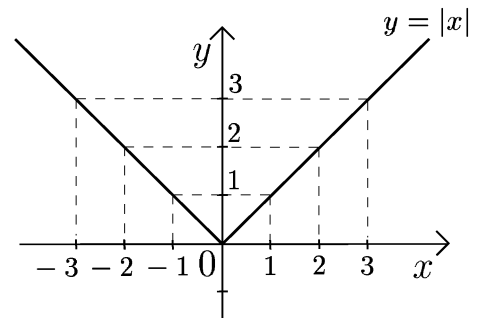


< ページ1. 絶対値 >

解答1

x	-3	-2	-1	0	1	2	3
y	3	2	1	0	1	2	3



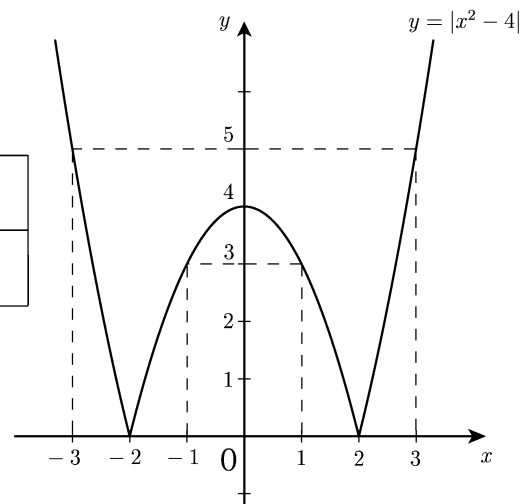
$x \geq 0$ の範囲では、直線 $y = \boxed{x}$ であり

$x < 0$ の範囲では、直線 $y = \boxed{-x}$ であることから、

$$y = |x| = \begin{cases} \boxed{x} & (x \geq 0) \\ \boxed{-x} & (x < 0) \end{cases} \text{ 分かる。}$$

解答2

x	-3	-2	-1	0	1	2	3
y	5	0	3	4	3	0	5



$$y = |x^2 - 4| = \begin{cases} \boxed{x^2 - 4} & (\boxed{2} \leq x) \\ \boxed{-x^2 + 4} & (\boxed{-2} < x < \boxed{2}) \\ \boxed{x^2 - 4} & (x \leq \boxed{-2}) \end{cases}$$

