＜ 1ページ. 三角関数 ＞

問1の解答

度数法	$-720°$	$-540°$	$-360°$	$-180°$	$-90°$	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$	$540°$	$720°$	$900°$
弧度法	-4π	-3π	-2π	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π	3π	4π	5π
$\sin\theta$	0	0	0	0	-1	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0	0	0	0
$\cos\theta$	1	-1	1	-1	0	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1	-1	1	-1

問2の解答

(1) $\sin\left(-\frac{3}{2}\pi\right) = \sin\left(\frac{\pi}{2}\right) = 1$

(2) $\cos\left(\frac{\pi}{2} + \frac{5}{6}\pi\right) = -\sin\left(\frac{5}{6}\pi\right) = -\frac{1}{2}$

(3) $\sin\left(\frac{\pi}{6} + 8\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

(4) $\cos\left(\frac{2}{3}\pi - 4\pi\right) = \cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$

(5) $\sin\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

(6) $\cos\left(\frac{\pi}{4} + 3\pi\right) = \cos\left(\frac{\pi}{4} + \pi\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

< 2ページ. 加法定理1 >

問1の解答

(1) $\cos\left(\theta - \frac{\pi}{2}\right) = \cos\theta\cos\left(\frac{\pi}{2}\right) + \sin\theta\sin\left(\frac{\pi}{2}\right) = \sin\theta$

(2) $\sin\left(\theta - \frac{\pi}{2}\right) = \sin\theta\cos\left(\frac{\pi}{2}\right) - \cos\theta\sin\left(\frac{\pi}{2}\right) = -\cos\theta$

問2の解答

(1)
$$\sin\left(\theta + \frac{\pi}{6}\right) = \sin\theta\cos\left(\frac{\pi}{6}\right) + \cos\theta\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$$

(2)
$$\sin\left(\theta + \frac{\pi}{4}\right) = \sin\theta\cos\left(\frac{\pi}{4}\right) + \cos\theta\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin\theta + \cos\theta)$$

(3)
$$\sin\left(\theta + \frac{2}{3}\pi\right) = \sin\theta\cos\left(\frac{2\pi}{3}\right) + \cos\theta\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$$

(4)
$$\sin\left(\theta - \frac{\pi}{6}\right) = \sin\theta\cos\left(\frac{\pi}{6}\right) - \cos\theta\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta$$

＜ 3ページ. 加法定理 2 ＞

解答

(1) $\sin\theta + \cos\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

(2) $\sqrt{3}\cos\theta + \sin\theta = 2\sin\left(\theta + \frac{\pi}{3}\right)$

(3) $\cos\theta - \sin\theta = \sqrt{2}\sin\left(\theta + \frac{3}{4}\pi\right)$

(4) $-4\cos\theta - 4\sqrt{3}\sin\theta = 8\sin\left(\theta + \frac{7}{6}\pi\right)$

$<$ 4ページ. 加法定理3 $>$

問1の解答

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

問2の解答

$$\begin{aligned}
\cos(3\theta) &= \cos(2\theta)\cos\theta - \sin(2\theta)\sin\theta \\
&= \left(2\cos^2\theta - 1\right)\cos\theta - 2\sin^2\theta\cos\theta \\
&= 2\cos^3\theta - \cos\theta - 2\left(1-\cos^2\theta\right)\cos\theta \\
&= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \\
&= 4\cos^3\theta - 3\cos\theta
\end{aligned}$$

＜ 5ページ. 積和公式 ＞

問1の解答

(1) $\sin\theta\cos\theta = \dfrac{1}{2}\sin(2\theta)$

(2) $\sin^2\theta = \dfrac{1}{2}\{1-\cos(2\theta)\}$

問2の解答

(1) $\sin(5t)\cos(4t) = \dfrac{1}{2}\{\sin(9t)+\sin(t)\}$

(2) $\sin(5t)\sin(4t) = \dfrac{1}{2}\{\cos(t)-\cos(9t)\}$

(3) $\cos(5t)\cos(4t) = \dfrac{1}{2}\{\cos(9t)+\cos(t)\}$

(4) $\sin(mt)\cos(nt) = \dfrac{1}{2}\Big\{\sin\big((m+n)\,t\big)+\sin\big((m-n)\,t\big)\Big\}$

(5) $\sin(mt)\sin(nt) = \dfrac{1}{2}\Big\{\cos\big((m-n)\,t\big)-\cos\big((m+n)\,t\big)\Big\}$

(6) $\cos(mt)\cos(nt) = \dfrac{1}{2}\Big\{\cos\big((m+n)\,t\big)+\cos\big((m-n)\,t\big)\Big\}$

