< ページ1. 部分積分法1 >

解答

$$\int f(x) \times g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

＜ ページ 2．部分積分法 2 ＞

解答

(1) $$\int (4x+3)\sin x dx = -(4x+3)\cos x + \int 4\cos x dx$$
$$= -(4x+3)\cos x + 4\sin x + C$$

(2) $$\int (5x-4)\cos x dx = (5x-4)\sin x + \int 5\sin x dx$$
$$= (5x-4)\sin x + 5\cos x + C$$

(3) $$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C$$

＜ ページ3．部分積分法3 ＞

解答

$$
\begin{aligned}
\int (\log x) \times x dx &= \frac{1}{2}x^2 \log x - \int \frac{1}{x} \times \frac{1}{2}x^2 dx \\
&= \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C
\end{aligned}
$$

< ページ4. 不定積分の検証 >

解答

(1) $\left(\dfrac{1}{10}(x^2+3)^5\right)' = \dfrac{1}{10} \times 5(x^2+3)^4 \times 2x = x(x^2+5)^4$ で正しい。

(2) $\left(\dfrac{1}{3}\log|x^3+1|\right)' = \dfrac{1}{3} \times \dfrac{3x^2}{x^3+1} = \dfrac{x^2}{x^3+1}$ で正しい。

(3) $\left(x^2e^x - 2xe^x + e^x\right)' = 2xe^x + x^2e^x - 2e^x - 2xe^x + e^x = x^2e^x - e^x$
で正しくない。

＜ ページ5．不定積分の練習1 ＞

解答1

(1) $\int dx = x + C$

(2) $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

(3) $\int \frac{1}{x} dx = \log|x| + C$

(4) $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$

(5) $\int \sqrt[3]{x} dx = \frac{3}{4} x \sqrt[3]{x} + C$

(6) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$

解答2

(1) $\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + C$

(2) $\int \frac{1}{ax+b} dx = \frac{1}{a} \log|ax+b| + C$

(3) $\int (4x+3)^5 dx = \frac{1}{24} (4x+3)^6 + C$

(4) $\int \frac{1}{(5x+6)^3} dx = -\frac{1}{10(5x+6)^2} + C$

(5) $\int \sqrt{3x-1} dx = \frac{2}{9} (3x-1)\sqrt{3x-1} + C$

(6) $\int \frac{1}{\sqrt[3]{4x+1}} dx = \frac{3}{8} \sqrt[3]{(4x+1)^2} + C$

(7) $\int \left(2x + 1 + \frac{1}{x}\right) dx = x^2 + x + \log|x| + C$

(8) $\int \frac{x^2+2x+1}{x} dx = \frac{1}{2} x^2 + 2x + \log|x| + C$

(9) $\int \frac{x^3+2x-1}{x^2} dx = \frac{x^2}{2} + 2\log|x| + \frac{1}{x} + C$

(10) $\int \frac{x^4+3x-1}{x^3} dx = \frac{1}{2} x^2 - \frac{3}{x} + \frac{1}{2x^2} + C$

(11) $\int \frac{4}{x+1} dx = 4\log|x+1| + C$

(12) $\int \frac{3}{(x+1)^2} dx = -\frac{3}{x+1} + C$

(13) $\int \frac{x+1}{\sqrt{x}} dx = \frac{2}{3} x\sqrt{x} + 2\sqrt{x} + C$

(14) $\int \frac{x^2+2x}{x+1} dx = \int \left(x + 1 - \frac{1}{x+1}\right) dx$

$= \frac{1}{2} x^2 + x - \log|x+1| + C$

＜ ページ 6. 不定積分の練習 2 ＞

解答 1

(1) $\int e^x dx = e^x + C$ (2) $\int \cos x dx = \sin x + C$

(3) $\int \sin x dx = -\cos x + C$ (4) $\int \frac{1}{\cos^2 x} dx = \tan x + C$

解答 2

(1) $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$ (2) $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$

(3) $\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$ (4) $\int \frac{1}{\cos^2(ax+b)} dx = \frac{1}{a} \tan(ax+b) + C$

