＜ 1ページ. 速度の合成 ＞

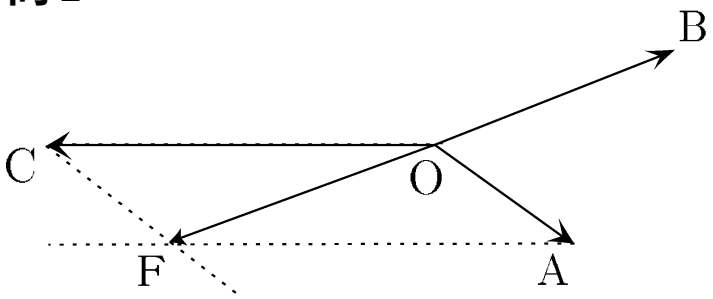
解答

$$\sin\theta = \frac{5}{13} \fallingdotseq 0.3846$$

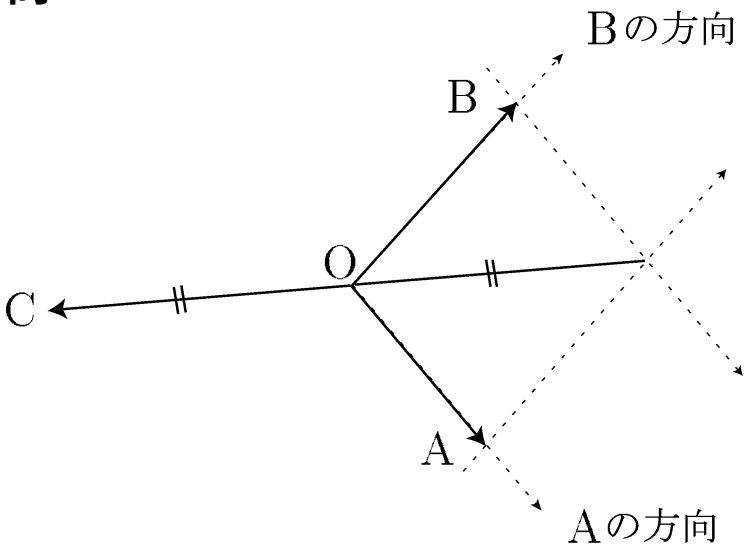
＜ 2ページ. 力の合成 ＞

解答

問1



問2



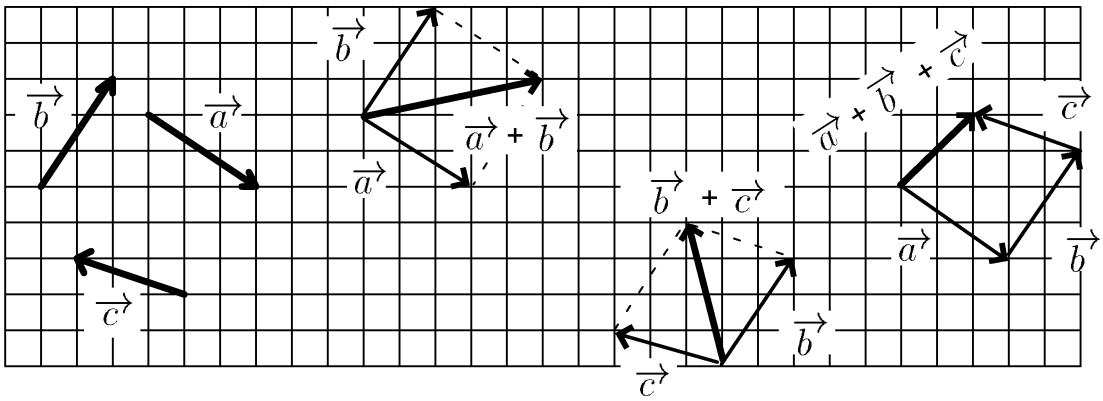
＜ 3ページ. 平面上のベクトル1 ＞

解答

$\overrightarrow{\mathrm{AF}} = \overrightarrow{\mathrm{BO}} = \overrightarrow{\mathrm{OE}} = \overrightarrow{\mathrm{CD}}$