＜ 6ページ. 偶関数と奇関数１ ＞

解答

(1) $f(x) = x^4$ 偶関数

(2) $f(x) = x^5$ 奇関数

(3) $f(x) = x^6$ 偶関数

(4) $f(x) = x^7$ 奇関数

(5) $f(x) = \cos(2x)$ 偶関数

(6) $f(x) = \sin(2x)$ 奇関数

(7) $f(x) = \cos(3x)$ 偶関数

(8) $f(x) = \sin(3x)$ 奇関数

(9) $f(x) = \sin^2(x)$ 偶関数

＜ 7ページ. 偶関数と奇関数2 ＞

問1の解答

(1) $x^2 \times x^4$　偶関数

(2) $x^2 \times x^5$　奇関数

(3) $x^3 \times x^7$　偶関数

(4) $x \sin(2x)$　偶関数

(5) $x^2 \cos(3x)$　偶関数

(6) $x^3 \cos(5x)$　奇関数

(7) $\sin(2x)\sin(3x)$　偶関数

(8) $\sin(4x)\cos(3x)$　奇関数

(9) $\cos(2x)\cos(5x)$　偶関数

問2の解答

(1) 奇関数 $\times$ 奇関数 $=$ 偶関数

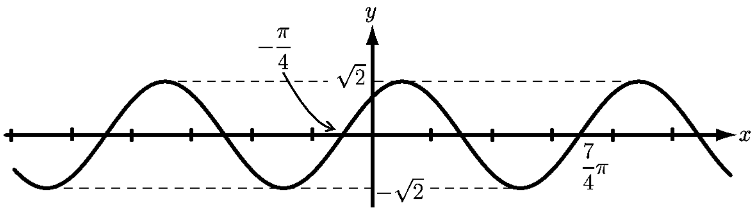
(2) 偶関数 $\times$ 偶関数 $=$ 偶関数

(3) 奇関数 $\times$ 偶関数 $=$ 奇関数

＜ 8 ページ. 正弦波のグラフ ＞

解答

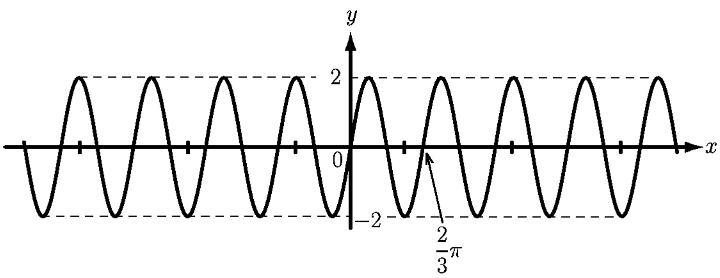
(1)



周期 2π ，　振幅 $\sqrt{2}$ ，　初期位相 $-\frac{\pi}{4}$

$$y = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

(2)



周期 $\frac{2\pi}{3}$ ，　振幅 2 ，　初期位相 0

$$y = 2\sin(3x)$$

＜ 9 ページ. 同周期正弦波の和 ＞

解答

(1) $\sin x + \cos x = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$

周期 2π ， 振幅 $\sqrt{2}$ ， 初期位相 $-\dfrac{\pi}{4}$

(2) $\sqrt{3}\sin(2x) + \cos(2x) = 2\sin\left(2x + \dfrac{\pi}{6}\right)$

周期 π ， 振幅 2 ， 初期位相 $-\dfrac{\pi}{12}$

(3) $\sin(3x) - \cos(3x) = \sqrt{2}\sin\left(3x - \dfrac{\pi}{4}\right)$

周期 $\dfrac{2\pi}{3}$ ， 振幅 $\sqrt{2}$ ， 初期位相 $\dfrac{\pi}{12}$

＜ 10 ページ. 異周期正弦波の和 ＞

解答

(1) $\cos(x) + \cos(3x)$ ， 周期 2π

(2) $\sin(2x) + \cos(5x)$ ， 周期 2π

(3) $\cos(3x) + \sin(6x)$ ， 周期 $\dfrac{2}{3}\pi$

$<$ 11ページ.周期 2π の関数１ $>$

解答

図 4：$y = 3 + \cos(2x) + \sin(3x)$

図 5：$y = 2 + \sin(2x) + \cos(2x)$

図 6：$y = -2 + \sin x + \sin(2x)$

< 12 ページ. 周期 2π の関数 2 >

解答

(1) $\sin^2 x = \dfrac{1}{2}\{1 - \cos(2x)\} = \dfrac{1}{2} - \dfrac{1}{2}\cos(2x)$

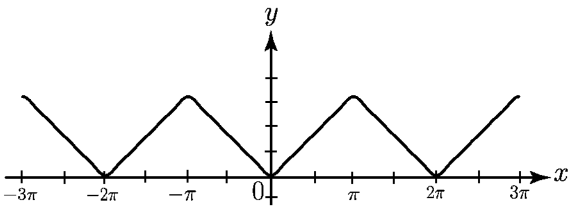
(2) $\cos^3 x = \dfrac{3}{4}\cos x + \dfrac{1}{4}\cos(3x)$

＜ 13ページ. 周期 2π の問題 3 ＞

解答

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\cos x - \frac{4}{9\pi}\cos(3x) - \frac{4}{25\pi}\cos(5x) - \frac{4}{49\pi}\cos(7x)$$

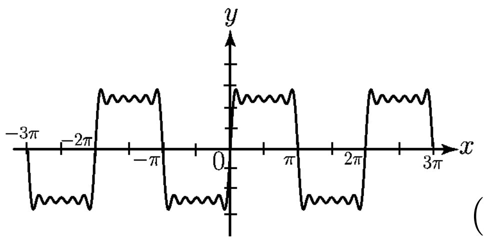
$f(x)$ のグラフ $\cdots$ 図 6



(図 6)

$$g(x) = 3\sin x + \sin(3x) + \frac{3}{5}\sin(5x) + \frac{3}{7}\sin(7x) + \frac{1}{3}\sin(9x) + \frac{3}{11}\sin(11x)$$