解答 3

(1) $\int \frac{1}{\sqrt{1-x^2}} dx$

$= \sin^{-1}(x) + C$

(2) $\int \left(-\frac{1}{\sqrt{1-x^2}} \right) dx$

$= \cos^{-1}(x) + C$

(3) $\int \frac{1}{1+x^2} dx$

$= \tan^{-1} x + C$

解答 4

(1) $\int \frac{x}{x^2+1} dx = \frac{1}{2} \log |x^2+1| + C$ (2) $\int \tan x dx = -\log |\cos x| + C$

解答 5

(1) $\int x e^x dx = x e^x - e^x + C$ (2) $\int \log x dx = x \log |x| - x + C$

＜ ページ7．不定積分の練習３ ＞

解答

(1) $$\int \frac{1}{x^2-x}dx = \int \left(\frac{1}{x-1} - \frac{1}{x}\right) dx$$
$$= \log|x-1| - \log|x| + C$$
$$= \log\left|\frac{x-1}{x}\right| + C$$

(2) $$\int \frac{1}{x^2+3x}dx = \int \frac{1}{3}\left(\frac{1}{x} - \frac{1}{x+3}\right) dx$$
$$= \frac{1}{3}\left(\log|x| - \log|x+3|\right) + C$$
$$= \frac{1}{3}\log\left|\frac{x}{x+3}\right| + C$$

＜ ページ8. 不定積分の練習4 ＞

解答

(1) $$\int \sin^2 x dx = \int \left\{ \frac{1}{2}\big(1 - \cos(2x)\big) \right\} dx$$
$$= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

(2) $$\int \cos(3x)\cos x dx = \int \frac{1}{2}\{\cos(4x) + \cos(2x)\} dx$$
$$= \frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x) + C$$

(3) $$\int \sin(3x)\sin(2x) dx = \int \frac{1}{2}\{\cos(x) - \cos(5x)\} dx$$
$$= \frac{1}{2}\sin(x) - \frac{1}{10}\sin(5x) + C$$

< ページ9. 和の記号 $\sum$ (シグマ) 1 >

解答

$$(1)\ \sum_{k=1}^{8} k = 1+2+3+4+5+6+7+8(=36)$$

$$(2)\ \sum_{k=1}^{5} k^3 = 1^3+2^3+3^3+4^3+5^3(=225)$$

$$(3)\ \sum_{k=1}^{6}(2k-1) = 1+3+5+7+9+11(=36)$$

$$(4)\ \sum_{k=1}^{4}(4k-3) = 1+5+9+13(=28)$$

$$(5)\ \sum_{k=1}^{6} = 1+1+1+1+1+1(=6)$$

$<$ ページ10．和の記号 $\sum$（シグマ）2 $>$

解答 1

(1) $1+2+3+4+\cdots+n=\sum_{k=1}^{n}k$

(2) $1\times3+3\times5+5\times7+\cdots+(2n-1)(2n+1)=\sum_{k=1}^{n}(2k-1)(2k+1)$

(3) $1+3+5+\cdots+19=\sum_{k=1}^{10}(2k-1)$

(4) $3+6+9+12+\cdots+300=\sum_{k=1}^{100}3k$

解答 2

(1) $\sum_{k=2}^{6}(k^2+3)=7+12+19+28+39$

(2)
$$\begin{aligned}\sum_{k=4}^{8}(2k-3)(3k-2)&=(2\times4-3)(3\times4-2)+(2\times5-3)(3\times5-2)\\&\quad+(2\times6-3)(3\times6-2)+(2\times7-3)(3\times7-2)\\&\quad+(2\times8-3)(3\times8-2)\\&=5\times10+7\times13+9\times16+11\times19+13\times22\\&=50+91+144+209+286\end{aligned}$$

(3) $\sum_{k=0}^{n}4^k=1+4+16+\cdots+4^n$

$<$ ページ 11. 和の記号 $\sum$ (シグマ) 3 $>$

解答 1

(1) $\displaystyle\sum_{k=1}^{n}(2k+4) = 2\sum_{k=1}^{n}k + 4\sum_{k=1}^{n}1 = 2\times\frac{n(n+1)}{2} + 4n = n(n+1)+4n$

$= n^2+5n$

(2) $\displaystyle\sum_{k=1}^{n}(6k-5) = 6\sum_{k=1}^{n}k - 5\sum_{k=1}^{n}1 = 3n(n+1)-5n = 3n^2-2n$

