$<$1 ページ　有理数 $>$

解答　(1) $\dfrac{1}{8} = 0.125$

(2) $\dfrac{7}{9} = 0.7777\cdots = 0.\dot{7}$

(3) $\dfrac{31}{60} = 0.516666\cdots = 0.51\dot{6}$

<2 ページ　実数 >

解答　(1) 無理数

(2) 無理数

(3) 有理数

(4) $\tan\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{3}$

無理数

(5) 有理数

<3 ページ　虚数の導入 1>

解答　(1) $x = \pm 2i$

(2) $x^2 = -\dfrac{4}{9}$

$x = \pm\dfrac{2}{3}i$

(3) $x = \pm\dfrac{\sqrt{2}}{2}i$

<4 ページ　虚数の導入 2 >

解答　(1) $x = \dfrac{3}{4} \pm \sqrt{3}i$

(2) $(x-1)^2 = 2$

$x = 1 \pm \sqrt{2}i$

(3) $x = a \pm \dfrac{c}{b}i$

$<$5 ページ　複素数の定義 $>$

解答　(1) $a = \dfrac{2}{5}$,　$b = \dfrac{3}{5}$

(2) $a = \dfrac{\sqrt{3}+2}{3}$,　$b = 0$

<6ページ　複素数の四則演算1>

解答　問1

(1) $(1+i)+(1-3i)$
$= 2-2i$

(2) $(3-2i)+(2+2i)$
$= 5$

(3) $\left(0.5+\frac{1}{2}i\right)+\left(\frac{1}{4}+0.5i\right)$
$= \frac{3}{4}+i$

(4) $\left(\frac{1}{3}-\frac{1}{5}i\right)-\left(\frac{1}{3}-\frac{1}{5}i\right)$
$= 0$

(5) $\left(\sqrt{2}-i\right)-\left(\sqrt{3}+2i\right)$
$= \sqrt{2}-\sqrt{3}-3i$

(6) $\left(\frac{1}{4}+\sqrt{2}i\right)-\left(\frac{1}{5}-\sqrt{3}i\right)$
$= \frac{1}{20}+(\sqrt{2}+\sqrt{3})i$

問2

(1) $2(3+2i)$
$= 6+4i$

(2) $\sqrt{3}\left(\frac{1}{3}-\frac{1}{2}i\right)$
$= \frac{\sqrt{3}}{3}-\frac{\sqrt{3}}{2}i$

(3) $4(1-3i)+2(2+2i)$
$= 4-12i+4+4i$
$= 8-8i$

(4) $\sqrt{2}\left(\sqrt{2}-\frac{1}{\sqrt{2}}i\right)-\left(2+\frac{1}{2}i\right)$
$= 2-i-2-\frac{1}{2}i$
$= -\frac{3}{2}i$

<7 ページ　複素数の四則演算 2>

解答　(1) $i^3 = -i$　　(2) $i^4 = +1$　　(3) $i^5 = i$

(4) $i^6 = -1$　　(5) $i^7 = -i$　　(6) $i^8 = 1$

(7) $(1+i)(1-i) = 1+1 = 2$

(8) $(\sqrt{2}+\sqrt{3}i)(\sqrt{2}-\sqrt{3}i) = 2+3 = 5$

(9)
$$\left(\frac{1-\sqrt{3}i}{3}\right)\left(\frac{1+\sqrt{3}i}{3}\right) = \frac{1+3}{9} = \frac{4}{9}$$

(10) $(2+i)^2 = 4+4i-1 = 3+4i$

(11) $(1-i)^2 = 1-2i-1 = -2i$

(12)
$$\begin{aligned}(1+2i)(3-3i) &= 3-3i+6i+6 \\ &= 9+3i\end{aligned}$$

(13)
$$\begin{aligned}(4-3i)(3-i) &= 12-4i-9i-3 \\ &= 9-13i\end{aligned}$$

(14)
$$\begin{aligned}(3+i)^3 &= 27+27i+9i^2+i^3 \\ &= 27+27i-9-i \\ &= 18+26i\end{aligned}$$

<8ページ　複素数の四則演算3>

解答

(1) $\dfrac{1}{1+i} = \dfrac{1-i}{(1+i)(1-i)}$
$= \dfrac{1-i}{2}$

(2) $\dfrac{1}{1-i} = \dfrac{1+i}{1^2-i^2}$
$= \dfrac{1+i}{2}$

(3) $\dfrac{i}{1-i} = \dfrac{i(1+i)}{(1-i)(1+i)}$
$= \dfrac{i-1}{2}$

(4) $\dfrac{2}{1-\sqrt{3}i} = \dfrac{2(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$
$= \dfrac{2(1+\sqrt{3}i)}{1+3} = \dfrac{1+\sqrt{3}i}{2}$

(5) $\dfrac{1}{2+i} = \dfrac{2-i}{(2+i)(2-i)}$
$= \dfrac{2-i}{5}$

(6) $\dfrac{3}{3+3i} = \dfrac{1}{1+i}$
$= \dfrac{1-i}{2}$

(7) $\dfrac{1}{i(i+1)} = \dfrac{i}{-(i+1)}$
$= -\dfrac{i(i-1)}{(i+1)(i-1)}$
$= -\dfrac{-1-i}{-1-1}$
$= -\dfrac{1+i}{2}$