で、表わされる。グラフをよく見ると、このグラフは2次関数

$$y = \boxed{x^2 - 4}$$

< ページ 2. ガウス記号 >

解答 1

(1) $[3.27] = 3$

(2) $[9.787] = 9$

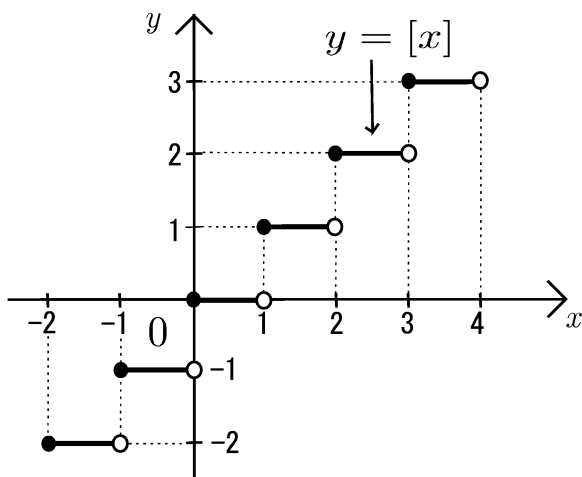
(3) $[0.9] = 0$

(4) $[-0.61] = -1$

(5) $[-5.73] = -6$

(6) $[-1.5] = -2$

解答 2



< ページ 3. 左極限・右極限 1 >

解答

$$(1) \lim_{x \rightarrow 3-0} [x] = 2$$

$$(2) \lim_{x \rightarrow 3+0} [x] = 3$$

$$(2) \lim_{x \rightarrow -0} [x] = -1$$

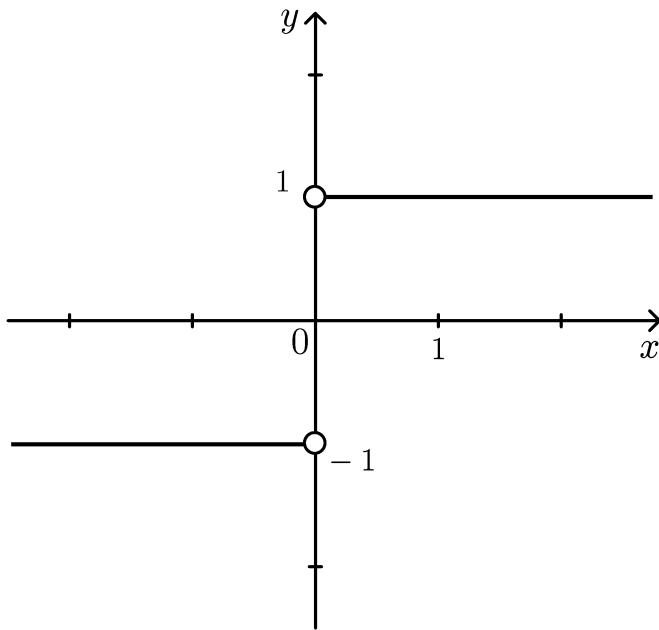
$$(4) \lim_{x \rightarrow +0} [x] = 0$$

< ページ 4. 左極限・右極限 2 >

解答

$$(1) \lim_{x \rightarrow -0} \frac{|x|}{x} = -1$$

$$(2) \lim_{x \rightarrow +0} \frac{|x|}{x} = 1$$



< ページ5. 三角関数の極限1 >

解答1

$$S_1 = \frac{1}{2} \times 1 \times \sin \theta = \frac{\sin \theta}{2}$$

解答2

$$S_2 = \frac{1}{2} \times 1 \times \tan \theta = \frac{1}{2} \tan \theta$$

解答3

$$\frac{1}{2} \sin \theta < \frac{1}{2} \theta < \frac{1}{2} \tan \theta$$

解答4

$$\boxed{\sin \theta} < \theta < \boxed{\frac{\sin \theta}{\cos \theta}}$$

< ページ 6. 三角関数の極限 2 >

解答 1

$$(**) \quad \boxed{1} < \frac{\theta}{\sin \theta} < \boxed{\frac{1}{\cos \theta}}$$

解答 2

$$\lim_{\theta \rightarrow +0} \frac{\theta}{\sin \theta} = 1$$

解答 3

$$\lim_{\theta \rightarrow +0} \frac{\sin \theta}{\theta} = \lim_{\theta \rightarrow +0} \frac{1}{\frac{\sin \theta}{\theta}} = \frac{1}{1} = 1$$

解答 4

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

< ページ 7. 三角関数の極限 3 >

解答

$$\begin{aligned} & \lim_{\theta \rightarrow 0} \frac{\cos(x + \theta) - \cos x}{\theta} \\ &= \lim_{\theta \rightarrow 0} \frac{\cos x \cos \theta - \sin x \sin \theta - \cos x}{\theta} \\ &= \lim_{\theta \rightarrow 0} \frac{\cos x(\cos \theta - 1) - \sin x \sin \theta}{\theta} \\ &= \lim_{\theta \rightarrow 0} \left\{ - \left(\frac{1 - \cos \theta}{\theta} \right) \cos x - \left(\frac{\sin \theta}{\theta} \right) \sin x \right\} \\ &= -0 \times \cos x - 1 \times \sin x = -\sin x \end{aligned}$$

< ページ 8. 三角関数の微分 >

解答 1

$$\begin{aligned}(\cos x)' &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \\ &= \lim_{\theta \rightarrow 0} \frac{\cos(x+\theta) - \cos x}{\theta} \\ &= -\sin x\end{aligned}$$

解答 2

$$\begin{aligned}(1) \quad (5 \sin x - 4 \cos x)' &= 5 \times (\sin x)' - 4 \times (\cos x)' \\ &= 5 \cos x + 4 \sin x\end{aligned}$$

$$(2) \quad (6 + 3x + x^2 - 2 \sin x + 5 \cos x)' = 3 + 2x - 2 \cos x - 5 \sin x$$

< ページ 9. 積の微分 1 >

解答

$$(f(x) \times g(x))' = f'(x)g(x) + f(x)g'(x)$$

< ページ 10. 積の微分 2 >

解答

$$(1) (x \sin x)' = (x)' \times \sin x + x \times (\sin x)' = \sin x + x \cos x$$

$$(2) (x^2 \cos x)' = 2x \cos x - x^2 \sin x$$

$$(3) (\sin^2 x)' = (\sin x \times \cos x)' = \cos x \sin x + \sin x \cos x \\ = 2 \sin x \cos x$$

