＜１ページ．正規直交系１＞

問の解答

$$\left|\overrightarrow{e_2}\right| = \sqrt{\left(\frac{2}{\sqrt{5}}\right)^2 + 0^2 + \left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{\frac{5}{5}} = 1$$

$$\left|\overrightarrow{e_3}\right| = \sqrt{\left(-\frac{3}{\sqrt{70}}\right)^2 + \left(\frac{5}{\sqrt{70}}\right)^2 + \left(\frac{6}{\sqrt{70}}\right)^2} = \sqrt{\frac{9+25+36}{70}} = 1$$

$$\overrightarrow{e_1} \cdot \overrightarrow{e_3} = \frac{2}{\sqrt{5}} \times \left(-\frac{3}{\sqrt{70}}\right) + 0 + \frac{1}{\sqrt{3}} \times \frac{6}{\sqrt{70}} = 0$$

$$\overrightarrow{e_2} \cdot \overrightarrow{e_3} = \frac{1}{\sqrt{14}} \times \left(-\frac{3}{\sqrt{70}}\right) + \frac{3}{\sqrt{14}} \times \frac{5}{\sqrt{70}} + \left(-\frac{2}{\sqrt{14}}\right) \times \frac{6}{\sqrt{70}} = \frac{-3+15-12}{\sqrt{14} \times \sqrt{70}} = 0$$

＜ 2ページ. 正規直交系2 ＞

問の解答

$$\overrightarrow{e_2} \times \overrightarrow{e_3} = \begin{pmatrix} \begin{vmatrix} \frac{3}{\sqrt{14}} & \frac{5}{\sqrt{70}} \\ \frac{-2}{\sqrt{14}} & \frac{6}{\sqrt{70}} \end{vmatrix} \\ \\ \begin{vmatrix} \frac{-2}{\sqrt{14}} & \frac{6}{\sqrt{70}} \\ \frac{1}{\sqrt{14}} & \frac{-3}{\sqrt{70}} \end{vmatrix} \\ \\ \begin{vmatrix} \frac{1}{\sqrt{14}} & \frac{-3}{\sqrt{70}} \\ \frac{3}{\sqrt{14}} & \frac{5}{\sqrt{70}} \end{vmatrix} \end{pmatrix} = \begin{pmatrix} \frac{18+10}{14\sqrt{5}} \\ \\ \frac{-6+6}{14\sqrt{5}} \\ \\ \frac{5+9}{14\sqrt{5}} \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ 0 \\ \frac{1}{\sqrt{5}} \end{pmatrix} = \overrightarrow{e_1}$$

$$\overrightarrow{e_3} \times \overrightarrow{e_1} = \begin{pmatrix} \begin{vmatrix} \frac{5}{\sqrt{70}} & 0 \\ \frac{6}{\sqrt{70}} & \frac{1}{\sqrt{5}} \end{vmatrix} \\ \\ \begin{vmatrix} \frac{6}{\sqrt{70}} & \frac{1}{\sqrt{5}} \\ \frac{-3}{\sqrt{70}} & \frac{2}{\sqrt{5}} \end{vmatrix} \\ \\ \begin{vmatrix} \frac{-3}{\sqrt{70}} & \frac{2}{\sqrt{5}} \\ \frac{5}{\sqrt{70}} & 0 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} \frac{5}{5\sqrt{14}} \\ \\ \frac{12+3}{5\sqrt{14}} \\ \\ \frac{-10}{5\sqrt{14}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \\ \frac{-2}{\sqrt{14}} \end{pmatrix} = \overrightarrow{e_2}$$

＜ 3ページ. 平行六面体の体積＞

問の解答

(1)

$$\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \overrightarrow{c} = \begin{pmatrix} \begin{vmatrix} 4 & 3 \\ 5 & 4 \end{vmatrix} \\ \\ \begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix} \\ \\ \begin{vmatrix} 3 & 2 \\ 4 & 3 \end{vmatrix} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 1 - 4 + 3 = 0$$

(2)

$$\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \overrightarrow{c} = \begin{pmatrix} \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} \\ \\ \begin{vmatrix} 3 & 5 \\ 1 & 1 \end{vmatrix} \\ \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} = 1 - 8 + 7 = 0$$

＜ 4ページ. スカラー三重積1 ＞

問の解答

(1) $$\left(\overrightarrow{b}\times\overrightarrow{a}\right)\cdot\overrightarrow{c}=\left\{\begin{pmatrix}2\\1\\0\end{pmatrix}\times\begin{pmatrix}1\\2\\0\end{pmatrix}\right\}\cdot\begin{pmatrix}0\\1\\2\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}1&2\\0&0\end{vmatrix}\\\begin{vmatrix}0&0\\2&1\end{vmatrix}\\\begin{vmatrix}2&1\\1&2\end{vmatrix}\end{pmatrix}\cdot\begin{pmatrix}0\\1\\2\end{pmatrix}=\begin{pmatrix}0\\0\\3\end{pmatrix}\cdot\begin{pmatrix}0\\1\\2\end{pmatrix}=6$$

(2) $$\left(\overrightarrow{a}\times\overrightarrow{c}\right)\cdot\overrightarrow{b}=\left\{\begin{pmatrix}1\\2\\0\end{pmatrix}\times\begin{pmatrix}0\\1\\2\end{pmatrix}\right\}\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}2&1\\0&2\end{vmatrix}\\\begin{vmatrix}0&2\\1&0\end{vmatrix}\\\begin{vmatrix}1&0\\2&1\end{vmatrix}\end{pmatrix}\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}=\begin{pmatrix}4\\-2\\-1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$=8-2+0=6$$