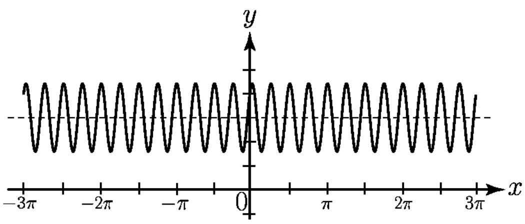
$g(x)$ のグラフ $\cdots$ 図 5



(図 5)

$$h(x) = 3 + \cos(8x) + \sin(8x)$$

$h(x)$ のグラフ $\cdots$ 図 4

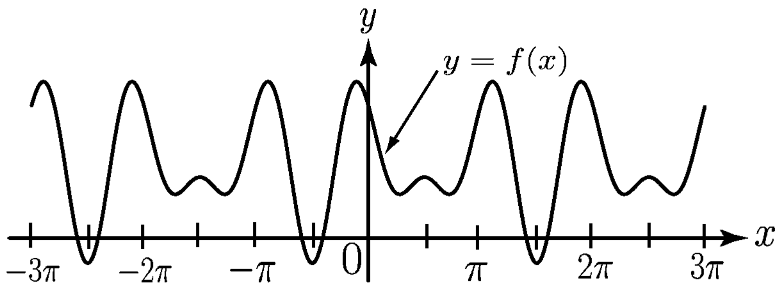


(図 4)

＜ 14 ページ. 周期 2π の関数 4 ＞

解答

$$f(x) = 2.1 + 1.6\cos(2x) - 1.2\sin(3x)$$



(図 3)

＜ 15 ページ. 積分 1 ＞

問 1 の解答

(1) $\displaystyle\int \cos(4x)dx = \frac{1}{4}\sin(4x) + C$

(2) $\displaystyle\int \sin(5x)dx = -\frac{1}{5}\cos(5x) + C$

(3) $\displaystyle\int \cos(nx)dx = \frac{1}{n}\sin(nx) + C$

(4) $\displaystyle\int \sin(nx)dx = -\frac{1}{n}\cos(nx) + C$

＜ 15 ページ. 積分 1 ＞

問 2 の解答

(1) $$\int \sin^2 x dx = \int \frac{1}{2}\big(1 - \cos(2x)\big) dx = \frac{x}{2} - \frac{1}{4}\sin(2x) + C$$

(2) $$\int \cos^2(2x) dx = \int \frac{1}{2}\big(1 + \cos(4x)\big) dx = \frac{x}{2} + \frac{1}{8}\sin(4x) + C$$

(3) $$\int \sin^2(3x) dx = \int \frac{1}{2}\big(1 - \cos(6x)\big) dx = \frac{x}{2} - \frac{1}{12}\sin(6x) + C$$

(4) $$\int \sin(3x)\sin(2x) dx$$
$$= \int \frac{1}{2}\left\{\cos x - \cos(5x)\right\} dx = \frac{1}{2}\sin x - \frac{1}{10}\sin(5x) + C$$

(5) $$\int \cos(2x)\cos(4x) dx$$
$$= \int \frac{1}{2}\left\{\cos(6x) + \cos(2x)\right\} dx = \frac{1}{12}\sin(6x) + \frac{1}{4}\sin(2x) + C$$

(6) $$\int \sin(4x)\cos(5x) dx$$
$$= \int \frac{1}{2}\left\{\sin(9x) - \sin x\right\} dx = -\frac{1}{18}\cos(9x) + \frac{1}{2}\cos x + C$$

＜ 16 ページ. 積分 2 ＞

解答

(1) $$\int_{-\pi}^{\pi} \sin^2 x dx = \int_{-\pi}^{\pi} \left\{ \frac{1}{2} - \frac{1}{2}\cos(2x) \right\} dx = \left[\frac{1}{2}x - \frac{1}{4}\sin(2x) \right]_{x=-\pi}^{x=\pi} = \pi$$

(2) $$\int_{-\pi}^{\pi} \cos^2(3x) dx = \int_{-\pi}^{\pi} \left\{ \frac{1}{2} + \frac{1}{2}\cos(6x) \right\} dx = \left[\frac{1}{2}x + \frac{1}{12}\sin(6x) \right]_{x=-\pi}^{x=\pi} = \pi$$

(3) $$\int_{-\pi}^{\pi} \sin^2(4x) dx = \int_{-\pi}^{\pi} \left\{ \frac{1}{2} - \frac{1}{2}\cos(8x) \right\} dx = \left[\frac{1}{2}x - \frac{1}{16}\sin(8x) \right]_{x=-\pi}^{x=\pi} = \pi$$

(4) $$\int_{-\pi}^{\pi} \sin x \sin(4x) dx = \int_{-\pi}^{\pi} \frac{1}{2} \left\{ \cos(3x) - \cos(5x) \right\} dx$$
$$= \left[\frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x) \right]_{x=-\pi}^{x=\pi} = 0$$

(5) $$\int_{-\pi}^{\pi} \cos(3x)\cos(4x) dx = \int_{-\pi}^{\pi} \frac{1}{2} \left\{ \cos(7x) + \cos x \right\} dx$$
$$= \left[\frac{1}{14}\sin(7x) + \frac{1}{2}\sin x \right]_{x=-\pi}^{x=\pi} = 0$$

(6) $$\int_{-\pi}^{\pi} \sin x \cos(4x) dx = \int_{-\pi}^{\pi} \frac{1}{2} \left\{ \sin(5x) - \sin(3x) \right\} dx$$
$$= \left[-\frac{1}{10}\cos(5x) + \frac{1}{6}\cos(3x) \right]_{x=-\pi}^{x=\pi} = 0$$