解答 2

(1) $\displaystyle 1+3+5+7+\cdots+(2n-1) = \sum_{k=1}^{n}(2k-1) = n(n+1)-n = n^2$

(2) $\displaystyle 2+5+8+11+\cdots+(3n-1) = \sum_{k=1}^{n}(3k-1) = \frac{3}{2}(n^2+n)-n$

$\displaystyle = \frac{3}{2}n^2+\frac{1}{2}n$

(3) $\displaystyle 3+9+15+21+\cdots+(6n-3) = \sum_{k=1}^{n}(6k-3) = 3(n^2+n)-3n$

$= 3n^2$

< ページ12. 和の記号 $\sum$ (シグマ) 4 >

解答1

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

解答2

(1) $1^2 + 2^2 + 3^2 + \cdots + 7^2 = \dfrac{7(7+1)(2\times 7+1)}{6} = \dfrac{7\times 8\times 15}{6}$

$= 7 \times 20 = 140$

(2) $1^2 + 2^2 + 3^2 + \cdots + n^2 + (n+1)^2 = \dfrac{(n+1)(n+2)(2n+3)}{6}$

$\left(= \dfrac{2n^3 + 9n^2 + 13n + 6}{6} \right)$

< ページ 13. 和の記号 $\sum$ (シグマ) 5 >

解答 1

$$\sum_{k=1}^{n} k^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

解答 2

(1) $1^3 + 2^3 + 3^3 + \cdots + 7^3 = \left(\frac{7(7+1)}{2}\right)^2 = \left(\frac{7 \times 8}{2}\right)^2$

$$= (7 \times 4)^2 = 784$$

(2) $1^3 + 2^3 + 3^3 + \cdots + (n-1)^3 = \left(\frac{(n-1)n}{2}\right)^2$

< ページ14. 和の記号 $\sum$ (シグマ) 6 >

解答 1

(1) $\displaystyle\sum_{i=2}^{4} x_i = x_2 + x_3 + x_4$

(2) $\displaystyle\sum_{j=3}^{6} y_j = y_3 + y_4 + y_5 + y_6$

(3) $\displaystyle\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$

(4) $\displaystyle\sum_{j=2}^{n} j^3 = 2^3 + 3^3 + 4^3 + \cdots + n^3$

解答 2

$$\sum_{i=2}^{4}\left\{\sum_{j=4}^{5}(x_i \times y_j)\right\} = \sum_{i=2}^{4}(x_i \times y_4 + x_i \times y_5)$$

$$= x_2 \times y_4 + x_2 \times y_5$$

$$+x_3 \times y_4 + x_3 \times y_5$$

$$+x_4 \times y_4 + x_4 \times y_5$$

< ページ15．区分求積法１ >

解答

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{6}\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right) = \frac{2}{6} = \frac{1}{3}$$

＜ページ16．区分求積法２＞

解答

(1) $$\begin{aligned}(x_1)^2+(x_2)^2+\cdots+(x_n)^2 &= h^2+(2h)^2+\cdots+(nh)^2 \\ &= h^2\left\{1^2+2^2+\cdots+n^2\right\} \\ &= h^2\sum_{k=1}^{n}k^2\end{aligned}$$

(2) $$(x_1)^2+(x_2)^2+\cdots+(x_n)^2 = h^2\sum_{k=1}^{n}k^2 = h^2\times\frac{1}{6}n(n+1)(2n+1)$$

(3) $$S_n^* = \left\{(x_1)^2+(x_2)^2+\cdots+(x_{n-1})^2+(x_n)^2\right\}h$$

$$= h^3\times\frac{1}{6}n(n+1)(2n+1)$$

$$= \left(\frac{1}{n}\right)^3\times\frac{1}{6}n(n+1)(2n+1)$$

$$= \frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

(4) $$\lim_{n\to\infty}S_n^* = \lim_{n\to\infty}\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = \frac{2}{6} = \frac{1}{3}$$

＜ ページ17. 区分求積法3 ＞

解答

$$S_n^* = h^3h + (2h)^3h + (3h)^3h + \cdots + (nh)^3h$$

$$= \{1^3 + 2^3 + 3^3 + \cdots + n^3\}h^4$$

$$= \left(\sum_{k=1}^{n} k^3\right) \times h^4 = \left(\frac{n(n+1)}{2}\right)^2 \times \left(\frac{1}{n}\right)^4 = \frac{1}{4}\left(1 + \frac{1}{n}\right)^2$$

$$\lim_{n\to\infty} S_n^* = \lim_{n\to\infty} \frac{1}{4}\left(1 + \frac{1}{n}\right)^2 = \frac{1}{4}$$