(8) $\dfrac{\sqrt{3}}{\sqrt{2}-i} = \dfrac{\sqrt{3}(\sqrt{2}+i)}{(\sqrt{2}-i)(\sqrt{2}+i)}$
$= \dfrac{\sqrt{6}+\sqrt{3}i}{2+1}$
$= \dfrac{\sqrt{6}+\sqrt{3}i}{3}$

(9) $\dfrac{1}{(1+i)^2} = \dfrac{1}{1+2i-1}$
$= \dfrac{1}{2i} = \dfrac{i}{2i^2} = -\dfrac{i}{2}$

(10) $\dfrac{i}{(1-i)^2} = \dfrac{i}{1-2i-1}$
$= \dfrac{i}{-2i} = -\dfrac{1}{2}$

＜9ページ　負の数の平方根＞

解答　(1) $=\sqrt{-24}=\sqrt{24}i=2\sqrt{6}i$

(2) $=\sqrt{2}i\times\sqrt{4}i\times\sqrt{3}i=2\sqrt{6}i^3=-2\sqrt{6}i$

(3) $\dfrac{\sqrt{5}}{\sqrt{2}i}=\dfrac{\sqrt{5}\times\sqrt{2}i}{2i^2}=-\dfrac{\sqrt{10}}{2}i$