＜ 17ページ. 積分3 ＞

解答

(1) (奇関数の積分より) $\displaystyle\int_{-\pi}^{\pi} x^2\sin(3x)dx = 0$

(2) (奇関数の積分より) $\displaystyle\int_{-\pi}^{\pi} \cos(x)\sin(2x)dx = 0$

(3) (偶関数の積分より)

$$\int_{-\pi}^{\pi} \sin(4x)\sin(3x)dx$$

$$= 2\int_{0}^{\pi} \sin(4x)\sin(3x)dx$$

$$= 2\int_{0}^{\pi} \frac{1}{2}\left\{\cos x - \cos(7x)\right\} dx$$

$$= \left[\sin x - \frac{1}{7}\sin(7x)\right]_0^{\pi}$$

$$= \left\{\sin\pi - \frac{1}{7}\sin(7\pi)\right\} - \left\{\sin 0 - \frac{1}{7}\sin 0\right\}$$

$$= 0$$

＜ 18 ページ. 積分 4 ＞

問 1 の解答

(1) $\displaystyle\int_{-\pi}^{\pi}\cos^2(4x)dx = 2\int_0^{\pi}\cos^2(4x)dx$

$\displaystyle = \int_0^{\pi}\big(1+\cos(8x)\big)dx = \left[x+\frac{1}{8}\sin(8x)\right]_0^{\pi} = \pi$

(2) $\displaystyle\int_{-\pi}^{\pi}\sin^2(5x)dx = 2\int_0^{\pi}\sin^2(5x)dx$

$\displaystyle = \int_0^{\pi}\big(1-\cos(10x)\big)dx = \left[x-\frac{1}{10}(10x)\right]_0^{\pi} = \pi$

(3) $\displaystyle\int_{-\pi}^{\pi}\sin(3x)\sin(4x)dx = 2\int_0^{\pi}\sin(3x)\sin(4x)dx$

$\displaystyle = \left[\sin x - \frac{1}{7}\sin(7x)\right]_0^{\pi} = 0$

(4) (奇関数の積分より) $\displaystyle\int_{-\pi}^{\pi}\sin(3x)\cos(4x)dx = 0$

(5) (奇関数の積分より) $\displaystyle\int_{-\pi}^{\pi}\sin(4x)\cos(4x)dx = 0$

(6) $\displaystyle\int_{-\pi}^{\pi}\cos(4x)\cos(5x)dx = 2\int_0^{\pi}\cos(4x)\cos(5x)dx$

$\displaystyle = \int_0^{\pi}\big(\cos(9x)+\cos x\big)dx = \left[\frac{1}{9}\sin(9x)+\sin x\right]_0^{\pi} = 0$

＜ 18ページ. 積分 4 ＞

問 2 の解答

(1) $\displaystyle\int_{-\pi}^{\pi}\cos^2(nx)dx = 2\int_0^{\pi}\cos^2(nx)dx = \left[x+\frac{1}{2n}\sin(2nx)\right]_0^{\pi} = \pi$

(2) $\displaystyle\int_{-\pi}^{\pi}\sin^2(nx)dx = 2\int_0^{\pi}\sin^2(nx)dx = \left[x-\frac{1}{2n}\sin(2nx)\right]_0^{\pi} = \pi$

(3) $\displaystyle\int_{-\pi}^{\pi}\sin(nx)\sin(mx)dx = 2\int_0^{\pi}\sin(nx)\sin(mx)dx$

$$= \left[\frac{\sin\big((n-m)x\big)}{n-m} - \frac{\sin\big((n+m)x\big)}{n+m}\right]_0^{\pi} = 0$$

(4) (奇関数の積分より) $\displaystyle\int_{-\pi}^{\pi}\sin(nx)\cos(nx)dx = 0$

(5) (奇関数の積分より) $\displaystyle\int_{-\pi}^{\pi}\sin(nx)\cos(mx)dx = 0$

(6) $\displaystyle\int_{-\pi}^{\pi}\cos(nx)\cos(mx)dx = 2\int_0^{\pi}\cos(nx)\cos(mx)dx$

$$= 2\int_0^{\pi}\frac{1}{2}\Big\{\cos\big((n+m)x\big)+\cos\big((n-m)x\big)\Big\}dx$$

$$= \left[\frac{\sin\big((n+m)x\big)}{n+m} + \frac{\sin\big((n-m)x\big)}{n-m}\right]_0^{\pi} = 0$$

＜ 19 ページ. 積分 5 ＞

解答

(1) $\displaystyle\int_0^{\pi} x\cos(4x)dx = \int_0^{\pi} x \times \left(\frac{1}{4}\sin(4x)\right)' dx = \left[\frac{x}{4}\sin(4x)\right]_0^{\pi} - \int_0^{\pi} \frac{1}{4}\sin(4x)dx$

$\displaystyle = 0 - \left[-\frac{1}{16}\cos(4x)\right]_0^{\pi} = \frac{1}{16}\cos(4\pi) - \frac{1}{16}\cos 0 = 0$

(2) $\displaystyle\int_0^{\pi} x\sin(4x)dx = \int_0^{\pi} x \times \left(-\frac{1}{4}\cos(4x)\right)' dx$

$\displaystyle = \left[-\frac{x}{4}\cos(4x)\right]_0^{\pi} - \int_0^{\pi}\left(-\frac{1}{4}\cos(4x)\right)dx$