(3) $$\left(\overrightarrow{c}\times\overrightarrow{b}\right)\cdot\overrightarrow{a}=\left\{\begin{pmatrix}0\\1\\2\end{pmatrix}\times\begin{pmatrix}2\\1\\0\end{pmatrix}\right\}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}1&1\\2&0\end{vmatrix}\\\begin{vmatrix}2&0\\0&2\end{vmatrix}\\\begin{vmatrix}0&2\\1&1\end{vmatrix}\end{pmatrix}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}-2\\4\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$=-2+8-0=6$$

(4) $$\left(\overrightarrow{b}\times\overrightarrow{c}\right)\cdot\overrightarrow{a}=\left\{\begin{pmatrix}2\\1\\0\end{pmatrix}\times\begin{pmatrix}0\\1\\2\end{pmatrix}\right\}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}1&1\\0&2\end{vmatrix}\\\begin{vmatrix}0&2\\2&0\end{vmatrix}\\\begin{vmatrix}2&0\\1&1\end{vmatrix}\end{pmatrix}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}2\\-4\\2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$=2-8+0=-6$$

＜ 5ページ. スカラー三重積2 ＞

問の解答

(1) $\left(\overrightarrow{c} \times 3\overrightarrow{a}\right) \cdot \overrightarrow{b} = 3\left(\overrightarrow{c} \times \overrightarrow{a}\right) \cdot \overrightarrow{b} = 3\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \overrightarrow{c} = 3 \times 3 = 9$

(2) $\left(\left(2\overrightarrow{b} + \overrightarrow{c}\right) \times \left(\overrightarrow{c} + \overrightarrow{b}\right)\right) \cdot \overrightarrow{a}$

$= \left(2\overrightarrow{b} \times \overrightarrow{c} + 2\overrightarrow{b} \times \overrightarrow{b} + \overrightarrow{c} \times \overrightarrow{c} + \overrightarrow{c} \times \overrightarrow{b}\right) \cdot \overrightarrow{a}$

$= \left(2\overrightarrow{b} \times \overrightarrow{c} - \overrightarrow{b} \times \overrightarrow{c}\right) \cdot \overrightarrow{a}$

$= \left(\overrightarrow{b} \times \overrightarrow{c}\right) \cdot \overrightarrow{a} = \left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \overrightarrow{c} = 3$

＜ 6ページ. 行列 ＞

問1の解答

3行4列 ((3 , 4)型) 行列

第1行 $(1\ 2\ 3\ 4)$

第2列 $\begin{pmatrix} 2 \\ 6 \\ 10 \end{pmatrix}$

(1 , 4) 成分　4

(2 , 3) 成分　7

問2の解答

4次の行ベクトル $(1\ 2\ 3\ 4)$

3次の列ベクトル $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

2次の正方行列 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

< 7ページ. 行列の計算1 >

問の解答

(1) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} + \begin{pmatrix} 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 7 & 7 \\ 7 & 7 & 7 \end{pmatrix}$

(2) $\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$

(3) $-2\begin{pmatrix} -1 & 0 & 1 \\ -3 & 3 & -5 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 6 & -6 & 10 \end{pmatrix}$

(4) $\begin{pmatrix} 5 & 6 \\ 8 & 4 \\ 9 & 8 \end{pmatrix} - 3\begin{pmatrix} 1 & 2 \\ 3 & -2 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 8 & 4 \\ 9 & 8 \end{pmatrix} - \begin{pmatrix} 3 & 6 \\ 9 & -6 \\ 12 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ -1 & 10 \\ -3 & 8 \end{pmatrix}$

＜ 8ページ. 行列の計算2 ＞

問の解答

(1) $3A + 5X - 2B = 0$

$$X = \frac{1}{5}\{-3A + 2B\}$$

$$= \frac{1}{5}\left\{-\begin{pmatrix} 9 & 3 \\ 6 & 12 \end{pmatrix} + \begin{pmatrix} 10 & 12 \\ 14 & 16 \end{pmatrix}\right\}$$

$$= \frac{1}{5}\begin{pmatrix} 1 & 9 \\ 8 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{5} & \frac{9}{5} \\ \frac{8}{5} & \frac{4}{5} \end{pmatrix}$$

(2) $3(2B - 4X) = -2(X - 5A)$

$$6B - 12X = -2X + 10A$$

$$-10A + 6B = 10X$$

$$X = \frac{1}{10}\{-10A + 6B\}$$

$$= -A + \frac{3}{5}B$$

$$= -\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} + \frac{3}{5}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} -3+3 & -1+\frac{18}{5} \\ -2+\frac{21}{5} & -4+\frac{24}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{13}{5} \\ \frac{11}{5} & \frac{4}{5} \end{pmatrix}$$

＜ 9ページ. 行列の積1 ＞

問の解答

(1) $(7 \ \ 5)\begin{pmatrix} 3 \\ -4 \end{pmatrix} = 21 - 20 = 1$

(2) $(5 \ \ 0 \ \ -6)\begin{pmatrix} 3 \\ 11 \\ 2 \end{pmatrix} = 15 + 0 - 12 = 3$

(3) $(1 \ \ 8 \ \ -7 \ \ 5)\begin{pmatrix} 6 \\ 0 \\ -3 \\ -2 \end{pmatrix} = 6 + 0 + 21 - 10 = 17$

＜ 10 ページ. 行列の積 2 ＞

問の解答

(1) $\begin{pmatrix} 3 & 6 \\ 9 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6+6 \\ 18+0 \end{pmatrix} = \begin{pmatrix} 12 \\ 18 \end{pmatrix}$

(2) $\begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 10+3 \\ 0-9 \end{pmatrix} = \begin{pmatrix} 13 \\ -9 \end{pmatrix}$

(3) $\begin{pmatrix} 0 & 3 & 1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 0-3+7 \\ 10+1+0 \end{pmatrix} = \begin{pmatrix} 4 \\ 11 \end{pmatrix}$

＜ 11 ページ. 行列の積 3 ＞

問の解答

(1) $\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 15 \\ 3 \end{pmatrix}$