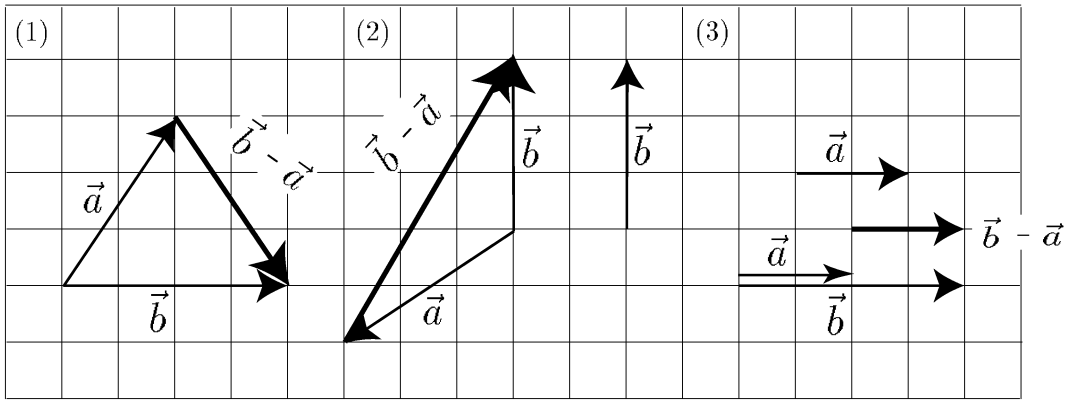
＜ 4ページ. 平面上のベクトル2 ＞

解答



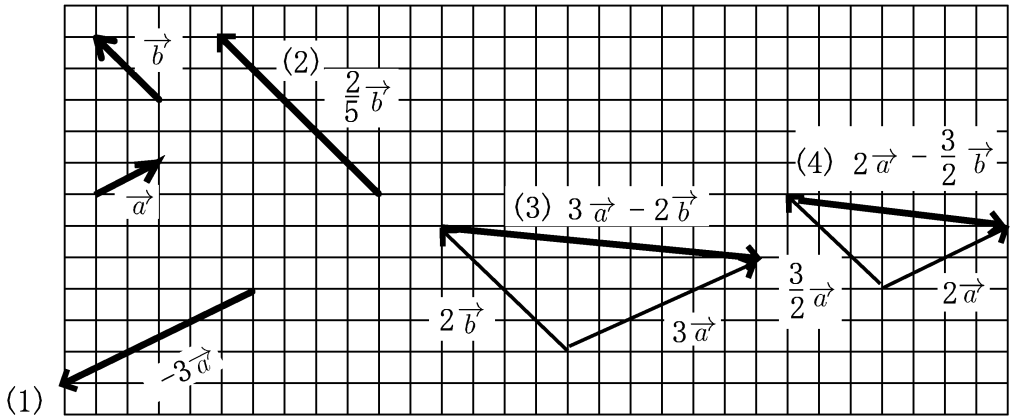
＜ 5ページ. 平面上のベクトル3 ＞

解答



＜ 6ページ. 平面上のベクトル４ ＞

解答



＜ 7ページ. 平面上のベクトル5 ＞

解答

問1　$\overrightarrow{\mathrm{OP}} = \overrightarrow{a} + \overrightarrow{\mathrm{AP}} = \overrightarrow{a} + \dfrac{m}{m+n}(\overrightarrow{b} - \overrightarrow{a})$

$$= \frac{n}{m+n}\overrightarrow{a} + \frac{m}{m+n}\overrightarrow{b}$$

問2　(1) $\overrightarrow{\mathrm{OM}} = \dfrac{\overrightarrow{a} + \overrightarrow{b}}{2}$

(2) $\overrightarrow{\mathrm{OG}} = \dfrac{2}{3}\overrightarrow{\mathrm{OM}} + \dfrac{1}{3}\overrightarrow{c}$

(3) $\overrightarrow{\mathrm{OG}} = \dfrac{3}{2}(\dfrac{\overrightarrow{a} + \overrightarrow{b}}{2}) + \dfrac{1}{3}\overrightarrow{c}$

$$= \frac{1}{3}\overrightarrow{a} + \frac{1}{3}\overrightarrow{b} + \frac{1}{3}\overrightarrow{c}$$

＜ 8ページ. ベクトルの成分1 ＞

解答

$$\overrightarrow{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\overrightarrow{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\overrightarrow{c} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

＜ 9ページ. ベクトルの成分2 ＞

解答

(1) $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$|\overrightarrow{\mathrm{AB}}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

(2) $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$|\overrightarrow{\mathrm{AB}}| = \sqrt{(-1)^2 + ^2} = \sqrt{5}$

＜ 10ページ. ベクトルの成分3 ＞

解答

問1 (1) $\overrightarrow{a}+\overrightarrow{b}=\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}+\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}=\begin{pmatrix} a_1+b_1 \\ a_2+b_2 \end{pmatrix}$

(2) $\overrightarrow{a}-\overrightarrow{b}=\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}-\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}=\begin{pmatrix} a_1-b_1 \\ a_2-b_2 \end{pmatrix}$

(3) $k\overrightarrow{a}=k\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}=\begin{pmatrix} ka_1 \\ ka_2 \end{pmatrix}$