$\displaystyle = -\frac{\pi}{4}\cos(4\pi) - 0 + \left[\frac{1}{16}\sin(4x)\right]_0^{\pi} = -\frac{\pi}{4}$

(3) $\displaystyle\int_0^{\pi} x\cos(5x)dx = \int_0^{\pi} x \times \left(\frac{1}{5}\sin(5x)\right)' dx = \left[\frac{x}{5}\sin(5x)\right]_0^{\pi} - \int_0^{\pi} \frac{1}{5}\sin(5x)dx$

$\displaystyle = 0 - \left[-\frac{1}{25}\cos(5x)\right]_0^{\pi} = \frac{1}{25}\cos(5\pi) - \frac{1}{25}\cos 0 = -\frac{2}{25}$

(4) $\displaystyle\int_0^{\pi} x\sin(5x)dx = \int_0^{\pi} x \times \left(-\frac{1}{5}\cos(5x)\right)' dx$

$\displaystyle = \left[-\frac{x}{5}\cos(5x)\right]_0^{\pi} - \int_0^{\pi}\left(-\frac{1}{5}\cos(5x)\right)dx$

$\displaystyle = -\frac{\pi}{5}\cos(5\pi) - 0 + \left[\frac{1}{25}\sin(5x)\right]_0^{\pi} = \frac{\pi}{5}$

＜ 20ページ. 積分6 ＞

解答

$$(1)\ I_n = \int_0^\pi \sin(nx)dx = \left[-\frac{1}{n}\cos(nx)\right]_0^\pi$$

$$= -\frac{1}{n}\cos(n\pi) + \frac{1}{n}\cos 0$$

$$= \begin{cases} 0 & ： n は偶数 \\ \frac{2}{n} & ： n は奇数 \end{cases}$$

$$(2)\ I_n = \int_0^\pi x\sin(nx)dx = \left[-\frac{x\cos(nx)}{n}\right]_0^\pi - \int_0^\pi 1 \times \left(-\frac{\cos(nx)}{n}\right) dx$$

$$= -\frac{\pi}{n}\cos(n\pi) + \int_0^\pi \frac{\cos(nx)}{n}dx$$

$$= -\frac{\pi}{n}\cos(n\pi) + \left[\frac{\sin(nx)}{n^2}\right]_0^\pi$$

$$= -\frac{\pi}{n}\cos(n\pi)$$

$$= \begin{cases} \frac{\pi}{n} & ： n は奇数 \\ -\frac{\pi}{n} & ： n は偶数 \end{cases}$$

＜ 21 ページ. 三角多項式の係数１ ＞

解答

(1) $f(x) = 5.2 - 3.1\cos x + 2.7\sin(5x)$

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} 5.2dx = 5.2 \times 2\pi = 10.4\pi$$

(2) $f(x) = -3.7 - 4.9\cos(3x) - 6.8\sin(7x)$

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} (-3.7)dx = -3.7 \times 2\pi = -7.4\pi$$

(3) $f(x) = a_0 + a_n\cos(nx) + b_n\sin(nx)$

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} a_0dx = 2\pi a_0$$

$<$ 22 ページ. 三角多項式の係数 2 $>$

解答

(1) $$\int_{-\pi}^{\pi} f(x)\sin x dx = \int_{-\pi}^{\pi} \left(-4\sin^2 x\right) dx = -4\pi$$

(2) $$\int_{-\pi}^{\pi} f(x)\cos(2x) dx = \int_{-\pi}^{\pi} 5\cos^2(2x) dx = 5\pi$$

(3) $$\int_{-\pi}^{\pi} f(x)\sin(2x) dx = 0$$

(4) $$\int_{-\pi}^{\pi} f(x)\cos(3x) dx = 0$$

(5) $$\int_{-\pi}^{\pi} f(x)\sin(3x) dx = \int_{-\pi}^{\pi} \left(-8\sin^2(3x)\right) dx = -8\pi$$