(2) $\begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 0+4 & 5-2 \\ 0+8 & 3-4 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 8 & -1 \end{pmatrix}$

(3) $\begin{pmatrix} 0 & 3 & 1 \\ 2 & -1 & 0 \end{pmatrix}\begin{pmatrix} 5 & 3 \\ -1 & 0 \\ 7 & 2 \end{pmatrix} = \begin{pmatrix} 0-3+7 & 0+0+2 \\ 10+1+0 & 6-0+0 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 11 & 6 \end{pmatrix}$

(4) $\begin{pmatrix} 0 & 1 & -1 \\ 2 & 0 & 3 \\ 1 & 5 & 0 \end{pmatrix}\begin{pmatrix} 3 & 5 & -1 \\ 2 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 0+2+1 & 0-1-2 & 0+0-1 \\ 6+0-3 & 10-0+6 & -2+0+3 \\ 3+10-0 & 5-5+0 & -1+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -3 & -1 \\ 3 & 16 & 1 \\ 13 & 0 & -1 \end{pmatrix}$$

＜ 12 ページ. 行列の積 4 ＞

問の解答

(1) $AB = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 6 & 7 \end{pmatrix}$

$BA = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 2 & 3 \end{pmatrix}$

(2) $AB = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 5 & 7 & 9 \\ 4 & 0 & 3 \\ 1 & -2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 5-4+2 & 7-0-4 & 9-3-2 \\ 0+12-1 & 0+0+2 & 0+9+1 \\ 0+4+2 & 0+0-4 & 0+3-2 \end{pmatrix}$

$= \begin{pmatrix} 3 & 3 & 4 \\ 11 & 2 & 10 \\ 6 & -4 & 1 \end{pmatrix}$

$BA = \begin{pmatrix} 5 & 7 & 9 \\ 4 & 0 & 3 \\ 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 5 & -5+21+9 & 10-7+18 \\ 4 & -4+0+3 & 8-0+6 \\ 1 & -1-6-1 & 2+2-2 \end{pmatrix}$

$= \begin{pmatrix} 5 & 25 & 21 \\ 4 & -1 & 14 \\ 1 & -8 & 2 \end{pmatrix}$

＜ 13 ページ. 行列の積 5 ＞

問の解答

(1) $C(B+A) = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} -5+2 & -3+1 \\ 10+6 & 6+3 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ 16 & 9 \end{pmatrix}$

(2) $BC - AC = (B-A)C = \left(\begin{pmatrix} 3 & 4 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \right) \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 5 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -1+10 & 1+15 \\ 0-10 & 0-15 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 16 \\ -10 & -15 \end{pmatrix}$$

(3) $AA + AC = A(A+C) = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \left(\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix} \right)$

$$= \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 2-3 & 0-6 \\ 1+9 & 0+18 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -6 \\ 10 & 18 \end{pmatrix}$$

＜ 14ページ. 行列の積 6 ＞

問の解答

(1) $A^2 - B^2 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} 4-1 & 2+0 \\ -2-0 & -1+0 \end{pmatrix} - \begin{pmatrix} 4+0 & 2+2 \\ 0+0 & 0+4 \end{pmatrix}$

$= \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} - \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix}$

$= \begin{pmatrix} -1 & -2 \\ -2 & -5 \end{pmatrix}$

(2) $A^2 - BA + AB - B^2 = (A-B)A + (A-B)B = (A-B)(A+B)$

$= \left(\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\right)\left(\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\right)$

$= \begin{pmatrix} 0 & 0 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 0-0 & 0+0 \\ -4+2 & -2-4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -2 & -6 \end{pmatrix}$

＜ 15 ページ.2 次の行列式 1 ＞

問の解答

(1) $\begin{vmatrix} k_1 & 2 \\ k_2 & -1 \end{vmatrix} = \begin{vmatrix} 3x+2y & 2 \\ x-y & -1 \end{vmatrix} = x\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} + y\begin{vmatrix} 2 & 2 \\ -1 & -1 \end{vmatrix} = -5x$

(2) $\begin{vmatrix} 3 & k_1 \\ 1 & k_2 \end{vmatrix} = \begin{vmatrix} 3 & 3x+2y \\ 1 & x-y \end{vmatrix} = x\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} + y\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = -5y$

＜ 16ページ.2次の行列式2 ＞

問の解答

(1) $AB = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ 8 & -1 \end{pmatrix}$

$\det(A) = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10$ $\det(AB) = \begin{vmatrix} 4 & 7 \\ 8 & -1 \end{vmatrix} = -4 - 56 = -60$

$BA = \begin{pmatrix} 0 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 12 \\ 5 & 0 \end{pmatrix}$

$\det(B) = \begin{vmatrix} 0 & 3 \\ 2 & -1 \end{vmatrix} = -6$ $\det(BA) = \begin{vmatrix} 3 & 12 \\ 5 & 0 \end{vmatrix} = -60$

(2) $AB = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x & y \\ y & w \end{pmatrix} = \begin{pmatrix} x+2y & y+2w \\ 3x-y & 3y-w \end{pmatrix}$

$\det(A) = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -1 - 6 = -7$

$\det(AB) = \begin{vmatrix} x+2y & y+2w \\ 3x-y & 3y-w \end{vmatrix} = (x+2y)(3y-w) - (y+2w)(3x-y)$

$= 3xy + 6y^2 - xw - 2yw - (3xy + 6xw - y^2 - 2yw) = 7y^2 - 7xw$

$BA = \begin{pmatrix} x & y \\ y & w \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} x+3y & 2x-y \\ y+3w & 2y-w \end{pmatrix}$