＜ページ18．面積関数 $S(x)$ 1 ＞

解答

$$S_n^*(x) = h^2h + (2h)^2h + (3h)^2h + \cdots + (nh)^2h$$

$$= \{1^2 + 2^2 + 3^2 + \cdots + n^2\}h^3$$

$$= \frac{1}{6}n(n+1)(2n+1) \times \left(\frac{x}{n}\right)^3 = \frac{1}{6}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)x^3$$

$$\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty} \frac{1}{6}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)x^3 = \frac{2}{6}x^3 = \frac{1}{3}x^3$$

< ページ19. 面積関数 $S(x)$ 2 >

解答

$$S_n^*(x) = h^3h + (2h)^3h + (3h)^3h + \cdots + (nh)^3h$$

$$= \{1^3 + 2^3 + 3^3 + \cdots + n^3\}h^4$$

$$= \left(\frac{n(n+1)}{2}\right)^2 \times \left(\frac{x}{n}\right)^4 = \frac{1}{4}\left(1 + \frac{1}{n}\right)^2 x^4$$

$$\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty} \frac{1}{4}\left(1 + \frac{1}{n}\right)^2 x^4 = \frac{1}{4}x^4$$

< ページ20. 面積関数 $S(x)$ 3 >

解答 1

(1) $f(x) = 1$ のとき $S(x) = x$

(2) $f(x) = x$ のとき $S(x) = \frac{1}{2}x^2$

(3) $f(x) = x^2$ のとき $S(x) = \frac{1}{3}x^3$

(4) $f(x) = x^3$ のとき $S(x) = \frac{1}{4}x^4$

解答 2

$$S(x) = \frac{1}{5}x^5$$

解答 3

$$S(x) = \frac{1}{n+1}x^{n+1}$$

解答 4

$$S'(x) = f(x)$$

＜ ページ21. 面積関数 $S(x)$ 4 ＞

解答

$$S(x) = \int \left(x^3 - 3x^2 + 4\right) dx = \frac{1}{4}x^4 - x^3 + 4x + C$$

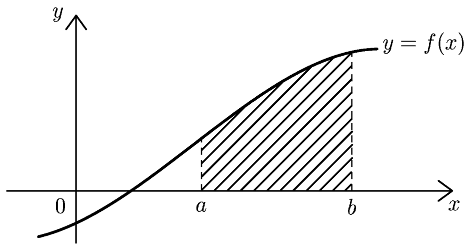
$S(0) = 0$ より $C = 0 \Rightarrow S(x) = \dfrac{1}{4}x^4 - x^3 + 4x$

$$\begin{aligned}
S &= S(3) - S(2) \\
&= \left(\frac{1}{4} \times 3^4 - 3^3 + 4 \times 3\right) - \left(\frac{1}{4} \times 2^4 - 2^3 + 4 \times 2\right) \\
&= \frac{81}{4} - 27 + 12 - \frac{16}{4} + 8 - 8 \\
&= \frac{81 - 16}{4} - 15 = \frac{65 - 60}{4} = \frac{5}{4}
\end{aligned}$$

< ページ22. 定積分の定義 >

解答

斜線部分の面積



< ページ 24. 定積分 1 >

解答

(1) $\displaystyle\int_4^7 1dx = [x]_4^7 = 7 - 4 = 3$

(2) $\displaystyle\int_{-1}^3 xdx = \left[\frac{1}{2}x^2\right]_{-1}^3 = \frac{9}{2} - \frac{1}{2} = 4$

(3) $\displaystyle\int_{-2}^1 x^2dx = \left[\frac{x^3}{3}\right]_{-2}^1 = \frac{1}{3} - \left(-\frac{8}{3}\right)$

$\displaystyle = 3$

(4) $\displaystyle\int_{-2}^2 x^3dx = \left[\frac{1}{4}x^4\right]_{-2}^2 = \frac{16}{4} - \frac{16}{4} = 0$