(6) $$\int_{-\pi}^{\pi} f(x)\cos(4x) dx = 0$$

(7) $$\int_{-\pi}^{\pi} f(x)\sin(4x) dx = 0$$

(8) $$\int_{-\pi}^{\pi} f(x)\cos(5x) dx = 0$$

< 23 ページ. 三角多項式の係数 3 >

解答

(1) $\displaystyle\int_{-\pi}^{\pi} f(x)\cos x dx = a_1\pi$

(2) $\displaystyle\int_{-\pi}^{\pi} f(x)\sin x dx = b_1\pi$

(3) $\displaystyle\int_{-\pi}^{\pi} f(x)\sin(2x) dx = b_2\pi$

(4) $\displaystyle\int_{-\pi}^{\pi} f(x)\cos(3x) dx = a_3\pi$

(5) $\displaystyle\int_{-\pi}^{\pi} f(x)\sin(3x) dx = b_3\pi$

(6) $\displaystyle\int_{-\pi}^{\pi} f(x)\cos(4x) dx = 0$

(7) $\displaystyle\int_{-\pi}^{\pi} f(x)\sin(4x) dx = 0$

(8) $\displaystyle\int_{-\pi}^{\pi} f(x)\cos(5x) dx = 0$

＜ 24ページ. 三角多項式の係 4 ＞

問 1 の解答

(1) $a_1 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos x dx$

(2) $b_1 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin x dx$

(3) $b_2 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin(2x) dx$

(4) $a_3 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos(3x) dx$

(5) $b_3 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin(3x) dx$

問 2 の解答

(1) $a_0 = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} f(x) dx$

(2) $a_k = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos(kx) dx$

(3) $b_k = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin(kx) dx$

＜ 25 ページ. 三角数列 ＞

問 1 の解答

(1) $a_n = \sin(n\pi)$

$$a_1 = 0\ ,\ a_2 = 0\ ,\ a_3 = 0\ ,\ a_4 = 0\ ,\ a_5 = 0$$

(2) $a_n = \dfrac{1}{n}\cos(n\pi)$

$$a_1 = -1\ ,\ a_2 = \frac{1}{2}\ ,\ a_3 = -\frac{1}{3}\ ,\ a_4 = \frac{1}{4}\ ,\ a_5 = -\frac{1}{5}$$

(3) $a_n = \dfrac{1}{n^2}\{1 - \cos(n\pi)\}$

$$a_1 = 2\ ,\ a_2 = 0\ ,\ a_3 = \frac{2}{9}\ ,\ a_4 = 0\ ,\ a_5 = \frac{2}{25}$$

問 2 の解答

(1) $$5^n \sin(n\theta) = 5^n\left(\frac{e^{in\theta} - e^{-in\theta}}{2i}\right)$$

$$= \frac{1}{2i}\left(5e^{i\theta}\right)^n - \frac{1}{2i}\left(5e^{-i\theta}\right)^n$$

$$= -\frac{i}{2}\left(5e^{i\theta}\right)^n + \frac{i}{2}\left(5e^{-i\theta}\right)^n$$

(2) $$r^n \cos(n\theta) = r^n\left(\frac{e^{in\theta} + e^{-in\theta}}{2}\right) = \frac{1}{2}\left(re^{i\theta}\right)^n + \frac{1}{2}\left(re^{-i\theta}\right)^n$$

(3) $$r^n \sin(n\theta) = r^n\left(\frac{e^{in\theta} - e^{-in\theta}}{2i}\right) = -\frac{i}{2}\left(re^{i\theta}\right)^n + \frac{i}{2}\left(re^{-i\theta}\right)^n$$

< 26 ページ. 無限級数 1 >

解答

(1) $4 + 4 \times \dfrac{2}{5} + 4 \times \left(\dfrac{2}{5}\right)^2 + \cdots + 4 \times \left(\dfrac{2}{5}\right)^{n-1} + \cdots$

$$= \lim_{n\to\infty} \frac{4 - 4 \times \left(\frac{2}{5}\right)^n}{1 - \frac{2}{3}} = \frac{4}{1 - \frac{2}{5}} = \frac{20}{5 - 2} = \frac{20}{3}$$

(2) $a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$

$$= \lim_{n\to\infty} \frac{a - ar^n}{1 - r} = \frac{a}{1 - r}$$

＜ 27 ページ. 無限級数 2 ＞

解答

$$r\cos\theta + r^2\cos(2\theta) + \cdots + r^n\cos(n\theta) + \cdots$$

$$= \sum_{k=1}^{\infty} r^k \cos(k\theta)$$

$$= \sum_{k=1}^{\infty} \left\{ \frac{1}{2}\left(re^{i\theta}\right)^k + \frac{1}{2}\left(re^{-i\theta}\right)^k \right\}$$

$$= \frac{\frac{1}{2}re^{i\theta}}{1 - re^{i\theta}} + \frac{\frac{1}{2}re^{-i\theta}}{1 - re^{-i\theta}}$$

$$= \frac{1}{2}r \times \left\{ \frac{e^{i\theta} - r + e^{-i\theta} - r}{1 - r\left(e^{i\theta} + e^{-i\theta}\right) + r^2 e^{ie} \times e^{-i\theta}} \right\}$$

$$= \frac{r(\cos\theta - r)}{1 - 2r\cos\theta + r^2}$$

< 28 ページ. フーリエ級数 1 >

解答

$$f(x) \sim \sum_{k=1}^{\infty} b_k \sin(kx)$$

＜ 29 ページ. フーリエ級数 2 ＞

解答

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{2\pi}\int_{-\pi}^{0} 1dx = \frac{1}{2\pi}\big[x\big]_{-\pi}^{0} = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \frac{1}{\pi}\int_{-\pi}^{0} \cos(nx)dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \frac{1}{\pi}\int_{-\pi}^{0} \sin(nx)dx$$

$$= \begin{cases} 0 & ：\ n \text{が偶数} \\ -\frac{2}{n\pi} & ：\ n \text{が奇数} \end{cases}$$

$$f(x) \sim a_0 + \sum_{n=1}^{\infty}\{a_n\cos(nx) + b_n\sin(nx)\}$$

$$= \frac{1}{2} - \frac{2}{\pi}\sin x - \frac{2}{3\pi}\sin(3x) - \frac{2}{5\pi}\sin(5x) - \frac{2}{7\pi}\sin(7x)\cdots$$

＜ 30 ページ. フーリエ級数 3 ＞

解答

$f(x)$ は奇関数より

$$a_0 = a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\big(-x\sin(nx)\big)dx = -\frac{2}{\pi}\int_0^{\pi} x\sin(nx)dx$$

$$= \begin{cases} -\frac{2}{\pi}\times\frac{\pi}{n} = -\frac{2}{n} & : \ n \text{が奇数} \\ -\frac{2}{\pi}\times(-\frac{\pi}{n}) = \frac{2}{n} & : \ n \text{が偶数} \end{cases}$$