問2 (1) $\frac{1}{2}\overrightarrow{a}=\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(2) $-\overrightarrow{b}=\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(3) $\overrightarrow{a}-\overrightarrow{b}=\begin{pmatrix} 2+1 \\ 6+3 \end{pmatrix}=\begin{pmatrix} 3 \\ 9 \end{pmatrix}$

(4) $\overrightarrow{a}+2\overrightarrow{b}=\begin{pmatrix} 2 \\ 6 \end{pmatrix}+\begin{pmatrix} -2 \\ -6 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

＜ 11ページ. 内分点 ＞

解答

問1　$\overrightarrow{\mathrm{OP}} = \dfrac{2}{3}\overrightarrow{\mathrm{OA}} + \dfrac{1}{3}\overrightarrow{\mathrm{OB}} = \begin{pmatrix} \dfrac{2a_1 + b_1}{3} \\ \\ \dfrac{2a_2 + b_2}{3} \end{pmatrix}$

(答)P の座標は $\left(\dfrac{2a_1 + b_1}{3} \ , \ \dfrac{2a_2 + b_2}{3} \right)$

問2　(答)P $\left(\dfrac{na_1 + mb_1}{m+n} \ , \ \dfrac{na_2 + mb_2}{m+n} \right)$

＜ 12ページ. ベクトルの内積1 ＞

解答

$\overrightarrow{a} \cdot \overrightarrow{b} = 2 \times 3 \times \cos 60^\circ = 3$

$\overrightarrow{c} \cdot \overrightarrow{d} = 4 \times 2 \times \cos 150^\circ = -4\sqrt{3}$

＜ 13ページ. 平面上のベクトル5 ＞

解答

(1) $\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{AC}} = 2 \times 2 \times \cos 60^\circ = 2$

(2) $\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{AM}} = 2 \times \sqrt{3} \times \cos 30^\circ = 3$

(3) $\overrightarrow{\mathrm{BC}} \cdot \overrightarrow{\mathrm{AM}} = 2 \times \sqrt{3} \times \cos 90^\circ = 0$

(4) $\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{BC}} = 2 \times 2 \times \cos 120^\circ = -2$

(5) $\overrightarrow{\mathrm{MB}} \cdot \overrightarrow{\mathrm{MC}} = 1 \times 1 \times \cos 180^\circ = -1$

＜ 14ページ. 内積の成分表示１ ＞

解答

問１

$$\frac{1}{2}\left\{\mathrm{OA}^2+\mathrm{OB}^2-\mathrm{AB}^2\right\}=\frac{1}{2}\left\{{a_1}^2+{a_2}^2+{b_1}^2+{b_2}^2-(b_1-a_1)^2-(b_2-a_2)^2\right\}$$

$$=\frac{1}{2}\left\{{a_1}^2+{a_2}^2+{b_1}^2+{b_2}^2-{b_1}^2+2b_1a_1-{a_1}^2-{b_2}^2+2b_2a_2-{a_2}^2\right\}$$

$$=\frac{1}{2}(2b_1a_1+2b_2a_2)=b_1a_1+b_2a_2=a_1b_1+a_2b_2$$

問２

$$\overrightarrow{a}\cdot\overrightarrow{b}=a_1b_1+a_2b_2$$

＜ 15ページ. 内積の成分表示2 ＞

解答

問1

(1) $\overrightarrow{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \overrightarrow{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \quad \overrightarrow{a} \cdot \overrightarrow{b} = 2 \times 4 + 3 \times 5 = 8 + 15 = 23$

(2) $\overrightarrow{a} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \quad \overrightarrow{b} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{a} \cdot \overrightarrow{b} = 4 \times (-3) + 6 \times 2 = 0 \Rightarrow \overrightarrow{a} \perp \overrightarrow{b}$

(3) $\overrightarrow{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \overrightarrow{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \overrightarrow{a} \cdot \overrightarrow{b} = 1 \times 0 + 0 \times 1 = 0 \Rightarrow \overrightarrow{a} \perp \overrightarrow{b}$

問2

$$\overrightarrow{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad , \quad \overrightarrow{c} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

＜ 16ページ. ベクトルのなす角 ＞

解答

問1

$$\cos\theta = \frac{\overrightarrow{a}\cdot\overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|} = \frac{a_1b_1+a_2b_2}{\sqrt{{a_1}^2+{a_2}^2}\times\sqrt{{b_1}^2+{b_2}^2}}$$

問2

(1) $\cos\theta = \dfrac{\overrightarrow{a}\cdot\overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|} = \dfrac{-\sqrt{3}}{1\times 2} = -\dfrac{\sqrt{3}}{2} \quad , \quad \theta = \dfrac{5}{6}\pi$