$\det(B) = \begin{vmatrix} x & y \\ y & w \end{vmatrix} = xw - y^2$

$\det(BA) = \begin{vmatrix} x+3y & 2x-y \\ y+3w & 2y-w \end{vmatrix} = (x+3y)(2y-w) - (2x-y)(y+3w)$

$= 2xy + 6y^2 - xw - 3yw - (2xy - y^2 + 6xw - 3yw) = 7y^2 - 7xw$

＜ 17ページ.3次の行列式1 ＞

問の解答

(1) $\begin{vmatrix} 2 & -1 & 10 \\ 1 & 1 & 10 \\ 0 & 3 & 10 \end{vmatrix} = 20 + 30 - 0 - (0 + 60 - 10) = 50 - 50 = 0$

(2) $\begin{vmatrix} 2 & 1 & 0 \\ 3 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0 + 1 + 0 - (0 + 4 + 9) = 1 - 13 = -12$

(3) $\begin{vmatrix} 5 & 1 & -2 \\ 0 & -1 & 3 \\ -1 & 2 & 0 \end{vmatrix} = 0 - 0 - 3 - (-2 + 30 + 0) = -3 - 28 = -31$

＜ 18 ページ.3 次の行列式 2 ＞

問の解答

(1) $\begin{vmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ 3 & 2 & -1 \end{vmatrix} = 1 \times \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5$

(2) $\begin{vmatrix} 0 & 3 & 1 \\ 2 & -1 & 2 \\ 0 & 1 & 4 \end{vmatrix} = -2 \times \begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix} = -2 \times (12 - 1) = -22$

(3) $\begin{vmatrix} 5 & 2 & -3 \\ -1 & 0 & 4 \\ 3 & -2 & 1 \end{vmatrix} = 5 \times \begin{vmatrix} 0 & 4 \\ -2 & 1 \end{vmatrix} - 2 \times \begin{vmatrix} -1 & 4 \\ 3 & 1 \end{vmatrix} - 3 \begin{vmatrix} -1 & 0 \\ 3 & -2 \end{vmatrix}$

$= 5 \times 8 - 2 \times (-13) - 3 \times 2$

$= 40 + 26 - 6 = 60$

＜ 19ページ.3次の行列式3 ＞

問の解答

(1) $\begin{vmatrix} 3 & 2 & 4 \\ 1 & 0 & 0 \\ 2 & -1 & 3 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 0 \\ 3 & 2 & 4 \\ 2 & -1 & 3 \end{vmatrix} = -\begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix} = -(6+4) = -10$

(2) $\begin{vmatrix} 2 & 1 & -1 \\ -1 & 3 & 0 \\ 4 & -2 & 0 \end{vmatrix} = -\begin{vmatrix} -1 & 1 & 2 \\ 0 & 3 & -1 \\ 0 & -2 & 4 \end{vmatrix} = -(-1)\begin{vmatrix} 3 & -1 \\ -2 & 4 \end{vmatrix} = 12 - 2 = 10$

(3) $\begin{vmatrix} 1 & -2 & 0 \\ -1 & 3 & 2 \\ 5 & 1 & 0 \end{vmatrix} = -\begin{vmatrix} 0 & -2 & 1 \\ 2 & 3 & -1 \\ 0 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 1 & 5 \end{vmatrix} = 2\begin{vmatrix} -2 & 1 \\ 1 & 5 \end{vmatrix}$

$= 2(-10-1) = -22$

＜ 20ページ.3次の行列式4 ＞

問の解答

(1) $\begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 3 & 4 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 4 \\ 0 & -1 & -3 \\ 0 & -2 & -6 \end{vmatrix} = \begin{vmatrix} -1 & -3 \\ -2 & -6 \end{vmatrix} = 6 - 6 = 0$

(2) $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 4 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$

(3) $\begin{vmatrix} 1 & 0 & -1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 3 & 3 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix} = 6 - 3 = 3$

＜ 21 ページ.3 次の行列式 5 ＞
問の解答

(1) $\begin{vmatrix} 1 & 8 & 11 \\ 2 & 10 & 13 \\ 3 & 12 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -6 & -9 \\ 3 & -12 & -26 \end{vmatrix} = \begin{vmatrix} -6 & -9 \\ -12 & -26 \end{vmatrix}$

$$= -6 \times (-1) \begin{vmatrix} 1 & 9 \\ 2 & 26 \end{vmatrix} = 6 \times (26 - 18) = 48$$

(2) $\begin{vmatrix} 1 & 2 & 3 \\ 10 & 21 & 33 \\ 45 & 93 & 160 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 3 & 25 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 3 & 25 \end{vmatrix} = 25 - 9 = 16$

(3) $\begin{vmatrix} 10 & 28 & 19 \\ 1 & 3 & 2 \\ 22 & 80 & 46 \end{vmatrix} = -\begin{vmatrix} 1 & 3 & 2 \\ 10 & 28 & 19 \\ 22 & 80 & 46 \end{vmatrix} = -\begin{vmatrix} 1 & 3 & 2 \\ 0 & -2 & -1 \\ 0 & 14 & 2 \end{vmatrix}$

$$= -\begin{vmatrix} -2 & -1 \\ 14 & 2 \end{vmatrix} = -(-4 + 14) = -10$$

(4) $\begin{vmatrix} 11 & 10 & 13 \\ 26 & 24 & 27 \\ 5 & 5 & 5 \end{vmatrix} = 5\begin{vmatrix} 11 & 10 & 13 \\ 26 & 24 & 27 \\ 1 & 1 & 1 \end{vmatrix} = -5\begin{vmatrix} 1 & 1 & 1 \\ 26 & 24 & 27 \\ 11 & 10 & 13 \end{vmatrix}$

$$= -5\begin{vmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = -5\begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= -5(-4 + 1) = -5 \times (-3) = 15$$

