= \int_0^1 \left\{ y^2 - \frac{1}{2}y^2 \right\} dy = \int_0^1 \frac{1}{2}y^2 dy$$

$$= \left[\frac{1}{6}y^3 \right]_0^1 = \frac{1}{6}$$

＜39 ページ　面積比＞

解答

問

(1) $\Delta(u,v) = ab$, $\Delta(x,y) = \begin{vmatrix} 3a & 2b \\ a & 4b \end{vmatrix} = 12ab - 2ab = 10ab$

$$\frac{\Delta(x,y)}{\Delta(u,v)} = \frac{10ab}{ab} = 10$$

(2) $J = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10$

<40 ページ　重積分の変数変換>

解答

問

$$\iint_D e^{-x^2-y^2}dxdy = \iint_\Omega e^{-r^2} r dr d\theta \qquad \left(\Omega=\left\{(r,\theta)\ :\ 0\leqq r\leqq R\ ,\ 0\leqq\theta\leqq\frac{\pi}{2}\right\}\right)$$

$$= \int_0^{\frac{\pi}{2}}\left\{\int_0^R e^{-r^2} r dr\right\} d\theta = \int_0^{\frac{\pi}{2}}\left\{\left[-\frac{1}{2}e^{r^2}\right]_{r=0}^{r=R}\right\} d\theta$$

$$= \int_0^{\frac{pi}{2}}\left\{-\frac{1}{2}e^{-R^2}+\frac{1}{2}\right\} d\theta = \frac{\pi}{2}\left(-\frac{1}{2}e^{-R^2}+\frac{1}{2}\right)$$

$$= \frac{\pi}{4}\left(1-e^{-R^2}\right)$$