＜ ページ 25．定積分 2 ＞

解答

(1) $\displaystyle\int_2^2 (x^4 - 5x^3)dx = 0$

(2) $\displaystyle\int_4^4 \sqrt{x}dx = 0$

(3) $\displaystyle\int_\pi^\pi \cos x dx = 0$

(4) $\displaystyle\int_2^1 x^3 dx = \left[\frac{1}{4}x^4\right]_2^1$

$\displaystyle = \frac{1}{4} - \frac{16}{4} = -\frac{15}{4}$

(5) $\displaystyle\int_3^0 x^4 dx = \left[\frac{1}{5}x^5\right]_3^0$

$\displaystyle = 0 - \frac{243}{5}$

$\displaystyle = -\frac{243}{5} (= -48.6)$

(6) $\displaystyle\int_1^{-1} (x^2 - 1)dx = \left[\frac{x^3}{3} - x\right]_1^{-1}$

$\displaystyle = -\frac{1}{3} + 1 - \left(\frac{1}{3} - 1\right)$

$\displaystyle = 2 - \frac{2}{3} = \frac{4}{3}$

(7) $\displaystyle\int_4^0 (x - 3)dx = \left[\frac{x^2}{2} - 3x\right]_4^0$

$\displaystyle = 0 - \left(\frac{16}{2} - 12\right)$

$= -(8 - 12) = 4$

(8) $\displaystyle\int_2^{-2} (x^3 + x)dx = \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_2^{-2}$

$\displaystyle = \frac{16}{4} + \frac{4}{2} - \left(\frac{16}{4} + \frac{4}{2}\right)$

$= 0$

＜ページ 26．定積分 3 ＞

解答 1

(1) $\displaystyle\int dx = x + C$　　(2) $\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + C$

(3) $\displaystyle\int \frac{dx}{x} = \log|x| + C$　　(4) $\displaystyle\int e^x dx = e^x + C$

(5) $\displaystyle\int \cos x dx = \sin x + C$　　(6) $\displaystyle\int \sin x dx = -\cos x + C$

(7) $\displaystyle\int \frac{1}{\cos^2 x}dx = \tan x + C$　　(8) $\displaystyle\int \frac{1}{x^3}dx = -\frac{1}{2x^2} + C$

(9) $\displaystyle\int \sqrt{x}dx = \frac{2}{3}x\sqrt{x} + C$　　(10) $\displaystyle\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + C$

解答 2

(1) $\displaystyle\int_{-1}^{3} dx = [\,x\,]_{-1}^{3} = 4$　　(2) $\displaystyle\int_0^2 x^7 dx = \left[\frac{1}{8}x^8\right]_0^2 = 32$

(3) $\displaystyle\int_1^e \frac{dx}{x} = \left[\log|x|\right]_1^e = 1$　　(4) $\displaystyle\int_0^1 e^x dx = [\,e^x\,]_0^1 = e - 1$

(5) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x dx = [-\sin(x)]_0^{\frac{\pi}{2}}$
$= 1$　　(6) $\displaystyle\int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi}$
$= 2$

(7) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = [\tan x]_0^{\frac{\pi}{4}} = 1$　　(8) $\displaystyle\int_1^2 \frac{1}{x^3}dx = \left[-\frac{1}{2x^2}\right]_1^2 = \frac{3}{8}$

(9) $\displaystyle\int_0^4 \sqrt{x}dx = \left[\frac{2}{3}x\sqrt{x}\right]_0^4 = \frac{16}{3}$　　(10) $\displaystyle\int_1^9 \frac{1}{\sqrt{x}}dx = \left[2\sqrt{x}\right]_1^9 = 4$