(4) $\sqrt{\dfrac{5}{2}}i=\dfrac{\sqrt{10}}{2}i$

<10ページ　2次方程式>

解答　(1) $x=-1$

(2) $x=\dfrac{-1\pm\sqrt{1-4}}{2}=\dfrac{-1\pm\sqrt{3}i}{2}$

(3) $x=\dfrac{5\pm\sqrt{25-48}}{4}=\dfrac{5\pm\sqrt{23}i}{4}$

<11 ページ　因数分解 1>

解答　(1) $3x^2+12x+9=3(x^2+4x+3)=3(x+3)(x+1)$

(2) $2x^2+3x+1=(2x+1)(x+1)$

(3) $4x^2+4x+4=4(x^2+x+1)$

$$=4\left(x-\frac{-1+\sqrt{3}i}{2}\right)\left(x-\frac{-1-\sqrt{3}i}{2}\right)$$

$$=(2x+1-\sqrt{3}i)(2x+1+\sqrt{3}i)$$

<12 ページ　因数分解 2>

解答　(1) $x^3+8=(x+2)(x^2-2x+4)$

$$=(x+2)(x-1+\sqrt{3}i)(x-1-\sqrt{3}i)$$

(2) $x^3-64=(x-4)(x^2+4x+16)$

$$=(x-4)(x+2+2\sqrt{3}i)(x+2-2\sqrt{3}i)$$

(3) $x^4-81=(x^2-9)(x^2+9)$

$$=(x-3)(x+3)(x-3i)(x+3i)$$

<13 ページ　因数分解 3>

解答　(1) $x=-2,\ 1-\sqrt{3}i,\ 1+\sqrt{3}i$

(2) $x=4,\ -2+2\sqrt{3}i,\ -2-2\sqrt{3}i$

(3) $x=\pm 3,\ \pm 3i$

<14 ページ　共役複素数 >

解答

問1　(1) $\overline{z} = 1$　(2) $\overline{z} = -i$

(3) $\overline{z} = 2 + 5i$　(4) $\overline{z} = \dfrac{1+i}{2}$

問2　(1) $\dfrac{1}{2}(z + \overline{z})$

$= \dfrac{1}{2}(2 + 3i + 2 - 3i) = 2$

(2) $\dfrac{1}{2i}(z - \overline{z})$

$= \dfrac{1}{2i}(2 + 3i - (2 - 3i)) = 3$

(3) $z\overline{z}$

$= (2 + 3i)(2 - 3i) = 2^2 + 3^2 = 13$

問3　(1) $\dfrac{1}{2}(z + \overline{z}) = a$

(2) $\dfrac{1}{2i}(z - \overline{z}) = b$

(3) $z\overline{z} = a^2 + b^2$

<15ページ　絶対値>

解答

問1　(1) $|z| = 2$

(2) $|z| = 3$

(3) $|z| = \sqrt{\dfrac{1+3}{4}} = 1$

問2

(1) $z = 1 + i$

$|z|^2 = 1 + 1 = 2$

$z^2 = (1+i)^2$

$= 1 + 2i + (-1) = 2i$

$|z^2| = |2i| = 2$

(2) $z = 2 - 3i$

$|z|^2 = 2^2 + 3^2 = 13$

$z^2 = (2-3i)^2 = 4 - 12i + (-9)$

$= -5 - 12i$

$|z^2| = |-5 - 12i| = \sqrt{5^2 + 12^2}$

$= \sqrt{169} = 13$

<16ページ　複素平面1>

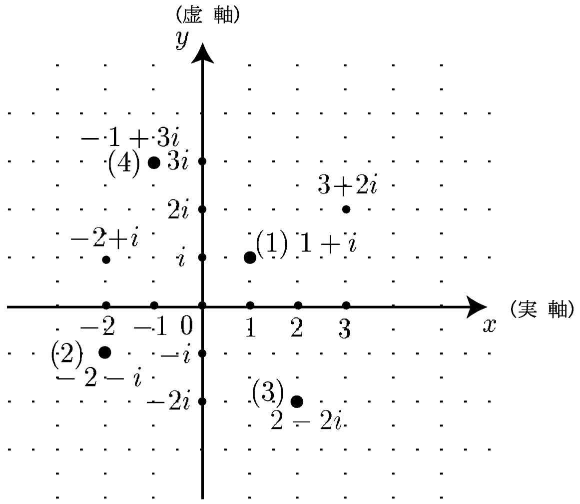
解答

問1　(1) $1+i$

(2) $-2-i$

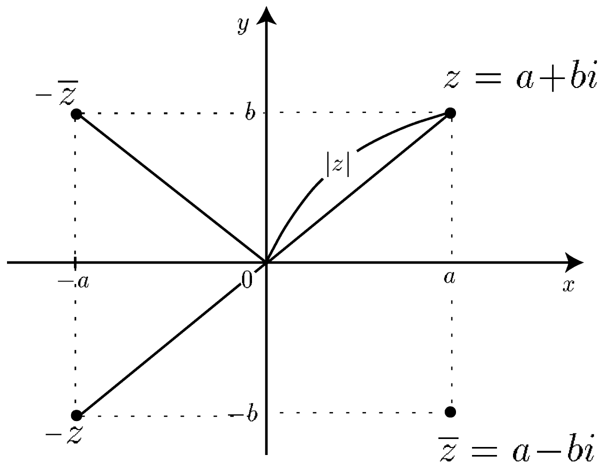
(3) $2-2i$

(4) $-1+3i$



問2　$-z=-a-bi$

$$-\overline{z}=-(a-bi)$$
$$=-a+bi$$



<17ページ　複素平面2>

解答

問　① $z_1 + z_2 = (3+i) + (1+3i)$

$$= 4 + 4i$$

② $z_1 - z_2 = (3+i) - (1+3i)$

$$= 2 - 2i$$

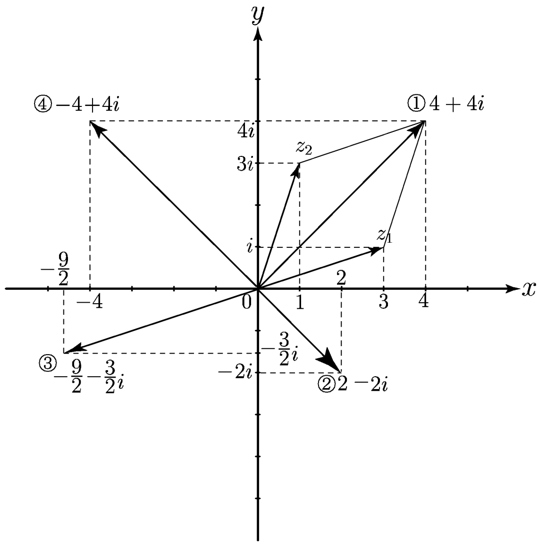
③ $-\dfrac{3}{2}z_1 = -\dfrac{3}{2}(3+i)$

$$= -\frac{9}{2} - \frac{3}{2}i$$

④ $2z_2 - 2z_1 = 2(1+3i) - 2(3+i)$

$$= 2 + 6i - (6+2i)$$

$$= -4 + 4i$$

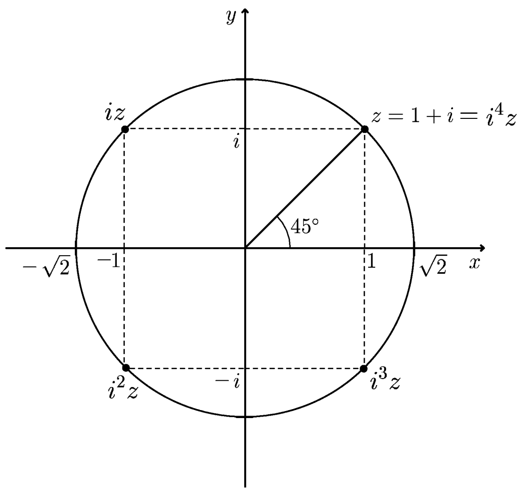


<18ページ　複素のi倍>

解答

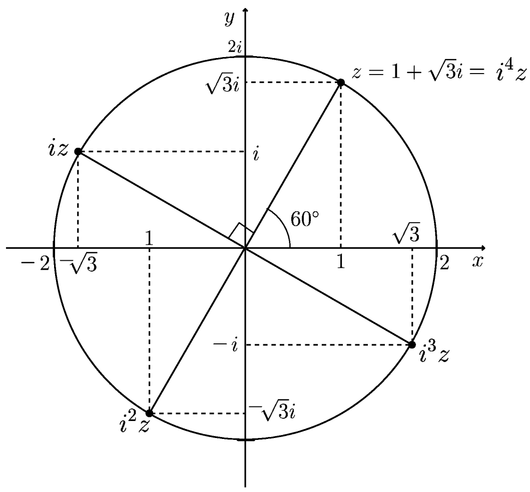
問　(1) $z = 1 + i$

$$iz = i(1+i) = i - 1$$
$$i^2 z = i(i-1) = -1 - i$$
$$i^3 z = i(-i-1) = -i + 1$$
$$i^4 z = i(-i+1) = 1 + i$$