よって

$$f(x) \sim -\frac{2}{1}\sin x + \frac{2}{2}\sin(2x) - \frac{2}{3}\sin(3x)$$

$$+\frac{2}{4}\sin(4x) - \frac{2}{5}\sin(5x) + \frac{2}{6}\sin(6x) + \cdots$$

< 31 ページ. フーリエ級数 4 >

解答

$f(x) = |x|$ は偶関数より $b_n = 0$

$$a_0 = \frac{2}{2\pi}\int_0^{\pi} |x|dx = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} |x|\cos(nx)dx = \begin{cases} \frac{2}{\pi}\times\left(-\frac{2}{n^2}\right) = -\frac{4}{n^2\pi} & : \ n\text{が奇数} \\ 0 & : \ n\text{が偶数} \end{cases}$$

よって

$$f(x) \sim \frac{\pi}{2} - \frac{4}{\pi}\left\{\cos x + \frac{1}{9}\cos(3x) + \frac{1}{25}\cos(5x) + \frac{1}{49}\cos(7x) + \cdots\right\}$$

＜ 32 ページ. 三角関数 ＞

解答

$$f(0) = 1 \quad , \quad S_\infty(0) = \frac{1}{2}$$

$$f(\pi) = 0 \quad , \quad S_\infty(\pi) = \frac{1}{2}$$

< **33** ページ. デルタ関数 **1** >

解答

$$\lim_{h\to 0}(f*\delta_h)(x) = \frac{1}{2}f(x+0) + \frac{1}{2}f(x-0)$$

＜ 34 ページ. デルタ関数 2 ＞

解答

(1) $\displaystyle \lim_{h\to+0}(f*\delta_h)(2) = \frac{f(2+0)+f(2-0)}{2} = \frac{2+1}{2} = \frac{3}{2}$

(2) $\displaystyle \lim_{h\to+0}(f*\delta_h)(2.5) = f(2.5) = 2$

＜ 35 ページ. ポアソン核 ＞

解答

(1) $\lim_{r\to 1-0}(f*P_r)(1) = \dfrac{f(1+0)+f(1-0)}{2} = \dfrac{1}{2}$

(2) $\lim_{r\to 1-0}(f*P_r)(1.5) = f(1.5) = 1$

< 36ページ.ポアソン積分 >

解答

$$
f(r,x) = \int_{-\pi}^{\pi} f(t) \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} r^n \cos\bigl(n(x-t)\bigr) \right\} dt
$$

$$
= \int_{-\pi}^{\pi} f(t) P_r(x-t) dt
$$

$$
= (f * P_r)(x)
$$

< 37 ページ. フーリエ級数の収束 >

解答

(1) $S_\infty\left(\frac{3}{2}\pi\right) = \frac{\pi}{2}$

(2) $S_\infty(0) = 0$

(3) $S_\infty\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

＜ 38 ページ. 一般の周期関数 1 ＞

解答

(1) $\sin\left(\frac{2\pi}{5}x\right)$ ：周期 5

(2) $\cos\left(\frac{2\pi}{7}x\right)$ ：周期 7

(3) $\sin\left(\frac{2\pi}{9}x\right)+\cos\left(\frac{2\pi}{9}x\right)$ ：周期 9

(4) $\sin\left(\frac{\pi}{3}x\right)$ ：周期 6

(5) $\cos\left(\frac{\pi}{2}x\right)$ ：周期 4

(6) $\sin(\pi x)$ ：周期 2

(7) $\cos(3\pi x)$ ：周期 $\frac{2}{3}$

(8) $\sin(n\pi x)$ ：周期 $\frac{2}{n}$

(9) $\cos\left(\frac{2\pi}{L}x\right)$ ：周期 L

(10) $\sin\left(\frac{\pi}{l}x\right)$ ：周期 $2l$

(11) $\cos\left(\frac{2n\pi}{L}x\right)$ ：周期 $\frac{L}{n}$

(12) $\sin\left(\frac{n\pi}{l}x\right)$ ：周期 $\frac{2l}{n}$

＜ 39 ページ. 一般の周期関数 2 ＞

解答

$$(1)\ y = a_0 + \sum_{k=1}^{n} \left\{ a_k \cos\left(\frac{2k\pi}{5}x\right) + b_k \sin\left(\frac{2k\pi}{5}x\right) \right\}$$

：周期 5 の周期関数

$$(2)\ y = a_0 + \sum_{k=1}^{n} \left\{ a_k \cos\left(\frac{2k\pi}{L}x\right) + b_k \sin\left(\frac{2k\pi}{L}x\right) \right\}$$

：周期 L の周期関数

$$(3)\ y = a_0 + \sum_{k=1}^{n} \left\{ a_k \cos\left(\frac{k\pi}{l}x\right) + b_k \sin\left(\frac{k\pi}{l}x\right) \right\}$$

：周期 $2l$ の周期関数

< 40ページ. 一般のフーリエ級数１ >

解答

(1) 周期 $2l$

$$a_0 = \frac{1}{2l}\int_{-l}^{l} f(x)dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$$

(2) 周期 $2\pi l$

$$a_0 = \frac{1}{2\pi l}\int_{-\pi l}^{\pi l} f(x)dx$$

$$a_n = \frac{1}{\pi l}\int_{-\pi l}^{\pi l} f(x)\cos\left(\frac{n}{l}x\right)dx$$

$$b_n = \frac{1}{\pi l}\int_{-\pi l}^{\pi l} f(x)\sin\left(\frac{n}{l}x\right)dx$$