＜ 22ページ.4次の行列式1 ＞

問の解答

(1) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & -2 \\ 0 & -1 & 0 & 5 \\ 2 & -1 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 0 & 5 \\ -1 & 3 & 0 \end{vmatrix} - 2 \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -2 \\ -1 & 0 & 5 \end{vmatrix}$

$$= (6 - 10 - 15) - 2(20 + 6 + 8 - 15 - 0)$$

$$= -19 - 2 \times 19 = -19 \times 3 = -57$$

(2) $\begin{vmatrix} 2 & 1 & 0 & 5 \\ 3 & -1 & 1 & 2 \\ 4 & 6 & 0 & -1 \\ 5 & 3 & 0 & -4 \end{vmatrix} = - \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & -1 & 3 & 2 \\ 0 & 6 & 4 & -1 \\ 0 & 3 & 5 & -4 \end{vmatrix} = -(-1) \begin{vmatrix} 1 & 2 & 5 \\ 6 & 4 & -1 \\ 3 & 5 & -4 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 2 & 5 \\ 0 & -8 & -31 \\ 0 & -1 & -19 \end{vmatrix} = \begin{vmatrix} 8 & 31 \\ 1 & 19 \end{vmatrix} = \begin{vmatrix} 8 & -1 \\ 1 & 15 \end{vmatrix} = 121$$

＜ 23ページ.4次の行列式2 ＞

問の解答

(1) $\begin{vmatrix} 1 & 5 & 6 & 7 \\ 0 & 2 & 8 & 9 \\ 0 & 0 & 3 & 10 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 1 \times 2 \times 3 \times 4 = 24$

(2) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 0 & 0 & 0 & 0 \\ 9 & 10 & 11 & 12 \end{vmatrix} = 0$

(3) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 5 & 5 & 5 & 5 \\ 4 & 5 & 6 & 5 \end{vmatrix} = 0$

(4) $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 8 & 6 \\ 0 & 0 & 7 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \times \begin{vmatrix} 8 & 6 \\ 7 & 5 \end{vmatrix} = (4-6) \times (40-42) = 4$

(5) $\begin{vmatrix} 2 & 4 & -1 & 1 \\ 3 & 0 & 0 & 2 \\ 3 & 2 & 1 & 3 \\ 4 & 1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 4 & -1 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 3 \\ 0 & 1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 4 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & 1 & 2 & 4 \end{vmatrix}$

$$= -\begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 4 & -1 & 0 \\ 0 & 2 & 1 & 3 \\ 0 & 1 & 2 & 4 \end{vmatrix} = -\begin{vmatrix} 4 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 4 & -1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= -(16-3-24+8) + (4-1) = 3+3 = 6$$

(6) $\begin{vmatrix} 4 & 1 & 0 & 3 \\ 3 & 0 & 2 & 0 \\ 2 & -1 & 4 & 1 \\ 1 & -2 & 6 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 0 & 3 \\ 3 & 0 & 2 & 0 \\ 3 & -1 & 4 & 1 \\ 3 & -2 & 6 & 2 \end{vmatrix} = 3 \times 2 \times \begin{vmatrix} 1 & 1 & 0 & 3 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 \\ 1 & -2 & 3 & 2 \end{vmatrix} = 6 \times \begin{vmatrix} 1 & 1 & 1 & 3 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 1 & -2 & 1 & 2 \end{vmatrix} = 0$

(7) $\begin{vmatrix} -1 & 2 & 0 & 1 \\ 1 & 4 & 3 & 2 \\ 2 & 5 & 3 & 4 \\ 3 & 0 & -1 & 1 \end{vmatrix} = -27$

＜ 24 ページ. 高次の行列式＞

問 1 の解答

$$\begin{vmatrix} a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 \\ a_5 & b_5 & c_5 & d_5 & e_5 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 & d_2 & e_2 \\ b_3 & c_3 & d_3 & e_3 \\ b_4 & c_4 & d_4 & e_4 \\ b_5 & c_5 & d_5 & e_5 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 & d_2 & e_2 \\ a_3 & c_3 & d_3 & e_3 \\ a_4 & c_4 & d_4 & e_4 \\ a_5 & c_5 & d_5 & e_5 \end{vmatrix}$$

$$+c_1 \begin{vmatrix} a_2 & b_2 & d_2 & e_2 \\ a_3 & b_3 & d_3 & e_3 \\ a_4 & b_4 & d_4 & e_4 \\ a_5 & b_5 & d_5 & e_5 \end{vmatrix} - d_1 \begin{vmatrix} a_2 & b_2 & c_2 & e_2 \\ a_3 & b_3 & c_3 & e_3 \\ a_4 & b_4 & c_4 & e_4 \\ a_5 & b_5 & c_5 & e_5 \end{vmatrix} + e_1 \begin{vmatrix} a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \\ a_5 & b_5 & c_5 & d_5 \end{vmatrix}$$

問 2 の解答

$$(1) \begin{vmatrix} 1 & 3 & 4 & 5 & 6 \\ 0 & 2 & 5 & 6 & 7 \\ 0 & 0 & 3 & 7 & 8 \\ 0 & 0 & 0 & 4 & 9 \\ 0 & 0 & 0 & 0 & 5 \end{vmatrix} = 1 \times 2 \times 3 \times 4 \times 5 = 120$$

$$(2) \begin{vmatrix} 0 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 4 & 0 \\ 3 & 0 & 1 & 0 & 2 \\ 4 & 0 & 2 & 0 & 3 \\ 5 & 0 & 3 & 0 & 4 \end{vmatrix} = 0$$

$$(3) \begin{vmatrix} 1 & 2 & 2 & 3 & 3 \\ 0 & 0 & 3 & 0 & 0 \\ -2 & 2 & 3 & 1 & -1 \\ 0 & -1 & 1 & -2 & 3 \\ 4 & 3 & 5 & 2 & 6 \end{vmatrix} = -315$$