(2) $\cos\theta = \dfrac{-2\times 3+1\times 1}{\sqrt{(-2)^2+1^2}\sqrt{3^2+1^2}} = \dfrac{-5}{\sqrt{5}\sqrt{10}} = -\dfrac{1}{\sqrt{2}} \quad , \quad \theta = \dfrac{3}{4}\pi$

(3) $\cos\theta = \dfrac{\sqrt{3}\times\sqrt{3}+3\times 1}{\sqrt{(\sqrt{3})^2+3^2}\sqrt{(\sqrt{3})^2+1}} = \dfrac{6}{\sqrt{12}\sqrt{4}}$

$= \dfrac{6}{2\sqrt{3}\times 2} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2} \quad , \quad \theta = \dfrac{\pi}{6}$

＜ 17ページ. 平行四辺形の面積１ ＞

解答

問１

(1) $S^2 = |\overrightarrow{a}|^2 \times |\overrightarrow{b}|^2 - (\overrightarrow{a} \cdot \overrightarrow{b})^2$

(2) $S^2 = ({a_1}^2 + {a_2}^2)({b_1}^2 + {b_2}^2) - (a_1b_1 + a_2b_2)^2$

$= {a_1}^2{b_1}^2 + {a_1}^2{b_2}^2 + {a_2}^2{b_1}^2 + {a_2}^2{b_2}^2 - {a_1}^2{b_1}^2 - 2a_1b_1a_2b_2 - {a_2}^2{b_2}^2$

$= {a_1}^2{b_2}^2 - 2a_1b_1a_2b_2 + {a_2}^2{b_2}^2 = (a_1b_2 - a_2b_1)^2$

$S = |a_1b_2 - a_2b_1|$

＜ 18ページ. 平行四辺形の面積2 ＞

解答

(1) $S = |2 \times 3 - (-1) \times 1| = 7$

(2) $S = |1 \times 2 - 4 \times 5| = |-18| = 18$

＜ 19ページ.2 次の行列式 ＞

解答

問1

(1) $\begin{vmatrix} 2 & 5 \\ 1 & 8 \end{vmatrix} = 16 - 5 = 11$

(2) $\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix} = -4$

(3) $\begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = 12 - 12 = 0$

問2

(1) 平行

$\overrightarrow{b} = 3\overrightarrow{a}$

(2) 平行でない

(3) 平行

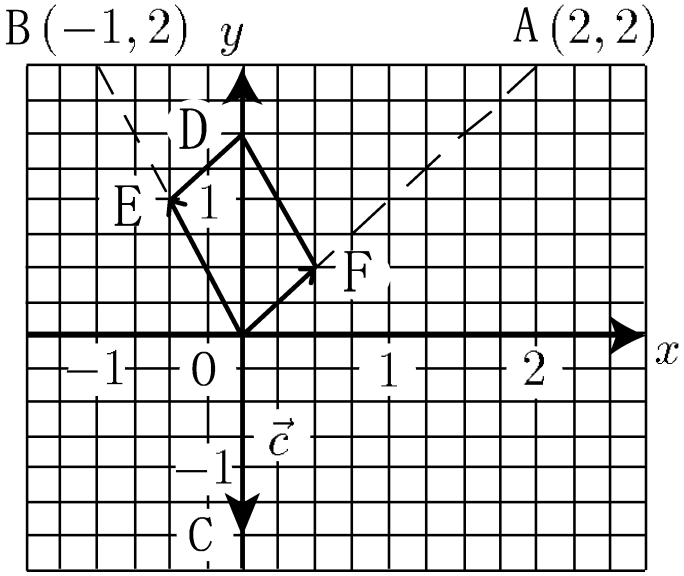
$\overrightarrow{b} = -2\overrightarrow{a}$

＜ 20 ページ. ベクトルの均衡 ＞

解答

(1) $D(0, 1.5)$

(2)



(3) $\overrightarrow{\mathrm{OF}} = \dfrac{1}{4}\overrightarrow{\mathrm{OA}}$

$\overrightarrow{\mathrm{OE}} = \dfrac{1}{2}\overrightarrow{\mathrm{OB}}$

＜ 21 ページ. 空間のベクトル 1 ＞

解答

問 1

(1) $\overrightarrow{\mathrm{OB}} = \overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{GF}} = \overrightarrow{\mathrm{CE}}$

(2) $\overrightarrow{\mathrm{OC}} = \overrightarrow{\mathrm{BE}} = \overrightarrow{\mathrm{DF}} = \overrightarrow{\mathrm{AG}}$

問 2

(1) $\overrightarrow{\mathrm{OC}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AG}} = \overrightarrow{a} + \overrightarrow{c}$