< ページ 11. 商の微分 1 >

解答

$$\left(\frac{1}{g(x)}\right)' = -\frac{g'(x)}{(g(x))^2}$$

< ページ 12. 商の微分 2 >

解答

$$(1) \quad \left(\frac{1}{x}\right)' = -\frac{(x)'}{(x)^2} = -\frac{1}{x^2}$$

$$(2) \quad \left(\frac{1}{x^2}\right)' = \frac{(x^2)'}{(x^2)^2} = -\frac{2x}{x^4} = -\frac{2}{x^3}$$

$$(3) \quad \left(\frac{1}{x^3}\right)' = \frac{(x^3)'}{(x^3)^2} = -\frac{3x^2}{x^6} = -\frac{3}{x^4}$$

$$(4) \quad \left(\frac{1}{\cos x}\right)' = -\frac{-\sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$$

< ページ 13. 分数関数の微分 >

解答

$$\begin{aligned}(1) \quad \left(\frac{x}{\cos x}\right)' &= \left(x \times \frac{1}{\cos x}\right)' \\ &= 1 \times \frac{1}{\cos x} + x \times \left(-\frac{\sin x}{\cos^2 x}\right) \\ &= \frac{\cos x + x \sin x}{\cos^2 x}\end{aligned}$$

$$\begin{aligned}(2) \quad \left(\frac{\cos x}{\sin x}\right)' &= \left(\cos x \times \frac{1}{\sin x}\right)' \\ &= -\sin x \times \frac{1}{\sin x} + \cos x \times \left(-\frac{\cos x}{\sin^2 x}\right) \\ &= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \\ &= -\frac{1}{\sin^2 x}\end{aligned}$$

< ページ 14. 速度 >

解答 1

$$\frac{f(2) - f(1)}{2 - 1} = \frac{19.6 - 4.9}{1} = 14.7 \text{ (m/s)}$$

解答 2

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{4.9 \times (2+h)^2 - 4.9 \times 2^2}{h} \\ &= \lim_{h \rightarrow 0} \{4.9 \times (4+h)\} \\ &= 19.6 \end{aligned}$$

解答 3

$$\begin{aligned} f'(t) &= \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} = \lim_{h \rightarrow 0} \frac{4.9 \times (t+h)^2 - 4.9 \times t^2}{h} \\ &= \lim_{h \rightarrow 0} \{4.9 \times (2t+h)\} \\ &= 9.8t \end{aligned}$$

< ページ 15. 微分記号 >

解答

$$(1) \quad y = 3x^2 - 4x + 5 \qquad \frac{dy}{dx} = 6x - 4$$

$$(2) \quad y = 4 - 9.8t \qquad \frac{dy}{dt} = -9.8$$

$$(3) \quad \ell = 5t^2 - 3t \qquad \frac{d\ell}{dt} = 10t - 3$$

$$(4) \quad S = \pi r^2 \quad (\pi \text{ は円周率}) \qquad \frac{dS}{dr} = 2\pi r$$

$$(5) \quad V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dr} = 4\pi r^2$$

< ページ 16. 増分記号 Δ (デルタ) >

解答

$$(1) \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^5 - x^5}{\Delta x} = (x^5)' = 5x^4$$

$$(2) \lim_{\Delta t \rightarrow 0} \frac{\sin(t + \Delta t) - \sin(t)}{\Delta t} = (\sin t)' = \cos t$$

$$(3) \lim_{\Delta u \rightarrow 0} \frac{\cos(u + \Delta u) - \cos(u)}{\Delta u} = (\cos u)' = -\sin u$$

< ページ 17. 合成関数 >

解答 1

$$(1) f(x) = x^2 - 3, g(x) = 2x \quad , g(f(x)) = 2(x^2 - 3) \\ , f(g(x)) = 4x^2 - 3$$

$$(2) f(x) = \cos x, g(x) = 2x + 3 \quad , g(f(x)) = 2(\cos x) + 3 \\ , f(g(x)) = \cos(2x + 3)$$

$$(3) f(x) = \sqrt{x}, g(x) = x^2 + 1 \quad , g(f(x)) = x + 1 \\ , f(g(x)) = \sqrt{x^2 + 1}$$

$$(4) f(x) = x^4 + 3, g(x) = \log_2 x \quad , g(f(x)) = \log_2(x^4 + 3) \\ , f(g(x)) = (\log_2 x)^4 + 3$$