(2) $z = 1 + \sqrt{3}i$

$$iz = i(1+\sqrt{3}i) = i - \sqrt{3}$$
$$i^2 z = i(i-\sqrt{3}) = -1 - \sqrt{3}i$$
$$i^3 z = i(-1-\sqrt{3}i) = -i + \sqrt{3}$$
$$i^4 z = i(-i+\sqrt{3}) = 1 + \sqrt{3}i$$



<19 ページ　絶対値 1 の複素数>

解答

問　(1) $\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$,　(2) $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$,　(3) $-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

(4) $-\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$,　(5) $-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$,　(6) -1

(7) $-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$,　(8) $-\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$,　(9) $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

(10) $-i$,　(11) $\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$,　(12) $\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i$

<20 ページ　極形式 1>

解答

問　(1) $i = \cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right)$

(2) $-2 = 2\left(\cos\pi + i\sin\pi\right)$

(3) $-\sqrt{2}i = \sqrt{2}\left(\cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right)\right)$

<21 ページ　極形式 2>

解答

問

(1) $z = 2\sqrt{3} - 2i$

$$= 4\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 4\left(\cos\left(\frac{11}{6}\pi\right) + i\sin\left(\frac{11}{6}\pi\right)\right)$$

$$= 4\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$$

(2) $z = 1 + i$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$$

(3) $z = -1 + i$

$$= \sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\cos\left(\frac{3}{4}\pi\right) + i\sin\left(\frac{3}{4}\pi\right)\right)$$

(4) $z = 4 - 4\sqrt{3}i$

$$= 8\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 8\left(\cos\left(\frac{5}{3}\pi\right) + i\sin\left(\frac{5}{3}\pi\right)\right)$$

$$= 8\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

(5) $z = -2 - 2\sqrt{3}i$

$$= 4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 4\left(\cos\left(\frac{4}{3}\pi\right) + i\sin\left(\frac{4}{3}\pi\right)\right)$$

$$= 4\left(\cos\left(-\frac{2}{3}\pi\right) + i\sin\left(-\frac{2}{3}\pi\right)\right)$$

<22 ページ　複素数の積>

解答

問

(1) $\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right)z=r\left(\cos\left(\theta+\dfrac{11}{6}\pi\right)+i\sin\left(\theta+\dfrac{11}{6}\pi\right)\right)$

$$\frac{11}{6}\pi\text{回転}$$

または

$$r\left(\cos\left(\theta-\frac{1}{6}\pi\right)+i\sin\left(\theta-\frac{1}{6}\pi\right)\right)$$

$$-\frac{\pi}{6}\text{回転}$$

(2) $\left(\dfrac{1}{\sqrt{2}}-\dfrac{1}{\sqrt{2}i}\right)z=r\left(\cos\left(\theta+\dfrac{7}{4}\pi\right)+i\sin\left(\theta+\dfrac{7}{4}\pi\right)\right)$

$$\frac{7}{4}\pi\text{回転}$$

または

$$r\left(\cos\left(\theta-\frac{\pi}{4}\right)+i\sin\left(\theta-\frac{\pi}{4}\right)\right)$$

$$-\frac{\pi}{4}\text{回転}$$

(3) $iz=r\left(\cos\left(\theta+\dfrac{\pi}{2}\right)+i\sin\left(\theta+\dfrac{\pi}{2}\right)\right)$

$$\frac{\pi}{2}\text{回転}$$

<23ページ　複素数の商>

解答

問

(1) $$\frac{\sqrt{3}-i}{\sqrt{3}+i}=\frac{2\Big(\cos\big(-\frac{\pi}{6}\big)+i\sin\big(-\frac{\pi}{6}\big)\Big)}{2\Big(\cos\big(\frac{\pi}{6}\big)+i\sin\big(\frac{\pi}{6}\big)\Big)}$$

$$=\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)$$

(2) $$\frac{1+i}{1-i}=\frac{\sqrt{2}\Big(\cos\big(\frac{\pi}{4}\big)+i\sin\big(\frac{\pi}{4}\big)\Big)}{\sqrt{2}\Big(\cos\big(-\frac{\pi}{4}\big)+i\sin\big(-\frac{\pi}{4}\big)\Big)}$$

$$=\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)$$

(3) $$\frac{\sqrt{3}+i}{1+i}=\frac{2\Big(\cos\big(\frac{\pi}{6}\big)+i\sin\big(\frac{\pi}{6}\big)\Big)}{\sqrt{2}\Big(\cos\big(\frac{\pi}{4}\big)+i\sin\big(\frac{\pi}{4}\big)\Big)}$$

$$=\sqrt{2}\Big(\cos\big(-\frac{\pi}{12}\big)+i\sin\big(-\frac{\pi}{12}\big)\Big)$$