< 25ページ.2元連立一次方程式1 >

問1の解答

$$\begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix} = x \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad \begin{vmatrix} a_1 & k_1 \\ a_2 & k_2 \end{vmatrix} = y \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$x = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix} \qquad y = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \begin{vmatrix} a_1 & k_1 \\ a_2 & k_2 \end{vmatrix}$$

問2の解答

(1) $\begin{cases} x - 3y = -1 \\ 3x + 2y = 1 \end{cases}$

$$\begin{vmatrix} 1 & -3 \\ 3 & 2 \end{vmatrix} = 2 + 9 = 11$$

$$x = \frac{1}{11} \begin{vmatrix} -1 & -3 \\ 1 & 2 \end{vmatrix} = \frac{1}{11}(-2+3) = \frac{1}{11}$$

$$y = \frac{1}{11} \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = \frac{1}{11}(1+3) = \frac{4}{11}$$

(2) $\begin{cases} 3x + 5y = k_1 \\ x + 2y = k_2 \end{cases}$

$$\begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix} = 6 - 5 = 1$$

$$x = \begin{vmatrix} k_1 & 5 \\ k_2 & 2 \end{vmatrix} = 2k_1 - 5k_2$$

$$y = \begin{vmatrix} 3 & k_1 \\ 1 & k_2 \end{vmatrix} = 3k_2 - k_1$$

< 26ページ.2元連立一次方程式2 >

問の解答

(1) $\begin{cases} x + \ \ 3y = 0 \\ 4x + 12y = 0 \end{cases}$ $\qquad$ $\begin{cases} x = 3t \\ y = -t \end{cases}$

(2) $\begin{cases} 2x - \ \ 5y = 0 \\ 10x - 25y = 0 \end{cases}$ $\qquad$ $\begin{cases} x = 5t \\ y = 2t \end{cases}$

＜ 27 ページ.3 元連立一次方程式＞

問 1 の解答

$$\begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix} = y \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix} = z \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$y = \frac{1}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix} \qquad z = \frac{1}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}$$

問 2 の解答

(1) $\begin{cases} x + 2y + 3z = 4 \\ 2x + y + 2z = 0 \\ 3x - y - 2z = -3 \end{cases}$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & -1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -3 & -4 \\ 3 & -7 & -11 \end{vmatrix} = \begin{vmatrix} -3 & -4 \\ -7 & -11 \end{vmatrix} = 32 - 28 = 5$$

$$\begin{vmatrix} 4 & 2 & 3 \\ 0 & 1 & 2 \\ -3 & -1 & -2 \end{vmatrix} = 4\begin{vmatrix} 1 & 2 \\ -1 & -2 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4(-2+2) - 3(4-3) = -3$$

$$\begin{vmatrix} 1 & 4 & 3 \\ 2 & 0 & 2 \\ 3 & -3 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -8 & -4 \\ 3 & -15 & -11 \end{vmatrix} = \begin{vmatrix} -8 & -4 \\ -15 & -11 \end{vmatrix} = 88 - 60 = 28$$

$$\begin{vmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \\ 3 & -1 & -3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -3 & -8 \\ 3 & -7 & -15 \end{vmatrix} = \begin{vmatrix} -3 & -8 \\ -7 & -15 \end{vmatrix} = 45 - 56 = -11$$

$$\underline{x = -\frac{3}{5}\ ,\ y = \frac{28}{5}\ ,\ z = -\frac{11}{5}}$$

(2) $\begin{cases} y - z = 2 \\ 3x + 2z = -1 \\ -2x - 5y = -3 \end{cases}$

$$\begin{vmatrix} 0 & 1 & -1 \\ 3 & 0 & 2 \\ -2 & -5 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -2 & -5 & -5 \end{vmatrix} = -\begin{vmatrix} 3 & 2 \\ -2 & -5 \end{vmatrix} = -(-15+4) = 11$$

$$\begin{vmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ -3 & -5 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -3 & -5 & -5 \end{vmatrix} = -\begin{vmatrix} 3 & 2 \\ -3 & -5 \end{vmatrix} = -(-15+6) = 9$$

$$\begin{vmatrix} 0 & 2 & -1 \\ 3 & -1 & 2 \\ -2 & -3 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -1 \\ 3 & 3 & 2 \\ -2 & -3 & 0 \end{vmatrix} = -\begin{vmatrix} 3 & 3 \\ -2 & -3 \end{vmatrix} = -(-9+6) = 3$$

$$\begin{vmatrix} 0 & 1 & 2 \\ 3 & 0 & -1 \\ -2 & -5 & -3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 3 & 0 & -1 \\ -2 & -5 & 7 \end{vmatrix} = -\begin{vmatrix} 3 & -1 \\ -2 & 7 \end{vmatrix} = -(21-2) = -19$$

$$\underline{x = \frac{9}{11}\ ,\ y = \frac{3}{11}\ ,\ z = -\frac{19}{11}}$$

< 28ページ. 単位行列 >

問1の解答

$$AB = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}\begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} = \begin{pmatrix} a_1b_1 & 0 \\ 0 & a_2b_2 \end{pmatrix}$$

$$BA = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} b_1a_1 & 0 \\ 0 & b_2a_2 \end{pmatrix}$$

問2の解答

(1) $AI = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = A \qquad IA = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = A$

(2) $$AI = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = A$$

$$IA = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = A$$

＜ 29 ページ. 零因子 ＞

問 1 の解答

(1) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(2) $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

問 2 の解答

(1) $AB = \begin{pmatrix} 6 & 2 \\ 9 & 3 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 18 & 14 \\ 27 & 21 \end{pmatrix}$

(2) $AC = \begin{pmatrix} 6 & 2 \\ 9 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 18 & 14 \\ 27 & 21 \end{pmatrix}$