解答 2

$$(1) f(x) = (x^2 + 2x + 3)^5 \quad , f(x) = x^2 + 2x + 3 \\ , g(x) = x^5$$

$$(2) f(x) = \sin(4x + 3) \quad , f(x) = 4x + 3 \\ , g(x) = \sin x$$

$$(3) f(x) = \sqrt{4 - x^2} \quad , f(x) = 4 - x^2 \\ , g(x) = \sqrt{x}$$

< ページ 18. 合成関数の微分 1 >

解答

$$\begin{aligned} \text{(解)} \quad \frac{dy}{dx} &= \left(\lim_{\Delta u \rightarrow 0} \frac{\cos(u + \Delta u) - \cos u}{\Delta u} \right) \times \left(\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x} \right) \\ &= (\cos u)' \times (x^4)' \\ &= -\sin u \times 4x^3 \\ &= -4x^3 \sin(x^4) \end{aligned}$$

< ページ 19. 合成関数の微分 2 >

解答 1

(答)
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

解答 2

(1) $u = x^2 + 3x - 1$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = (u^5)' \times (x^2 + 3x - 1)' \\ &= 5u^4 \times (2x + 3) \\ &= 5(x^2 + 3x - 1)^4 (2x + 3) \end{aligned}$$

(2) $u = 3x + 4$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = (\cos u)' \times (3x + 4)' \\ &= -\sin u \times 3 \\ &= -3 \sin(3x + 4) \end{aligned}$$

(3) $u = x^3 + 5x^2$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = (\sin u)' \times (x^3 + 5x^2)' \\ &= \cos u \times (3x^2 + 10x) \\ &= \cos(x^3 + 5x^2)(3x^2 + 10x) \end{aligned}$$

< ページ 20. ネピアの数 1 >

解答

$$\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

< ページ 21. ネピアの数 2 >

解答

$$(1) \lim_{h \rightarrow 0} \frac{e^{3+h} - e^3}{h} = \lim_{h \rightarrow 0} \frac{e^3 (e^h - 1)}{h} = e^3$$

$$(2) \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x (e^h - 1)}{h} = e^x$$

< ページ 22. 指数関数の微分 >

解答 1

$$(1) \log e = \log_e e = 1 \quad (2) \log(e^2) = 2 \quad (3) \log 1 = 0$$

解答 2

$$(1) (3^x)' = 3^x \log 3 \quad (2) (4^x)' = 4^x \log 4$$

< ページ 23. 逆関数の微分 >

解答

$$(1) y = \cos^{-1} x \Leftrightarrow x = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\cos y)'} = \frac{1}{-\sin y} = \frac{1}{-\sqrt{1 - \cos^2 y}} = -\frac{1}{\sqrt{1 - x^2}}$$

$$(2) y = \tan^{-1} x \Leftrightarrow x = \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\tan y)'} = \frac{1}{\frac{1}{\cos^2 y}} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

< ページ 25. 対数関数の微分 2 >

解答 1

(1) $u = x^4 + 3x + 1$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = (\log u)' \times (x^4 + 3x + 1)' \\ &= \frac{1}{u} \times (4x^3 + 3) \\ &= \frac{4x^3 + 3}{x^4 + 3x + 1}\end{aligned}$$

(2) $\frac{dy}{dx} = \frac{\cos x}{2 + \sin x}$

(3) $\frac{dy}{dx} = \frac{\sin x}{3 - \cos x}$

解答 2

(答) $\left(\log(f(x))\right)' = \frac{f'(x)}{f(x)}$

< ページ 26. 対数微分法 >

解答 1

(解) $\log y = \log 3^x = x \log 3$

$$\frac{y'}{y} = \log 3 \Rightarrow y' = y \times \log 3 = 3^x \log 3$$

解答 2

(答) $(a^x)' = a^x \log a$

解答 3

(答) $(e^x)' = e^x \log e = e^x$

< ページ 27. x^r の微分 >

解答

(解) $\log y = r \log x$

$$\frac{y'}{y} = \frac{r}{x} \Rightarrow y' = y \times \frac{r}{x} = x^r \times \frac{r}{x} = rx^{r-1}$$