(2) $\overrightarrow{\mathrm{OE}} = \overrightarrow{\mathrm{OC}} + \overrightarrow{\mathrm{CE}} = \overrightarrow{c} + \overrightarrow{b}$

(3) $\overrightarrow{\mathrm{OF}} = \overrightarrow{\mathrm{OG}} + \overrightarrow{\mathrm{GF}} = \overrightarrow{a} + \overrightarrow{c} + \overrightarrow{b}$

(4) $\overrightarrow{\mathrm{DG}} = \overrightarrow{\mathrm{DA}} + \overrightarrow{\mathrm{AG}} = -\overrightarrow{\mathrm{AD}} + \overrightarrow{\mathrm{AG}} = -\overrightarrow{b} + \overrightarrow{c}$

(5) $\overrightarrow{\mathrm{FB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OF}} = \overrightarrow{b} - (\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}) = -\overrightarrow{a} - \overrightarrow{c}$

(6) $\overrightarrow{\mathrm{CD}} = \overrightarrow{\mathrm{OD}} - \overrightarrow{\mathrm{OC}} = \overrightarrow{a} + \overrightarrow{b} - \overrightarrow{c}$

＜ 22ページ. 空間のベクトル2 ＞

解答

問1

$$\overrightarrow{\mathrm{OA}_1} = \begin{pmatrix} a_1 \\ 0 \\ 0 \end{pmatrix} \quad , \quad \overrightarrow{\mathrm{OA}_2} = \begin{pmatrix} 0 \\ a_2 \\ 0 \end{pmatrix} \quad , \quad \overrightarrow{\mathrm{OA}_3} = \begin{pmatrix} 0 \\ 0 \\ a_3 \end{pmatrix}$$

問2

(1) $|\overrightarrow{a}| = \sqrt{4^2 + 4^2 + 2^2} = \sqrt{16 + 16 + 4} = \sqrt{36} = 6$

(2) $|\overrightarrow{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

＜ 23ページ. 空間のベクトル3 ＞

解答

(1) $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

(2) $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}$

＜ 24ページ. 空間のベクトル4 ＞

解答

(1) $\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{OB}} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix}$

$\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

$2\overrightarrow{\mathrm{OB}} = 2\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}$

(2) $\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{OB}} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$

$\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}$

$3\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 3a_1 \\ 3a_2 \\ 3a_3 \end{pmatrix}$

＜ 25ページ. 空間のベクトル5 ＞

解答

問1

(1) $\overrightarrow{a} + \overrightarrow{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$

(2) $\overrightarrow{a} - \overrightarrow{b} = \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{pmatrix}$

(3) $k\overrightarrow{a} = \begin{pmatrix} ka_1 \\ ka_2 \\ ka_3 \end{pmatrix}$

問2

(1) $\overrightarrow{a} + \overrightarrow{b} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix}$

$|\overrightarrow{a} + \overrightarrow{b}| = \sqrt{4^2 + 0^2 + 4^2} = 4\sqrt{2}$

(2) $\overrightarrow{a} - \overrightarrow{b} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

$|\overrightarrow{a} - \overrightarrow{b}| = \sqrt{2^2 + 2^2 + 0^2} = 2\sqrt{2}$

(3) $3\overrightarrow{a} = 3\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 6 \end{pmatrix}$

$3|\overrightarrow{a}| = \sqrt{9^2 + 3^2 + 6^2} = \sqrt{81 + 9 + 36} = \sqrt{126} = 3\sqrt{14}$

(4) $\overrightarrow{a} - 3\overrightarrow{b} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ -4 \end{pmatrix}$

$|\overrightarrow{a} - 3\overrightarrow{b}| = \sqrt{0^2 + 4^2 + (-4)^2} = 4\sqrt{2}$

＜ 26 ページ. 空間座標と距離 ＞

解答

問 1

(1) $\mathrm{AB} = \sqrt{(-2)^2 + (4)^2 + 0^2} = \sqrt{20} = 2\sqrt{5}$

(2) $\mathrm{AB} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}$

問 2

(1) 中心 (　1 ,　2 ,　3) ,　半径 $= 5$

(2) 中心 (　2 ,　-1 ,　0) ,　半径 $= \sqrt{3}$

(3) 中心 (　0 ,　0 ,　0) ,　半径 $= 4$

＜ 27ページ. ベクトルの内積1 ＞

解答

(1) $\overrightarrow{\mathrm{AD}} \cdot \overrightarrow{\mathrm{AF}} = 0$

(2) $\overrightarrow{\mathrm{AD}} \cdot \overrightarrow{\mathrm{AB}} = 1 \times \sqrt{2} \times \cos 45^\circ = 1$