＜ ページ 27．定積分 4 ＞

解答 1

(1) $\displaystyle\int (ax+b)^n dx$

$\displaystyle = \frac{1}{a(n+1)}(ax+b)^{n+1} + C$

(2) $\displaystyle\int \frac{1}{(ax+b)^2}dx$

$\displaystyle = -\frac{1}{a(ax+b)} + C$

(3) $\displaystyle\int \sqrt{ax+b}\,dx$

$\displaystyle = \frac{2}{3a}(ax+b)\sqrt{ax+b} + C$

(4) $\displaystyle\int \frac{1}{\sqrt{ax+b}}dx$

$\displaystyle = \frac{2}{a}\sqrt{ax+b} + C$

(5) $\displaystyle\int \frac{1}{ax+b}dx$

$\displaystyle = \frac{1}{a}\log|ax+b| + C$

(6) $\displaystyle\int \cos(ax+b)dx$

$\displaystyle = \frac{1}{a}\sin(ax+b) + C$

(7) $\displaystyle\int \sin(ax+b)dx$

$\displaystyle = -\frac{1}{a}\cos(ax+b) + C$

(8) $\displaystyle\int e^{ax+b}dx$

$\displaystyle = \frac{1}{a}e^{ax+b} + C$

解答 2

(1) $\displaystyle\int_0^1 (3x+1)^4 dx = \left[\frac{1}{15}(3x+1)^5\right]_0^1$

$\displaystyle = \frac{341}{15}(=68.2)$

(2) $\displaystyle\int_1^2 \frac{dx}{(3x+4)^2} = \left[-\frac{1}{3(3x+4)}\right]_1^2$

$\displaystyle = \frac{1}{70}$

(3) $\displaystyle\int_1^3 \sqrt{4x-3}\,dx = \left[\frac{2}{12}(4x-3)\sqrt{4x-3}\right]_1^3$

$\displaystyle = \frac{13}{3}$

(4) $\displaystyle\int_1^4 \frac{1}{\sqrt{3x-3}}dx = \left[\frac{2}{3}\sqrt{3x-3}\right]_1^4$

$= 2$

(5) $\displaystyle\int_0^1 \frac{1}{2x+1}dx = \left[\frac{1}{2}\log|2x+1|\right]_0^1$

$\displaystyle = \frac{1}{2}\log|3|$

(6) $\displaystyle\int_0^{\frac{\pi}{4}} \cos(4x)dx = \left[\frac{1}{4}\sin(4x)\right]_0^{\frac{\pi}{4}}$

$= 0$

(7) $\displaystyle\int_0^{\pi} \sin\left(\frac{1}{2}x\right)dx = \left[-2\cos\left(\frac{1}{2}x\right)\right]_0^{\pi}$

$= 2$

(8) $\displaystyle\int_0^1 e^{2x-1}dx = \left[\frac{1}{2}e^{2x-1}\right]_0^1$

$\displaystyle = \frac{1}{2}e - \frac{1}{2e}$

＜ページ28. 定積分5＞

解答

(1) $$\int_1^3 (5-9.8t)dt = \left[\,5t-4.9t^2\,\right]_1^3$$
$$= 15-4.9\times 9-(5-4.9)$$
$$= 10-4.9\times 8 = -29.2$$

(2) $$\int_2^3 (2\pi r)dr = \left[\,\pi r^2\,\right]_2^3 = \pi\times 9-\pi\times 4 = 5\pi$$

(3) $$\int_0^{\pi} \sin^2\theta d\theta = \int_0^{\pi} \left\{\frac{1}{2}-\frac{1}{2}\cos(2\theta)\right\} d\theta$$
$$= \left[\frac{\theta}{2}-\frac{1}{4}\sin(2\theta)\right]_0^{\pi}$$
$$= \frac{\pi}{2}$$

(4) $$\int_a^b t^n dt = \left[\frac{1}{n+1}t^{n+1}\right]_a^b = \frac{b^{n+1}}{n+1}-\frac{a^{n+1}}{n+1}$$

(5) $$\int_0^4 t\sqrt{t}dt = \int_0^4 t^{\frac{3}{2}}dt = \left[\frac{3}{5}t^{\frac{5}{2}}\right]_0^4 = \frac{2}{5}\times 4^{\frac{5}{2}}$$
$$= \frac{2}{5}\times 2^5$$
$$= \frac{64}{5}(=12.8)$$

＜ ページ 29．定積分 6 ＞

解答

(1) $\displaystyle\int (2x+3)\sqrt{x^2+3x-4}dx = \int \sqrt{u}\ du = \frac{2}{3}u^{\frac{3}{2}} + C$

$\displaystyle = \frac{2}{3}(x^2+3x-4)^{\frac{3}{2}} + C$

$(u = x^2+3x-4)$

(2) $\displaystyle\int \frac{3x^2}{x^3+1}dx = \int \frac{1}{u}\ du = \log|u| + C = \log|x^3+1| + C$