(答) $(x^r)' = rx^{r-1}$

解答 2

$$(1) \left(\sqrt[3]{x^4}\right)' = \left(x^{\frac{4}{3}}\right)' = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3}\sqrt[3]{x}$$

$$(2) (\sqrt{x})' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$(3) \left(\frac{1}{\sqrt{x}}\right)' = \left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x\sqrt{x}}$$

$$(4) (x\sqrt{x})' = \left(x^{\frac{3}{2}}\right)' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

< ページ 28. 微分の練習 >

解答 1

$$(1) (k)' = 0 \quad (k \text{ は定数})$$

$$(2) (x^r)' = rx^{r-1} \quad (r \text{ は定数})$$

$$(3) (\sin x)' = \cos x$$

$$(4) (\cos x)' = -\sin x$$

$$(5) (\tan x)' = \frac{1}{\cos^2 x}$$

$$(6) (\log x)' = \frac{1}{x}$$

$$(7) (a^x)' = a^x \log a \quad (a > 0, a \neq 1)$$

$$(8) (e^x)' = e^x$$

$$(9) (f(x) \times g(x))' \\ = f'(x)g(x) + f(x)g'(x) \quad (\text{積の微分})$$

$$(10) \left(\frac{1}{g(x)}\right)' = -\frac{g'(x)}{(g(x))^2} \\ (\text{商の微分})$$

解答 2

$$(1) y = e^{-x^2} \quad \frac{dy}{dx} = -2xe^{-x^2}$$

$$(2) y = (2 + \sin x)^4 \quad \frac{dy}{dx} = 4 \cos x (2 + \sin x)^3$$

< ページ 29. $\log |x|$ の微分 >

解答

$$(1) y = \log |\sin x|, \quad \frac{dy}{dx} = \frac{\cos x}{\sin x}$$

$$(2) y = \log |x^3 - 3x|, \quad \frac{dy}{dx} = \frac{3x^2 - 3}{x^3 - 3x}$$

$$(3) y = \log |f(x)|, \quad \frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

< ページ 30. 原始関数 >

解答

$$(1) x^4 \text{ の原始関数の一般形} = \frac{1}{5}x^5 + C$$

$$(2) x^5 \text{ の原始関数の一般形} = \frac{1}{6}x^6 + C$$

$$(3) x^6 \text{ の原始関数の一般形} = \frac{1}{7}x^7 + C$$

< ページ 31. 不定積分 1 >

解答 1

$$(1) \int x^4 dx = \frac{1}{5}x^5 + C$$

$$(2) \int x^5 dx = \frac{1}{6}x^6 + C$$

$$(3) \int x^6 dx = \frac{1}{7}x^7 + C$$

解答 2

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

解答 3

$$n \neq -1$$

< ページ 32. 不定積分 2 >

解答 1

$$(1) \int x dx = \frac{1}{2}x^2 + C$$

$$(2) \int 1 dx = x + C$$

$$(3) \int \sqrt{x} dx = \frac{2}{3}x^{\frac{3}{2}} + C = \frac{2}{3}x\sqrt{x} + C$$

$$(4) \int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

解答 2

$$\int x^{-1} dx = \int \frac{1}{x} dx = \log |x| + C$$

< ページ 33. 不定積分 3 >

解答 1

$$(1) \int (-\sin x) dx = \cos x + C$$

$$(2) \int \frac{1}{\cos^2 x} dx = \tan x + C$$

$$(3) \int e^x dx = e^x + C$$

$$(4) \int a^x \log a dx = a^x + C$$

$$(5) \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

解答 2

$$(1) \int \sin x dx = -\cos x + C$$

$$(2) \int a^x dx = \frac{1}{\log a} a^x + C$$

< ページ 34. 不定積分 4 >

解答 1

$$(1) \int 5x^2 dx = \frac{5}{3}x^3 + C \quad (2) \int (3 - 6x) dx = 3x - 3x^2 + C$$

$$(3) \int (6x^2 - 8x + 1) dx = 2x^3 - 4x^2 + x + C$$

解答 2

$$\begin{aligned} & \int (6x^2 - 6x - 3) dx + 2 \int (-3x^2 + 4x - 1) dx \\ &= \int (6x^2 - 6x - 3 - 6x^2 + 8x - 2) dx \\ &= \int (2x - 5) dx = x^2 - 5x + C \end{aligned}$$