<24 ページ　ド・モアブルの定理>

解答

問

(1) $$\left(\frac{\sqrt{3}+i}{2}\right)^3 = \left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)^3$$
$$= \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$$

(2) $$\left(\frac{-\sqrt{3}+i}{2}\right)^6 = \left(\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)\right)^6$$
$$= \cos(5\pi) + i\sin(5\pi) = -1$$

(3) $$(-1-i) = \left(\sqrt{2}\left(\cos\left(\frac{5}{4}\pi\right)+i\sin\left(\frac{5}{4}\pi\right)\right)\right)^8$$
$$= 16\big(\cos(10\pi) + i\sin(10\pi)\big) = 16$$

(4) $$\left(\frac{\sqrt{3}+i}{1+i}\right)^6 = \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)\right)\right)^6$$
$$= 2^3\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right) = -8i$$

<25ページ　1の累乗根>

解答

問 (1) $r^3\big(\cos(3\theta)+i\sin(3\theta)\big)=1$

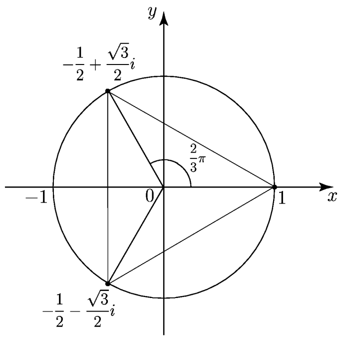
$r=1,\quad 3\theta=2n\pi$

$\theta=\frac{2n}{3}\pi\quad {\scriptstyle (n=0,1,2)}$

$n=0\ \Rightarrow\ \theta=0\ \Rightarrow\ z=1$

$n=1\ \Rightarrow\ \theta=\dfrac{2}{3}\pi\ \Rightarrow\ z=-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

$n=2\ \Rightarrow\ \theta=\dfrac{4}{3}\pi\ \Rightarrow\ z=-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$



(答) $1,\ \frac{-1\pm\sqrt{3}i}{2}$

(2) $r^4\big(\cos(4\theta)+i\sin(4\theta)\big)=1$

$r=1,\quad 4\theta=2n\pi$

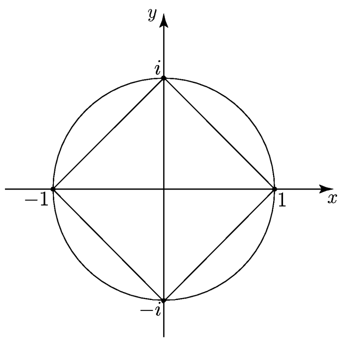
$\theta=\frac{1}{2}n\pi\quad {\scriptstyle (n=0,1,2,3)}$

$n=0\ \Rightarrow\ \theta=0\ \Rightarrow\ z=1$

$n=1\ \Rightarrow\ \theta=\dfrac{\pi}{2}\ \Rightarrow\ z=i$

$n=2\ \Rightarrow\ \theta=\pi\ \Rightarrow\ z=-1$

$n=3\ \Rightarrow\ \theta=\dfrac{3}{2}\pi\ \Rightarrow\ z=-i$

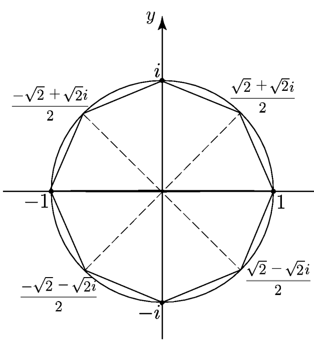


(答) $\pm1,\ \pm i$

(3) $r^8\big(\cos(8\theta)+i\sin(8\theta)\big)=1$

$r=1,\quad \theta=\frac{2}{8}n\pi=\frac{1}{4}n\pi\ {\scriptstyle (n=0,1,2,\cdots)}$

(答) $\frac{\sqrt{2}\pm\sqrt{2}i}{2},\ \frac{-\sqrt{2}\pm\sqrt{2}i}{2},\ \pm1,\ \pm i$



<26 ページ　平面上の回転移動>

解答

問

(1) $\begin{cases} x' = x\cos\left(\frac{\pi}{2}\right) - y\sin\left(\frac{\pi}{2}\right) = -y \\ y' = x\sin\left(\frac{\pi}{2}\right) + y\cos\left(\frac{\pi}{2}\right) = x \end{cases}$

(2) $\begin{cases} x' = x\cos\left(\frac{\pi}{6}\right) - y\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}x - \frac{1}{2}y \\ y' = x\sin\left(\frac{\pi}{6}\right) + y\cos\left(\frac{\pi}{6}\right) = \frac{1}{2}x + \frac{\sqrt{3}}{2}y \end{cases}$

(3) $\begin{cases} x' = x\cos\left(\frac{\pi}{4}\right) - y\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \\ y' = x\sin\left(\frac{\pi}{4}\right) + y\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \end{cases}$