$(u = x^3+1)$

(3) $\displaystyle\int_1^5 (2x+3)\sqrt{x^2+3x-4}dx = \left[\frac{2}{3}(x^2+3x-4)^{\frac{3}{2}}\right]_1^5$

$\displaystyle = \frac{2}{3}6^3 - 0 = 144$

(4) $\displaystyle\int_0^2 \frac{3x^2}{x^3+1}dx = \big[\log|x^3+1|\big]_0^2 = \log 9$

< ページ30．定積分の置換積分1 >

解答

$$(1)\quad \int_0^2 4x^3\sqrt{x^4+1}\,dx = \int_{u=1}^{u=17} \sqrt{u}\,du = \left[\frac{2}{3}u^{\frac{3}{2}}\right]_{u=1}^{u=17}$$

$$= \frac{2}{3}\times 17\sqrt{17} - \frac{2}{3}$$

$$= \frac{34\sqrt{17}-2}{3}$$

$(u = x^4 + 1\,,\ x = 0 \Rightarrow u = 1\,,\ x = 2 \Rightarrow u = 17)$

$$(2)\quad \int_0^2 \frac{4x^3}{x^4+1}\,dx = \int_{u=1}^{u=17} \sqrt{u}\,du = [\,\log|u|\,]_{u=1}^{u=17}$$

$$= \log 17$$

＜ ページ31．定積分の置換積分2 ＞

解答

(1) $$\int_0^1 (x^2+x-1)^4\,(2x+1)\,dx = \int_{u=-1}^{u=1} u^4\,du = \left[\frac{1}{5}u^5\right]_{u=-1}^{u=1} = \frac{1}{5} - \left(-\frac{1}{5}\right) = \frac{2}{5}$$

$(u = x^2+x-1\,,\ x=0 \Rightarrow u=-1\,,\ x=1 \Rightarrow u=1)$

(2) $$\int_0^2 \frac{2x}{x^2+1}\,dx = \int_{u=1}^{u=5} \frac{1}{u}\,du = [\,\log|u|\,]_{u=1}^{u=5} = \log 5$$

$(u = x^2+1\,,\ x=0 \Rightarrow u=1\,,\ x=2 \Rightarrow u=5)$

(3) $$\int_0^3 2xe^{x^2}\,dx = \int_{u=0}^{u=9} e^u\,du = [\,e^u\,]_{u=0}^{u=9} = e^9-1$$

$(u = x^2\,,\ x=0 \Rightarrow u=0\,,\ x=3 \Rightarrow u=9)$

< ページ 32. 定積分の置換積分法 3 >

解答

$$
\begin{aligned}
\int_0^{\frac{1}{2}} \sqrt{1-x^2}\, dx &= \left[\frac{u}{2} + \frac{1}{4}\sin(2u) \right]_{u=0}^{u=\frac{\pi}{6}} \\
&= \frac{\pi}{12} + \frac{1}{4}\sin\frac{\pi}{3} - 0 \\
&= \frac{\pi}{12} + \frac{\sqrt{3}}{8}
\end{aligned}
$$

＜ ページ 33. 定積分の部分積分 ＞

解答

(1) $$\int_0^1 x(x-1)^4\,dx = \left[x \times \frac{(x-1)^5}{5}\right]_0^1 - \int_0^1 \frac{(x-1)^5}{5}dx$$
$$= 0 - \left[\frac{(x-1)^6}{30}\right]_0^1$$
$$= -\left(0 - \frac{1}{30}\right)$$
$$= \frac{1}{30}$$

(2) $$\int_0^{\frac{\pi}{2}} x\sin x\,dx = \left[\,x \times (-\cos x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 1 \times (-\cos x)\,dx$$
$$= 0 + \left[\,\sin x\,\right]_0^{\frac{\pi}{2}}$$
$$= \sin\frac{\pi}{2} - \sin 0 = 1$$

(3) $$\int_0^1 xe^x\,dx = \left[\,xe^x\right]_0^1 - \int_0^1 e^x\,dx$$
$$= 1e^1 - 0 - \left[\,e^x\,\right]_0^1$$
$$= e - (e^1 - e^0) = 1$$