< ページ 35. 不定積分 5 >

解答 1

$$(1) \int (3x + 2)^5 dx = \frac{1}{18}(3x + 2)^6 + C$$

$$(2) \int (4x - 3)^7 dx = \frac{1}{32}(4x - 3)^8 + C$$

解答 2

$$(1) \int (ax + b)^3 dx = \frac{1}{4a}(ax + b)^4 + C$$

$$(2) \int (ax + b)^n dx = \frac{1}{(n + 1)a}(ax + b)^{n+1} + C$$

解答 3

$$(1) \int \sqrt{4x - 3} dx = \int (4x - 3)^{\frac{1}{2}} dx = \frac{1}{4 \left(\frac{1}{2} + 1 \right)} (4x - 3)^{\frac{3}{2}} + C \\ = \frac{1}{6} (4x - 3) \sqrt{4x - 3} + C$$

$$(2) \int \frac{1}{(3x + 1)^2} dx = \int (3x + 1)^{-2} dx = \frac{1}{3(-2 + 1)} (3x + 1)^{-1} + C \\ = -\frac{1}{3(3x + 1)} + C$$

< ページ 36. 不定積分 6 >

解答 1

$$(1) \int \cos(4x - 1) dx = \frac{1}{4} \sin(4x - 1) + C$$

$$(2) \int \sin(3x + 2) dx = -\frac{1}{3} \cos(3x + 2) + C$$

$$(3) \int \frac{1}{2x - 1} dx = \frac{1}{2} \log |2x - 1| + C$$

$$(4) \int e^{3x+1} dx = \frac{1}{3} e^{3x+1} + C$$

解答 2

$$(1) \int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C$$

$$(2) \int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$$

$$(3) \int \frac{1}{ax + b} dx = \frac{1}{a} \log |ax + b| + C$$

$$(4) \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

< ページ 37. 積分記号 >

解答

$$(1) \int (10 - 9.8t)dt = 10t - 4.9t^2 + C$$

$$(2) \int 4\pi r^2 dr = \frac{4}{3}\pi r^3 + C$$

$$(3) \int e^u du = e^u + C$$

$$(4) \int \frac{1}{y} dy = \log |y| + C$$

$$(5) \int \cos u du = \sin u + C$$

< ページ 38. 置換積分法 1 >

解答

(1) $u = x^4$

$$\begin{aligned}\int \cos(x^4) \times 4x^3 dx &= \int \cos(u) \frac{du}{dx} dx = \int \cos u du = \sin u + C \\ &= \sin(x^4) + C\end{aligned}$$

(2) $\int \cos(x^5) \times 5x^4 dx = \sin(x^5) + C$

(3) $\int \cos(g(x)) \times g'(x) dx = \sin(g(x)) + C$

< ページ 39. 置換積分法 2 >

解答 1

$$(1) \int \sin(x^3) \times 3x^2 dx = -\cos(x^3) + C$$

$$(2) \int \sin(g(x)) \times g'(x) dx = -\cos(g(x)) + C$$

解答 2

$$(1) \int e^{g(x)} \times g'(x) dx = e^{g(x)} + C$$

$$(2) \int \frac{1}{g(x)} \times g'(x) dx = \log |g(x)| + C$$

$$(3) \int \{g(x)\}^n \times g'(x) dx = \frac{1}{n+1} \{g(x)\}^{n+1} + C$$

解答 3

$$u = g(x)$$

$$\int f(g(x)) \times g'(x) dx = \int f(u) \frac{du}{dx} dx = \int f(u) du$$

< ページ 40. 置換積分法 3 >

解答

$$(1) \int x (x^2 + 1)^5 dx = \frac{1}{12}(x^2 + 1)^6 + C$$

$$(2) \int x^3 \cos (x^4 + 6) dx = \frac{1}{4} \sin (x^4 + 6) + C$$

$$(3) \int \frac{x^2}{x^3 + 1} dx = \frac{1}{3} \log |x^3 + 1| + C$$

$$(4) \int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\log |\cos x| + C$$