(3) $\overrightarrow{\mathrm{FE}} \cdot \overrightarrow{\mathrm{FD}} = \sqrt{2} \times \sqrt{2} \times \dfrac{1}{2} = 1$

(4) $\overrightarrow{\mathrm{AD}} \cdot \overrightarrow{\mathrm{OC}} = 0$

(5) $\overrightarrow{\mathrm{AD}} \cdot \overrightarrow{\mathrm{CE}} = 1$

(6) $\overrightarrow{\mathrm{AD}} \cdot \overrightarrow{\mathrm{GF}} = -1$

＜ 28ページ. ベクトルの内積2 ＞

解答

問1

$\mathrm{OA}^2 = {a_1}^2 + {a_2}^2 + {a_3}^2$

$\mathrm{OB}^2 = {b_1}^2 + {b_2}^2 + {b_3}^2$

$\mathrm{AB}^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2$

問2

$\dfrac{1}{2}\left\{\mathrm{OA}^2 + \mathrm{OB}^2 - \mathrm{AB}^2\right\}$

$= \dfrac{1}{2}\left\{{a_1}^2 + {a_2}^2 + {a_3}^2 + {b_1}^2 + {b_2}^2 + {b_3}^2 - (b_1 - a_1)^2 - (b_2 - a_2)^2 - (b_3 - a_3)^2\right\}$

$= \dfrac{1}{2}\left\{2b_1a_1 + 2b_2a_2 + 2b_3a_3\right\} = b_1a_1 + b_2a_2 + b_3a_3 = a_1b_1 + a_2b_2 + a_3b_3$

問2

$\overrightarrow{a} \cdot \overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3$

＜ 29ページ. ベクトルの内積3 ＞

解答

(1) $\cos\theta = \dfrac{2\times 3+0\times 0+1\times -1}{\sqrt{2^2+0^2+1^2}\sqrt{3^2+0^2+(-1)^2}} = \dfrac{5}{\sqrt{5}\sqrt{10}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\theta = 45^\circ = \dfrac{\pi}{4}$

(2) $\cos\theta = \dfrac{5\times 3+1\times 2+4\times 1}{\sqrt{5^2+1^2+4^2}\sqrt{3^2+2^2+1^2}}$

$= \dfrac{21}{\sqrt{42}\sqrt{14}} = \dfrac{21}{14\sqrt{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$\theta = 30^\circ = \dfrac{\pi}{6}$

(3) $\cos\theta = \dfrac{-1-1-1}{\sqrt{1+1+1}\sqrt{1+1+1}} = \dfrac{-3}{3} = -1$

$\theta = 180^\circ = \pi$

＜ 30ページ. 平面の方程式1 ＞

解答

(1) $\overrightarrow{n} \cdot \overrightarrow{\mathrm{QP}} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y+5 \\ z-3 \end{pmatrix} = 0$

$(x-1)+2(y+5)+3(z-3)=0$

$x+2y+3z-1+10-9=0$

$\underline{x+2y+3z=0}$

(2) $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x-q_1 \\ y-q_2 \\ z-q_3 \end{pmatrix} = 0$

$\underline{a(x-q_1)+b(y-q_2)+c(z-q_3)=0}$

＜ 31ページ. 平面の方程式2 ＞

解答

(1) 原点 $(0,0,0)$ を通り

$\overrightarrow{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ に垂直な平面

(2) $3(x-1)+2(y-1)-z=0$

点 $(1,1,0)$ を通り

$\overrightarrow{n} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ に垂直な平面

(3) $\dfrac{5}{11}x+\dfrac{7}{11}y+z=\dfrac{23}{11}$

$\dfrac{5}{11}(x-1)+\dfrac{7}{11}(y-1)+z=\dfrac{22}{11}-\dfrac{12}{11}=\dfrac{11}{11}=1$

$\dfrac{5}{11}(x-1)+\dfrac{7}{11}(y-1)+z-1=0$

点 $(1,1,1)$ を通り

$\overrightarrow{n} = \begin{pmatrix} \frac{5}{11} \\ \frac{7}{11} \\ 1 \end{pmatrix}$ に垂直な平面

＜ 32ページ. 空間の平行四辺形１ ＞

解答

(1) $S^2 = \left|\overrightarrow{a}\right|^2 \times \left|\overrightarrow{b}\right|^2 - (\overrightarrow{a} \cdot \overrightarrow{b})^2$

$= (1^2 + 3^2 + 5^2) \times (3^2 + 4^2 + 5^2) - (1 \times 3 + 3 \times 4 + 5 \times 5)^2$