(4) $\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases}$

$<$27 ページ　オイラーの公式 1$>$

解答

問

(1) $e^{0i} = \cos 0 + i\sin 0 = 1$

(2) $e^{\pi i} = \cos\pi + i\sin\pi = -1$

(3) $e^{\frac{2}{3}\pi i} = \cos\left(\dfrac{2}{3}\pi\right) + i\sin\left(\dfrac{2}{3}\pi\right) = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

(4) $e^{\frac{3}{2}\pi i} = \cos\left(\dfrac{3}{2}\pi\right) + i\sin\left(\dfrac{3}{2}\pi\right) = -i$

(5) $e^{-\frac{\pi}{4}i} = \cos\left(-\dfrac{\pi}{4}\right) + i\sin\left(-\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i$

(6) $e^{-\frac{5}{2}\pi i} = \cos\left(-\dfrac{5}{2}\pi\right) + i\sin\left(-\dfrac{5}{2}\pi\right) = -i$

<28ページ　オイラーの公式2>

解答

問

(1) $e^{1-\frac{\pi}{2}i} = e^1\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right) = -ei$

(2) $e^{0+\pi i} = \cos\pi + i\sin\pi = -1$

(3) $$e^{\frac{1}{2}+\frac{2}{3}\pi i} = e^{\frac{1}{2}}\left(\cos\left(\frac{2}{3}\pi\right) + i\sin\left(\frac{2}{3}\pi\right)\right)$$
$$= \sqrt{e}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

(4) $$e^{2-\frac{2}{3}\pi i} = e^2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

(5) $$e^{\frac{1}{2}\log 4+\frac{\pi}{3}i} = e^{\log 2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$
$$= 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$
$$= 1 + \sqrt{3}i$$

(6) $$e^{\log 2+\frac{\pi}{4}i} = 2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$$
$$= \sqrt{2} + \sqrt{2}i$$

<29 ページ　複素数の指数表示>

解答

問 1

$e^{i\theta_1} \times e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$

問 2

(1) $e^{\frac{2}{3}\pi i} \times e^{\frac{1}{3}\pi i} = e^{\pi i} = \cos\pi + i\sin\pi = -1$

(2) $e^{\frac{5}{6}\pi i} \div e^{\frac{1}{3}\pi i} = e^{\left(\frac{5}{6}-\frac{1}{3}\right)\pi i} = e^{\frac{3}{6}\pi i} = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$

(3) $\left(e^{\frac{\pi}{5}i}\right)^5 = e^{\pi i} = -1$

(4) $\left(e^{\frac{2}{3}\pi i}\right)^6 = e^{4\pi i} = 1$

<30 ページ　指数法則>

解答

問 1

(1) $e^{z_1}\ e^{z_2} = e^{z_1+z_2}$　　(2) $\dfrac{e^{z_1}}{e^{z_2}} = e^{\boxed{z_1+z_2}}$

(3) $\left(e^{z}\right)^{n} = e^{\boxed{nz}}$　（ド・モアブルの定理）$\left(\begin{array}{c} z_1,\ z_2,\ z \text{ は複素数} \\ n \text{ は整数} \end{array}\right)$

問 2　(1) $e^{2+3\pi i} \times e^{1-3\pi i} = e^{3}$

(2) $e^{-2-\frac{\pi}{3}i} \div e^{-3-\frac{\pi}{3}i} = e^{-2-\frac{\pi}{3}i-\left(-3-\frac{\pi}{3}i\right)} = e$

(3) $\left(e^{\frac{3}{5}+\frac{\pi}{6}i}\right)^{5} = e^{3+\frac{5}{6}\pi i} = e^{3}\left(-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)$

問 3

$$\frac{\left(1-\sqrt{3}i\right)^{7}}{(1-i)^{6}} = \frac{\left(2e^{-\frac{\pi}{3}i}\right)^{7}}{\left(\sqrt{2}e^{-\frac{\pi}{4}i}\right)^{6}} = \frac{2^{7}e^{-\frac{7}{3}\pi i}}{2^{3}e^{-\frac{3}{2}\pi i}} = 2^{7-3} \times e^{\left(-\frac{7}{3}+\frac{3}{2}\right)\pi i}$$

$$= 2^{4} \times e^{-\frac{5}{6}\pi i} = 16\left(\cos\left(-\frac{5}{6}\pi\right) + i\sin\left(-\frac{5}{6}\pi\right)\right)$$

$$= 16\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = -8\sqrt{3} - 8i$$

<31 ページ　円関数>

解答

問1　(1) $\cos(i\theta) = \dfrac{e^{i(i\theta)} + e^{-i(i\theta)}}{2} = \dfrac{e^{-\theta} + e^{\theta}}{2}$

(2) $(\cos z)^2 = \left(\dfrac{e^{iz} + e^{-iz}}{2}\right)^2 = \dfrac{e^{2iz} + 2 + e^{-2iz}}{4}$