＜ ページ34．面積1 ＞

解答

$$
\begin{aligned}
S &= \int_{-1}^{2} (-x^2 + 2x + 4 - x^2)\ dx \\
&= \int_{-1}^{2} (-2x^2 + 2x + 4)\ dx \\
&= \left[-\frac{1}{3}x^3 + x^2 + 4x \right]_{-1}^{2} \\
&= -\frac{16}{3} + 4 + 8 - \left(+\frac{2}{3} + 1 - 4 \right) \\
&= 9
\end{aligned}
$$

＜ページ35．面積2＞

解答1

$$S = \int_a^b \{f(x) - g(x)\}\, dx$$

解答2

$$\begin{aligned} S &= \int_{-1}^{1} \left\{(-x^2 + 2x + 1) - (x^2 + 2x - 1)\right\} dx \\ &= \int_{-1}^{1} \{-2x^2 + 2\}\ dx \\ &= \left[-\frac{2}{3}x^3 + 2x \right]_{-1}^{1} \\ &= -\frac{2}{3} + 2 - \left(+\frac{2}{3} - 2 \right) \\ &= \frac{8}{3} \end{aligned}$$

＜ ページ36．体積１ ＞

解答

$$
\begin{aligned}
V = \lim_{n\to\infty} V_n &= \lim_{n\to\infty} \frac{5^2\times 7}{12}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) \\
&= \frac{5^2\times 7}{12}\times 2 \\
&= \frac{175}{6}
\end{aligned}
$$

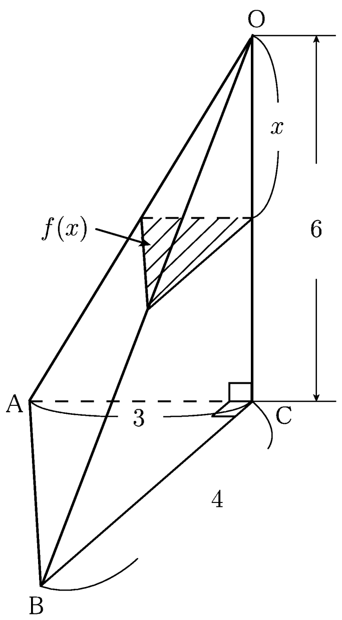
＜ ページ37．体積2 ＞

解答

$$f(x) : \frac{1}{2} \times 3 \times 4 = x^2 : 6^2$$

$$f(x) = \frac{1}{6}x^2$$

$$V = \int_0^6 f(x)\ dx = \int_0^6 \frac{1}{6}x^2\ dx = \left[\frac{1}{18}x^3\right]_0^6 = \frac{6^3}{18} = 12$$



＜ ページ38．体積3 ＞

解答1

$$f(x) = \pi \times \left(\frac{x}{3}\right)^2 = \frac{\pi}{9}x^2$$

$$, V = \int_0^6 \frac{\pi}{9}x^2\, dx = \left[\frac{\pi}{27}x^3\right]_0^6 = 8\pi$$

解答2

$$f(x) = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$$

$$, V = \int_0^8 \frac{x^2}{4}\, dx = \left[\frac{x^3}{12}\right]_0^8 = \frac{8^3}{12} = \frac{128}{3}$$

＜ ページ39．体積4 ＞

解答

$$f(x) = \pi\left(\sqrt{r^2 - x^2}\right)^2 = \pi\left(r^2 - x^2\right)$$

$$\begin{aligned} V &= \int_{-r}^{r} \pi(r^2 - x^2)dx \\ &= \left[\pi r^2 x - \frac{\pi}{3}x^3\right]_{-r}^{r} \\ &= \pi r^3 - \frac{\pi}{3}r^3 - \left(-\pi r^3 + \frac{\pi}{3}r^3\right) \\ &= 2\pi r^3 - \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3 \end{aligned}$$

< ページ 40. 重量と重心 >

解答

$$M = \int_0^2 \left(-x^2 + 2x\right)\, dx = \left[-\frac{x^3}{3} + x^2\right]_0^2 = -\frac{8}{3} + 4 = \frac{4}{3}$$

$$g = \frac{1}{\frac{4}{3}} \int_0^2 (-x^3 + 2x^2) dx$$

$$= \frac{3}{4}\left[-\frac{x^4}{3} + \frac{2}{3}x^3\right]_0^2$$

$$= \frac{3}{4}\left\{-\frac{16}{4} + \frac{16}{3}\right\}$$

$$= 1$$