(3) $A(B-C) = \begin{pmatrix} 6 & 2 \\ 9 & 3 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

問 3 の解答

(1) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} ad-bc & -ab+ab \\ cd-dc & -cb+ad \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(2) $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} da-bc & db-bd \\ -ca+ac & -cb+ad \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

< 30ページ. 正則行列 >

問1の解答

(1) $AB = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$BA = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(2) $AB = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$BA = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

問2の解答

もし正則行列であると仮定すると B^{-1} が存在する。

$$A = (B^{-1}B)A = B^{-1}(BA) = B^{-1}O = O$$

となって $A = O$ となり矛盾する。従って正則行列ではない。

＜ 31 ページ. 逆行列 1 ＞

問の解答

$A^{-1} = \begin{pmatrix} x & z \\ y & w \end{pmatrix}$ とする

$$AA^{-1} = \begin{pmatrix} 5 & -2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x & z \\ y & w \end{pmatrix} = \begin{pmatrix} 5x-2y & 5z-2w \\ 3x+y & 3z+w \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} 5x - 2y = 1 \\ 3x + y = 0 \end{cases} \qquad \begin{cases} 5z - 2w = 0 \\ 3z + w = 1 \end{cases}$$

この連立方程式を解くと、

$$x = \frac{1}{11}\ ,\ y = -\frac{3}{11} \quad , \quad z = \frac{2}{11}\ ,\ w = \frac{5}{11} \qquad \text{よって } A^{-1} = \begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ -\frac{3}{11} & \frac{5}{11} \end{pmatrix}$$

$$A^{-1}A = \frac{1}{11} \begin{pmatrix} 1 & 2 \\ -3 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ 3 & 1 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 5+6 & -2+2 \\ -15+15 & 6+5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

< 32ページ. 逆行列2 >

問の解答

$A^{-1} = \begin{pmatrix} x & z \\ y & w \end{pmatrix}$ とする

$$AA^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x & z \\ y & w \end{pmatrix} = \begin{pmatrix} ax+by & az+bw \\ cx+dy & cz+dw \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} ax+by=1 \\ cx+dy=0 \end{cases} \qquad \begin{cases} az+bw=0 \\ cz+dw=1 \end{cases}$$

この連立方程式を解くと、

$$x = \frac{d}{ad-bc}\ ,\ y = \frac{-c}{ad-bc}\ ,\ z = \frac{-b}{ad-bc}\ ,\ w = \frac{a}{ad-bc}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1}A = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{ad-bc} \begin{pmatrix} da-bc & 0 \\ 0 & -cb+ad \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

＜ 33ページ. 逆行列3 ＞

問の解答

(1) $\det A = 7 - 6 = 1$ より A は正則行列

$$A^{-1} = \begin{pmatrix} 1 & -3 \\ -2 & 7 \end{pmatrix}$$

(2) $\det A = 12 - 12 = 0$ より A は正則行列ではない

(3) $\det A = 21 - 20 = 1$ より A は正則行列

$$A^{-1} = \begin{pmatrix} 7 & 5 \\ 4 & 3 \end{pmatrix}$$

(4) $\det A = 24 + 24 = 48$ より A は正則行列

$$A^{-1} = \frac{1}{48}\begin{pmatrix} 4 & -3 \\ 8 & 6 \end{pmatrix} = \begin{pmatrix} \frac{4}{48} & -\frac{3}{48} \\ \frac{8}{48} & \frac{6}{48} \end{pmatrix} = \begin{pmatrix} \frac{1}{12} & -\frac{1}{16} \\ \frac{1}{6} & \frac{1}{8} \end{pmatrix}$$

＜ 34ページ. 固有値1 ＞

問の解答

(1) $\det(A-\lambda I)=\begin{vmatrix} 3-\lambda & 1 \\ 2 & 4-\lambda \end{vmatrix}=(3-\lambda)(4-\lambda)-2=0$

$\lambda^2-7\lambda+10=0$

$(\lambda-2)(\lambda-5)=0$ $\underline{\lambda=2\ ,\ 5}$

(2) $\det(A-\lambda I)=\begin{vmatrix} 1-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix}$

$=(1-\lambda)(3-\lambda)+2=\lambda^2-4\lambda+5=0$

$\underline{\lambda=2\pm i}$

(3) $\det(A-\lambda I)=\begin{vmatrix} 1-\lambda & -2 \\ 2 & 5-\lambda \end{vmatrix}$

$=(1-\lambda)(5-\lambda)+4=\lambda^2-6\lambda+9=0$

$\underline{\lambda=3}$

(4) $\det(A-\lambda I)=\begin{vmatrix} 1-\lambda & 3 \\ -2 & -1-\lambda \end{vmatrix}=-(1-\lambda)(1+\lambda)+6$

$=-(1-\lambda^2)+6=\lambda^2+5=0$

$\underline{\lambda=\pm\sqrt{5}i}$

< 35ページ. 行列の積4 >

問の解答

(1) $\begin{vmatrix} 3-\lambda & 2 \\ 1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda) - 2 = \lambda^2 - 5\lambda + 4 = 0$