問2　(1) $\cos(-iz) + i\sin(-iz)$

$$= \frac{e^{i(-iz)} + e^{-i(-iz)}}{2} + i \times \frac{e^{i(-iz)} + e^{-i(-iz)}}{2i}$$

$$= \frac{e^{z} + e^{-z} + e^{z} - e^{-z}}{2} = e^{z}$$

(2) $(\cos z)^2 + (\sin z)^2$

$$= \left(\frac{e^{iz} + e^{-iz}}{2}\right)^2 + \left(\frac{e^{iz} - e^{-iz}}{2i}\right)^2$$

$$= \frac{e^{2iz} + 2 + e^{-2iz}}{4} + \frac{e^{2iz} - 2 + e^{-2iz}}{-4}$$

$$= \frac{4}{4} = 1$$

<32 ページ　双曲線関数 >

解答

問

(1) $\cosh(i\theta) = \dfrac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$

(2) $\cosh(z) + \sinh(z) = \dfrac{e^{z} + e^{-z}}{2} + \dfrac{e^{z} - e^{-z}}{2} = e^{z}$

(3) $\cosh(a+bi) - \cosh(a-bi) + \sinh(a+bi) - \sinh(a-bi)$

$$= \frac{e^{a+bi} + e^{-a-bi}}{2} - \frac{e^{a-bi} + e^{-a+bi}}{2} + \frac{e^{a+bi} - e^{-a-bi}}{2} - -\frac{e^{a-bi} - e^{-a+bi}}{2}$$

$$= \frac{e^{a+bi} + e^{-a-bi} - e^{a-bi} - e^{-a+bi} + e^{a+bi} - e^{-a-bi} - e^{a-bi} + e^{-a+bi}}{2}$$

$$= e^{a+bi} - e^{a-bi} = e^{a}(e^{bi} - e^{-bi}) = 2e^{a}\sin(b)i$$

<33 ページ　微分の復習 1>

解答

問

(1) $\dfrac{d}{dx}(e^x) = e^x$

(2) $\dfrac{d}{dx}(e^{5x}) = 5e^{5x}$

(3) $\dfrac{d}{dx}(xe^x) = e^x + xe^x$

(4) $\dfrac{d}{dx}(2xe^{-3x}) = 2e^{-3x} - 6xe^{-3x}$

(5) $\dfrac{d}{dx}(\sin x) = \cos x$

(6) $\dfrac{d}{dx}(\sin(3x)) = 3\cos(3x)$

(7) $\dfrac{d}{dx}(\cos x) = -\sin x$

(8) $\dfrac{d}{dx}(\cos(2x)) = -2\sin(2x)$

(9) $\dfrac{d}{dx}(e^x \sin(2x)) = e^x \sin(2x) + 2e^x \cos(2x)$

(10) $\dfrac{d}{dx}(e^{4x} \cos x) = 4e^{4x} \cos(x) - e^{4x} \sin(x)$

(11) $\dfrac{d}{dx}(e^{-x} \sin(2x)) = -e^{-x} \sin(2x) + 2e^{-x} \cos(2x)$

(12) $\dfrac{d}{dx}(e^{-2x} \cos(3x)) = -2e^{-2x} \cos(3x) - 3e^{-2x} \sin(3x)$

(13) $\dfrac{d}{dx}(e^{-3x} \sin(5x)) = -3e^{-3x} \sin(5x) + 5e^{-3x} \cos(5x)$

(14) $\dfrac{d}{dx}(e^{-4x} \cos(7x)) = -4e^{-4x} \cos(7x) - 7e^{-4x} \sin(7x)$

<34 ページ　微分の復習 2>

解答

問

(1) $\dfrac{d}{dt}\left\{\dfrac{e^{4t}}{41}\big(4\cos(5t)+5\sin(5t)\big)\right\}$

$$=\frac{4}{41}e^{4t}\big(4\cos(5t)+5\sin(5t)\big)+\frac{1}{41}e^{4t}\big(-20\sin(5t)+25\cos(5t)\big)$$

$$=e^{4t}\cos(5t)$$

(2) $\dfrac{d}{dt}\left\{\dfrac{e^{4t}}{41}\big(-5\cos(5t)+4\sin(5t)\big)\right\}$

$$=\frac{4}{41}e^{4t}\big(-5\cos(5t)+4\sin(5t)\big)+\frac{1}{41}e^{4t}\big(25\sin(5t)+20\cos(5t)\big)$$

$$=e^{4t}\sin(5t)$$

(3) $\dfrac{d}{dt}\left\{\dfrac{e^{at}}{a^2+b^2}\big(a\cos(bt)+b\sin(bt)\big)\right\}$

$$=\frac{a}{a^2+b^2}e^{at}\big(a\cos(bt)+b\sin(bt)\big)+\frac{1}{a^2+b^2}e^{at}\big(-ab\sin(bt)+b^2\cos(bt)\big)$$

$$=e^{at}\cos(bt)$$

(4) $\dfrac{d}{dt}\left\{\dfrac{e^{at}}{a^2+b^2}\big(-b\cos(bt)+a\sin(bt)\big)\right\}$

$$=\frac{a}{a^2+b^2}e^{at}\big(-b\cos(bt)+a\sin(bt)\big)+\frac{1}{a^2+b^2}e^{at}\big(b^2\sin(bt)+ab\cos(bt)\big)$$