＜ 41 ページ. 一般のフーリエ級数 2 ＞

問 1 の解答

$$a_0 = 0 \quad , \quad a_n = 0 \quad , \quad b_n = \frac{4}{L}\int_0^{L/2} f(x)\sin\left(\frac{2n\pi}{L}x\right)dx$$

問 2 の解答

$$a_0 + \sum_{n=1}^{\infty}\left\{a_n\cos\left(\frac{n\pi}{l}x\right) + b_n\sin\left(\frac{n\pi}{l}x\right)\right\} = \frac{f(x+0)+f(x-0)}{2}$$

$$a_0 = \frac{1}{2l}\int_{-l}^{l} f(x)dx \quad , \quad a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$$

問 3 の解答

(1) $f(x)$ が偶関数の時

$$a_0 = \frac{1}{l}\int_0^{l} f(x)dx \quad , \quad a_n = \frac{2}{l}\int_0^{l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx \quad , \quad b_n = 0$$

(2) $f(x)$ が奇関数の時

$$a_0 = 0 \quad , \quad a_n = 0 \quad , \quad b_n = \frac{2}{l}\int_0^{l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$$

< 42ページ. フーリエ級数の複素数表示1 >

解答

$$a_n \cos(n\omega x) + b_n \sin(n\omega x) = C_n e^{in\omega x} + C_{-n} e^{-in\omega x}$$

＜ 43 ページ. フーリエ級数の複素数表示 2 ＞

解答

(1) 周期 2π

$$f(x) \sim \sum_{k=-\infty}^{\infty} C_k e^{ikx} \qquad : \quad (\omega = 1)$$

$$C_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-ikx} dx$$

(2) 周期 $2\pi l$

$$f(x) \sim \sum_{k=-\infty}^{\infty} C_k e^{\frac{ikx}{l}} \qquad : \quad (\omega = \frac{1}{l})$$

$$C_k = \frac{1}{2\pi l}\int_{-\pi l}^{\pi l} f(x) e^{\frac{-ikx}{l}} dx$$

＜ 44 ページ. 広義積分 1 ＞

解答

(1) $\displaystyle \int_0^\infty e^{-tx}dx = \lim_{b\to\infty}\int_0^b e^{-tx}dx$

$$= \lim_{b\to\infty}\left[-\frac{1}{t}e^{-tx}\right]_{x=0}^{x=b} = \lim_{b\to\infty}\left(-\frac{1}{t}e^{-tb}+\frac{1}{t}\right) = \frac{1}{t}$$

(2) $\displaystyle \int_1^\infty \frac{1}{x^2}dx = \lim_{b\to\infty}\int_1^b x^{-2}dx$

$$= \lim_{b\to\infty}\left[-x^{-1}\right]_1^b = \lim_{b\to\infty}\left(-\frac{1}{b}+1\right) = 1$$

(3) $\displaystyle \int_1^\infty \frac{1}{x^r}dx = \lim_{b\to\infty}\int_1^b x^{-r}dx$

$$= \lim_{b\to\infty}\left[\frac{1}{-r+1}x^{-r+1}\right]_1^b = \lim_{b\to\infty}\left(-\frac{1}{(r-1)b^{r-1}}+\frac{1}{r-1}\right) = \frac{1}{r-1}$$

＜ 45 ページ. 広義積分 2 ＞

解答

$$(1)\ \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\lambda}} dx = 2\int_0^{\infty} e^{-\frac{x^2}{2\lambda}} dx = 2\int_0^{\infty} e^{-t^2}\sqrt{2\lambda}dt$$

$$= 2\sqrt{2\lambda}\int_0^{\infty} e^{-t^2}dt = 2\sqrt{2\lambda}\frac{\sqrt{\pi}}{2} = \sqrt{2\pi\lambda}$$

$$(2)\ \int_0^{\infty} \frac{\sin(\lambda x)}{x}dx = \int_0^{\infty} \frac{\sin(t)}{\frac{t}{\lambda}} \cdot \frac{1}{\lambda}dt = \int_0^{\infty} \frac{\sin t}{t}dt = \frac{\pi}{2}$$

$$(3)\ \int_0^{\infty} \frac{\sin(-\lambda x)}{x}dx = -\int_0^{\infty} \frac{\sin(\lambda x)}{x}dx = -\frac{\pi}{2}$$

＜ 48 ページ. フーリエ変換 3 ＞

解答

f は偶関数より

$$F(x) = \frac{1}{\pi}\int_0^{\infty} f(t)\cos(xt)dt = \frac{1}{\pi}\int_0^{8} 6\cos(xt)dt$$

$$= \frac{6}{\pi}\left[\frac{1}{x}\sin(xt)\right]_{t=0}^{t=8} = \frac{6}{\pi}\left(\frac{1}{x}\sin(8x)\right) = \frac{6\sin(8x)}{\pi x}$$

＜ 50 ページ. フーリエ変換 5 ＞

解答

① $t = -8$ のとき

$$I(t) = \frac{6}{\pi}\left\{0 + \frac{\pi}{2}\right\} = 3$$

② $t < -8$ のとき

$$I(t) = \frac{6}{\pi}\left\{-\frac{\pi}{2} + \frac{\pi}{2}\right\} = 0$$