$(\lambda-1)(\lambda-4) = 0$ $\qquad \underline{\lambda = 1\ ,\ 4}$

(2) $\underline{\lambda = 1\ ,\ 6\ ,\ 5}$

(3) $\begin{vmatrix} 5-\lambda & -2 & 7 \\ 2 & 1-\lambda & -1 \\ 0 & 0 & 3-\lambda \end{vmatrix} = -\begin{vmatrix} 0 & 0 & 3-\lambda \\ 2 & 1-\lambda & -1 \\ 5-\lambda & -2 & 7 \end{vmatrix}$

$$= -(3-\lambda)\begin{vmatrix} 2 & 1-\lambda \\ 5-\lambda & -2 \end{vmatrix}$$

$$= -(3-\lambda)(-4-(1-\lambda)(5-\lambda))$$

$$= -(3-\lambda)(-4-(\lambda^2-6\lambda+5))$$

$$= (3-\lambda)(\lambda^2-6\lambda+9) = (3-\lambda)(\lambda-3)^2 = 0$$

$\underline{\lambda = 3}$

(4) $\begin{vmatrix} 1-\lambda & 7 & 3 \\ 0 & -\lambda & 0 \\ 2 & -11 & -1-\lambda \end{vmatrix} = -\begin{vmatrix} 0 & -\lambda & 0 \\ 1-\lambda & 7 & 3 \\ 2 & -11 & -1-\lambda \end{vmatrix}$

$$= (-\lambda)\begin{vmatrix} 1-\lambda & 3 \\ 2 & -1-\lambda \end{vmatrix}$$

$$= -\lambda\{(1-\lambda)(-1-\lambda) - 6\}$$

$$= -\lambda\{-(1-\lambda^2) - 6\} = -\lambda\{\lambda^2 - 7\} = 0$$

$\underline{\lambda = 0\ ,\ \pm\sqrt{7}}$

$<$ 36ページ. 固有ベクトル $>$

問の解答

(1) $\lambda = 2$, 5

(ア) $\lambda = 2$ のとき

$(A-\lambda I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ 2x+2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\lambda = 2$ に対する固有ベクトルは $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(イ) $\lambda = 5$ のとき

$(A-\lambda I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2x+y \\ 2x-y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\lambda = 5$ に対する固有ベクトルは $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(2) $\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 8 = \lambda^2 - 4\lambda + 3 - 8 = \lambda^2 - 4\lambda - 5 = (\lambda-5)(\lambda+1) = 0$

(ア) $\lambda = 5$ のとき

$(A-\lambda I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4x+4y \\ 2x-2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

固有値 $\lambda = 5$ に対する固有ベクトルは $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(イ) $\lambda = -1$ のとき

$(A-\lambda I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+4y \\ 2x+4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

固有値 $\lambda = -1$ に対する固有ベクトルは $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

(3) $\begin{vmatrix} 5-\lambda & 3 \\ 2 & 4-\lambda \end{vmatrix} = (5-\lambda)(4-\lambda) - 6 = \lambda^2 - 9\lambda + 20 - 6 = \lambda^2 - 9\lambda + 14 = (\lambda-2)(\lambda-7) = 0$

(ア) $\lambda = 2$ のとき

$(A-\lambda I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+3y \\ 2x+2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

固有値 $\lambda = 2$ に対する固有ベクトルは $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(イ) $\lambda = 7$ のとき

$(A-\lambda I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2x+3y \\ 2x-3y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

固有値 $\lambda = 7$ に対する固有ベクトルは $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$<$ 37 ページ. 行列の対角化 $>$

問の解答

(1) $P = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$

$\det P = 2 + 1 = 3 \qquad P^{-1} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$

$$P^{-1}AP = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ -2 & 10 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 6 & 0 \\ 0 & 15 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$$

(2) $P = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$

$\det P = 1 + 2 = 3 \qquad P^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$

$$P^{-1}AP = \frac{1}{3}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 5 & -1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 15 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$$

(3) $P = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$

$\det P = 2 + 3 = 5 \qquad P^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$

$$P^{-1}AP = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 21 \\ -2 & 14 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 10 & 0 \\ 0 & 35 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}$$

＜ 38ページ.行列のn乗＞

問の解答

(1) $A^n = P\begin{pmatrix} 2^n & 0 \\ 0 & 5^n \end{pmatrix}P^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2^n & 0 \\ 0 & 5^n \end{pmatrix}\frac{1}{3}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 2^n & 5^n \\ -2^n & 2\times 5^n \end{pmatrix}\frac{1}{3}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2^{n+1}+5^n}{3} & \frac{-2^n+5^n}{3} \\ \frac{-2^{n+1}+2\times 5^n}{3} & \frac{2^n+2\times 5^n}{3} \end{pmatrix}$$

(2) $A^n = P\begin{pmatrix} 5^n & 0 \\ 0 & (-1)^n \end{pmatrix}P^{-1} = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5^n & 0 \\ 0 & (-1)^n \end{pmatrix}\frac{1}{3}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 5^n & -2\times(-1)^n \\ 5^n & (-1)^n \end{pmatrix}\frac{1}{3}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 5^n+2\times(-1)^n & 2\times 5^n-2\times(-1)^n \\ 5^n-(-1)^n & 2\times 5^n+(-1)^n \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5^n+2\times(-1)^n}{3} & \frac{2\times 5^n-2\times(-1)^n}{3} \\ \frac{5^n-(-1)^n}{3} & \frac{2\times 5^n+(-1)^n}{3} \end{pmatrix}$$

(3) $A^n = P\begin{pmatrix} 2^n & 0 \\ 0 & 7^n \end{pmatrix}P^{-1} = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2^n & 0 \\ 0 & 7^n \end{pmatrix}\frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 2^n & 3\times 7^n \\ -2^n & 2\times 7^n \end{pmatrix}\frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 2^{n+1}+3\times 7^n & -3\times 2^n+3\times 7^n \\ -2^{n+1}+2\times 7^n & 3\times 2^n+2\times 7^n \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2^{n+1}+3\times 7^n}{5} & \frac{-3\times 2^n+3\times 7^n}{5} \\ \frac{-2^{n+1}+2\times 7^n}{5} & \frac{3\times 2^n+2\times 7^n}{5} \end{pmatrix}$$

＜ 39 ページ. 平面の一次変換１ ＞

問の解答

$$\begin{cases} x' = -x \\ y' = y \end{cases} \Leftrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

＜ 40 ページ. 平面の一次変換 2 ＞

問の解答

(1) $\begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

(2) $\begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

(3) $\begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(4) $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$