$= 35 \times 50 - 40^2 = 150$

$S = \sqrt{150} = 5\sqrt{6}$

(2) $S^2 = \left|\overrightarrow{a}\right|^2 \times \left|\overrightarrow{b}\right|^2 - (\overrightarrow{a} \cdot \overrightarrow{b})^2$

$= ({a_1}^2 + {a_2}^2 + {a_3}^2) \times (1^2 + 0^2 + (-1)^2) - (a_1 - a_2)^2$

$= 2({a_1}^2 + {a_2}^2 + {a_3}^2) - ({a_1}^2 - 2a_1a_3 + {a_3}^2)$

$= {a_1}^2 + 2{a_2}^2 + {a_3}^2 + 2a_1a_3$

$S = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2 + 2a_1a_3}$

＜ 33 ページ. 空間の平行四辺形 2 ＞

解答

問 1

$$S^2 = \{a_1 b_2 - a_2 b_1\}^2 + \{a_2 b_3 - a_3 b_2\}^2 + \{a_3 b_1 - a_1 b_3\}^2$$

問 2

$$S^2 = \left\{ \left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right|^2 + \left| \begin{array}{cc} a_2 & b_2 \\ a_3 & b_3 \end{array} \right|^2 + \left| \begin{array}{cc} a_3 & b_3 \\ a_1 & b_1 \end{array} \right|^2 \right\}$$

問 3

$$S = \sqrt{\left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right|^2 + \left| \begin{array}{cc} a_2 & b_2 \\ a_3 & b_3 \end{array} \right|^2 + \left| \begin{array}{cc} a_3 & b_3 \\ a_1 & b_1 \end{array} \right|^2}$$

＜ 34ページ. 外積1 ＞

解答

問1

(1)

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{pmatrix} \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} \\ \begin{vmatrix} 3 & 5 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\overrightarrow{a} \cdot \overrightarrow{b} = 1 \times 1 + 2 \times 3 + 3 \times 5 = 22$$

(2)

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{pmatrix} \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}$$

$$\overrightarrow{a} \cdot \overrightarrow{b} = 3 \times 1 + 2 \times 0 + 1 \times (-1) = 2$$

＜ 35ページ. 外積2 ＞

解答

問1

$$(1)\ \left(\overrightarrow{a}\times\overrightarrow{b}\right)\cdot\overrightarrow{a}=\begin{pmatrix}1\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\3\end{pmatrix}=1-4+3=0$$

$$\left(\overrightarrow{a}\times\overrightarrow{b}\right)\cdot\overrightarrow{b}=\begin{pmatrix}1\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}1\\3\\5\end{pmatrix}=1-6+5=0$$

$$(2)\ \left(\overrightarrow{a}\times\overrightarrow{b}\right)\cdot\overrightarrow{a}=\begin{pmatrix}-2\\4\\-2\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}=-6+8-2=0$$

$$\left(\overrightarrow{a}\times\overrightarrow{b}\right)\cdot\overrightarrow{b}=\begin{pmatrix}-2\\4\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-1\end{pmatrix}=-2+0+2=0$$

問2

$$\left(\overrightarrow{a}\times\overrightarrow{b}\right)\cdot\overrightarrow{b}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}\cdot\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

$$=(a_2b_3-a_3b_2)b_1+(a_3b_1-a_1b_3)b_2+(a_1b_2-a_2b_1)b_3$$

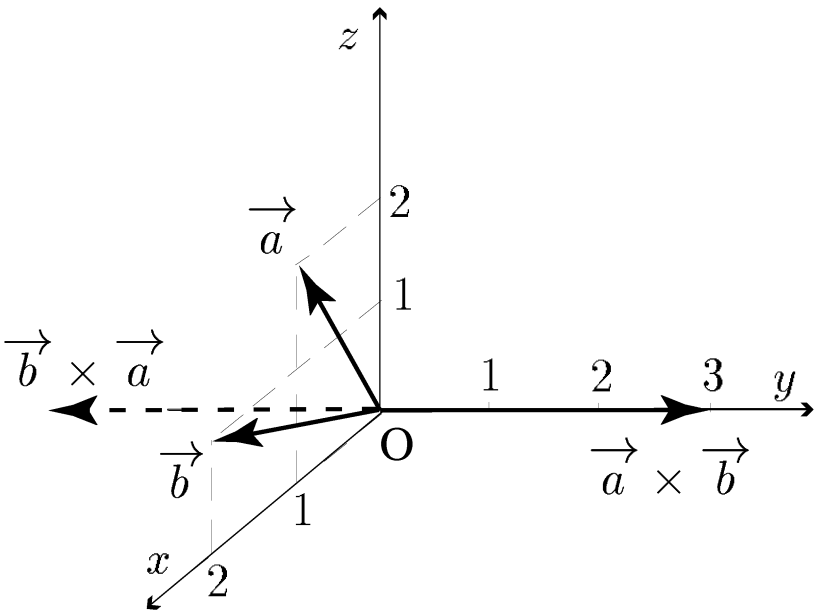
$$=a_2b_1b_3+a_3b_1b_2+a_1b_2b_3-a_3b_1b_2-a_1b_2b_3-a_2b_1b_3=0$$