$$=e^{at}\sin(bt)$$

<35 ページ　複素数値関数の微分 1>

解答

問

(1) $\dfrac{dz}{dt} = 6t^5 + 3t^2 i$

(2) $\dfrac{dz}{dt} = -b\sin(bt) + bi\cos(bt)$

(3) $z(t) = e^{(2+5i)t} = e^{2t}(\cos(5t) + i\sin(5t))$

$$\frac{dz}{dt} = 2e^{2t}(\cos(5t) + i\sin(5t)) + e^{2t}(-5\sin(5t) + 5i\cos(5t))$$

$$= (2e^{2t}\cos(5t) - 5e^{2t}\sin(5t)) + i(2e^{2t}\sin(5t) + 5e^{2t}\cos(5t))$$

(4) $z(t) = e^{(a+bi)t} = e^{at}(\cos(bt) + i\sin(bt))$

$$\frac{dz}{dt} = ae^{at}(\cos(bt) + i\sin(bt)) + e^{at}(-b\sin(bt) + bi\cos(bt))$$

$$= (ae^{at}\cos(bt) - be^{at}\sin(bt)) + i(ae^{at}\sin(bt) + be^{at}\cos(bt))$$

<36 ページ　複素数値関数の微分 2>

解答

問

(1) $\dfrac{d}{dt}(e^{5it}) = \dfrac{d}{dt}\Big(\cos(5t) + i\sin(5t)\Big) = -5\sin(5t) + 5i\cos(5t)$

$$= 5i(i\sin(5t) + \cos(5t)) = 5ie^{5it}$$

(2) $\dfrac{d}{dt}(e^{bit}) = bie^{bit}$

(3) $\dfrac{d}{dt}(e^{(2+5i)t}) = \Big(2e^{2t}\cos(5t) - 5e^{2t}\sin(5t)\Big) + i\Big(2e^{2t}\sin(5t) + 5e^{2t}\cos(5t)\Big)$

$$= e^{2t}\Big\{2\cos(5t) - 5\sin(5t) + i\Big(2\sin(5t) + 5\cos(5t)\Big)\Big\}$$

$$= e^{2t}\{(2+5i)\cos(5t) + (-5+2i)\sin(5t)\}$$

$$= e^{2t}(2+5i)\Big(\cos(5t) + i\sin(5t)\Big) = (2+5i)e^{(2+5i)t}$$

(4) $\dfrac{d}{dt}(e^{(a+bi)t}) = (a+bi)e^{(a+bi)t}$

<37 ページ　積分の復習 1>

解答

問

(1) $\displaystyle\int dt = t + C$

(2) $\displaystyle\int t^n dt = \frac{1}{n+1}t^{n+1} + C$

(3) $\displaystyle\int \frac{1}{y}dy = \log|y| + C$

(4) $\displaystyle\int e^u du = e^u + C$

(5) $\displaystyle\int \cos v dv = \sin v + C$

(6) $\displaystyle\int \sin t dt = -\cos t + C$

<38 ページ　積分の復習 2>

解答

問

(1) $\displaystyle\int \cos(3x+4)dx = \frac{1}{3}\sin(3x+4) + C$

(2) $\displaystyle\int \sin(5x-2)dx = -\frac{1}{5}\cos(5x-2) + C$

(3) $\displaystyle\int e^{3x+5}dx = \frac{1}{3}e^{3x+5} + C$

(4) $\displaystyle\int \frac{1}{5x-3}dx = \frac{1}{5}\log|5x-3| + C$

(5) $\displaystyle\int (8x+7)^5 = \frac{1}{48}(8x+7)^6 + C$

(6) $\displaystyle\int \cos(3t)dt = \frac{1}{3}\sin(3t) + C$

(7) $\displaystyle\int e^{2t-3}dt = \frac{1}{2}e^{2t-3} + C$

(8) $\displaystyle\int \cos(at+b)dt = \frac{1}{a}\sin(at+b) + C$

(9) $\displaystyle\int \sin(at+b)dt = -\frac{1}{a}\cos(at+b) + C$

(10) $\displaystyle\int e^{at+b}dt = \frac{1}{a}e^{at+b} + C$

<39 ページ　複素数値関数の積分 1>

解答

問

(1) $$\int (t^3 + t^5 i)dt = \frac{1}{4}t^4 + \frac{1}{6}t^6 i + C$$

(2) $$\int (\cos t + i \sin t)dt = \sin t - i \cos t + C$$

(3) $$\int (e^{2t} + i \cos(3t))dt = \frac{1}{2}e^{2t} + \frac{1}{3}\sin(3t)i + C$$

(4) $$\int (\sin(2t) + e^{3t} i)dt = -\frac{1}{2}\cos(2t) + \frac{1}{3}e^{3t} i + C$$

<40 ページ　複素数値関数の積分 2>

解答

問

(1) $\displaystyle\int e^{5it}dt = \frac{1}{5i}e^{5it} + C$

(2) $\displaystyle\int e^{bit}dt = \frac{1}{bi}e^{bit} + C$

(3) $\displaystyle\int e^{(2+5i)}dt = \frac{1}{2+5i}e^{(2+5i)t} + C$

(4) $\displaystyle\int e^{(a+bi)}dt = \frac{1}{a+bi}e^{(a+bi)t} + C$