＜ 36 ページ. 外積 3 ＞

解答

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{pmatrix} \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$$

$$\overrightarrow{b} \times \overrightarrow{a} = \begin{pmatrix} \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 0 & 0 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}$$



＜ 37ページ. 外積4 ＞

解答

$$(1)\ (3\overrightarrow{a}) \times \overrightarrow{b} = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 6 & 0 \\ 9 & 4 \end{vmatrix} \\ \begin{vmatrix} 9 & 4 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 6 & 0 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 24 \\ -21 \\ 6 \end{pmatrix}$$

$$(2)\ (-\overrightarrow{a}) \times \overrightarrow{b} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} -2 & 0 \\ -3 & 4 \end{vmatrix} \\ \begin{vmatrix} -3 & 4 \\ -1 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & -1 \\ -2 & 0 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \\ -2 \end{pmatrix}$$

$$(3)\ (k\overrightarrow{a}) \times \overrightarrow{b} = \begin{pmatrix} 8k \\ -7k \\ 2k \end{pmatrix}$$

$$(4)\ \overrightarrow{a} \times (k\overrightarrow{b}) = k \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \overrightarrow{0}$$

＜ 38ページ. 外積の計算1 ＞

解答

$$\overrightarrow{a} \times \overrightarrow{b} - \overrightarrow{a} \times \overrightarrow{c} = \begin{pmatrix} -2 \\ -11 \\ 8 \end{pmatrix} - \begin{pmatrix} -4 \\ 8 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ -19 \\ 12 \end{pmatrix}$$

$$\overrightarrow{a} \times \left(\overrightarrow{b} - \overrightarrow{c}\right) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} -9 \\ -6 \\ -8 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 2 & -6 \\ 3 & -8 \end{vmatrix} \\ \begin{vmatrix} 3 & -8 \\ 1 & -9 \end{vmatrix} \\ \begin{vmatrix} 1 & -9 \\ 2 & -6 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -19 \\ 12 \end{pmatrix}$$

＜ 39ページ. 外積の計算2 ＞

解答

$$-(\overrightarrow{a}\times\overrightarrow{c})-2\overrightarrow{c}\times\overrightarrow{b}=\overrightarrow{c}\times\overrightarrow{a}-\overrightarrow{c}\times 2\overrightarrow{b}=\overrightarrow{c}\times(\overrightarrow{a}-2\overrightarrow{b})$$

$$=\begin{pmatrix}1\\-2\\-3\end{pmatrix}\times\left\{\begin{pmatrix}1\\-1\\0\end{pmatrix}-2\begin{pmatrix}3\\4\\5\end{pmatrix}\right\}=\begin{pmatrix}1\\-2\\-3\end{pmatrix}\times\begin{pmatrix}1-6\\-1-8\\0-10\end{pmatrix}$$

$$=\begin{pmatrix}1\\-2\\-3\end{pmatrix}\times\begin{pmatrix}-5\\-9\\-10\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}-2&-9\\-3&-10\end{vmatrix}\\\\\begin{vmatrix}-3&-10\\1&-5\end{vmatrix}\\\\\begin{vmatrix}1&-5\\-2&-9\end{vmatrix}\end{pmatrix}=\begin{pmatrix}-7\\25\\-19\end{pmatrix}$$

＜ 40ページ. 直行系 ＞

解答

(1) $\overrightarrow{a} \cdot \overrightarrow{b} = 6 \times 1 + 0 \times 2 + (-3) \times 3 = 6 - 6 = 0$

(2) $\overrightarrow{c} = \overrightarrow{a} \times \overrightarrow{b} = \begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 0 & 2 \\ -2 & 3 \end{vmatrix} \\ \begin{vmatrix} -2 & 3 \\ 6 & 1 \end{vmatrix} \\ \begin{vmatrix} 6 & 1 \\ 0 & 2 \end{vmatrix} \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -20 \\ 12 \end{pmatrix}$

(3) $\overrightarrow{c} \cdot \overrightarrow{a} = 4 \times 6 - 20 \times 0 + 12(-2) = 24 - 24 = 0$

$\overrightarrow{c} \cdot \overrightarrow{b} = 4 \times 1 - 20 \times 2 + 12 \times 3 = 4 - 40 + 36 = 0$